Poisson Shifted Gompertz Distribution: Properties and Applications

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Abstract: ***A novel distribution using Poisson-Generating family of distribution with parent distribution as shifted Gompertz distribution called Poisson shifted Gompertz distribution with relevant properties has been introduced. The estimation of unknown parameters is carried out via established methods including Maximum likelihood estimation (MLE). R software is applied for computational purposes. The application of the proposed model has been illustrated considering a real set of data and investigated the goodness-of-fit attained by the Poisson shifted Gompertz model through different graphical methods and test statistics where better fit was observed for the set of real data.***

Keywords: ***Estimation method, LSE, MLE, Poisson-Generating family, Shifted Gompertz distribution***

I. INTRODUCTION

In the statistical literature it has been noticed that the many life-time distributions have been generated but the real data sets related to engineering, life sciences, biology, hydrology do not present a better fit in these models So, the generation of new modified models appears to be necessary to deal with the problems in these fields. For achieving a better fit for the data we encounter in survival analysis different distributions are created making changes to the baseline distribution.\ The extended family Poisson-Weibull distribution, introduced by (Bereta,at al., 2011) demonstrates failure rate functions with decreasing and increasing nature, also exponential-Poisson distribution, presented by (Kus, 2007) with zero truncated Poisson distribution and exponential distribution compounded together. Exponential Poisson distribution's CDF is,

$$G(t;\beta,\lambda)=\frac{1}{\left(1-e^{-\lambda}\right)}\left[1-\exp\left\{-\lambda\left(1-e^{-\beta t}\right)\right\}\right]$$

$$;t>0,(\beta,\lambda)>0$$

Cribari-Neto and Barreto-Souza (2009) have generated generalized exponential Poisson distribution as generalization of exponential-Poisson distribution (Kus, 2007) with insertion of power parameter to this model. Using the similar approach, Cancho (2011) presented Poisson exponential (PE) distribution based on exponential distribution. PE distribution's CDF is

$$G(t;\alpha,\theta)=\frac{e^{-\alpha}-\exp\left\{-\theta\left(1-e^{-\alpha t}\right)\right\}}{\left(1-e^{-\alpha}\right)}$$

$$;t>0,(\alpha,\theta)>0$$

Similarly Louzada-Neto et al., (2011) introduced Poisson-exponential having two parameters via Bayesian approach. Alkarni and Oraby (2012) have presented Poisson family class obtained via a lifetime distribution and truncated Poisson distribution compounded together

The Poisson family's CDF is as follows,

$$W(y;\beta,\underline{\omega})=\frac{1-\exp\left\{-\beta\left[1-G(y;\underline{\omega})\right]\right\}}{\left(1-e^{-\beta}\right)}\quad;\ \beta>0 \tag{1.1}$$

And its corresponding PDF can be expressed as

$$w(y;\beta,\underline{\omega})=\frac{\beta g(y;\underline{\omega})\exp\left\{-\beta\left[1-G(y;\underline{\omega})\right]\right\}}{\left(1-e^{-\beta}\right)}\quad;\ \beta>0 \tag{1.2}$$

Where $\underline{\omega}$ the parameter is space and $g(t;\underline{\omega})$ and $G(t;\underline{\omega})$ are the PDF and CDF.

Employing same approach the Poisson Weibull power series class of distributions was given by (Morais & Barreto-Souza, 2011). Exponentiated Weibull–Poisson model with four parameters with increasing, decreasing, bathtub-shaped, and uni-modal failure rate has been presented by (Mahmoudi & Sepahdar, 2013) generated compounding exponentiated Weibull and Poisson distributions. Weibull–Poisson distribution is introduced by (Lu & Shi, 2012). Further Kaviayarasu and Fawaz (2017) made an extensive study on Weibull–Poisson distribution through a reliability sampling plan. Kyurkchiev et al. (2018) used the exponentiated exponential-Poisson as the software reliability model. Joshi & Kumar (2020) presented Poisson exponential power distribution and used different estimation methods to estimate the model parameter. Chaudhary & Kumar (2020) have introduced a new distribution using Poisson-G family called Poisson inverse NHE distribution. Chaudhary & Kumar(2020) introduced a new distribution generated by using the Poisson-G-family with parent distribution as NHE distribution named Poisson NHE distribution. Chaudhary & Kumar(2020) used Markov chain Monte Carlo (MCMC) method is used to estimate the parameters of the Gompertz extension distribution based on a complete sampleJoshi & Kumar (2020) have introduced a new model using Gompertz distribution called

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Lindley Gompertz distribution which is more flexible than Gompertz distribution. In this paper we have taken base distribution of shifted Gompertz (Bemmaor, 1992) the PDF and CDF of shifted Gompertz model can be expressed as

$$g(x) = \beta \exp(-\beta x + \alpha e^{-\beta x})\left\{1+\alpha(1-e^{-\beta x})\right\}$$

$$;x \geq 0, (\alpha\beta) > 0 \tag{1.3}$$

$$\mathrm{G}(x) = (1-e^{-\beta x})\exp\left\{-\alpha\, \mathrm{e}^{-\beta x}\right\}; x \geq 0, (\alpha,\beta) > 0 \tag{1.4}$$

This paper aims to present a model which can provide better fit to the data we encounter in lifetime distribution. In Section 2 Poisson inverted Lomax distribution with its statistical and mathematical properties has been presented. In section 3 we discuss the estimation models. In Section 4 using a real dataset, parameter's estimated values and their corresponding asymptotic confidence intervals and Fisher information matrix is given and the different test criteria to assess the potentiality of the proposed model is discussed. In section 5 we give conclusion

II. THE POISSON SHIFTED GOMPERTZ DISTRIBUTION

A new distribution Poisson shifted Gompertz (PSG) distribution is presented using the Poisson-G family defined by (Alkarni and Oraby, 2012). The PDF and CDF of PSG distribution is obtained by taking (1.3) and (1.4) as baseline distribution and is given by

$$f(x) = \frac{\beta\lambda}{(1-e^{-\lambda})}\exp(-\beta x - \alpha e^{-\beta x})\{1+\alpha(1-e^{-\beta x})\}\exp\left[-\lambda\{1-(1-e^{-\beta x})\exp(-\alpha\, e^{-\beta x}\}\right] \tag{2.1}$$

$$F(x) = \frac{1}{(1-e^{-\lambda})}\{1-\exp[-\lambda\{1-(1-e^{-\beta x})\exp(-\alpha e^{-\beta x})\}]\}$$

$$;x \geq 0, (\alpha,\beta,\lambda) > 0 \tag{2.2}$$

PSG distribution's reliability function is

$$S(x) = 1 - F(x)$$

$$= \frac{\exp[-\lambda\{1-(1-e^{-\beta x})\exp(-\alpha e^{-\beta x})\}] - \mathrm{e}^{-\lambda}}{(1-e^{-\lambda})} \tag{2.3}$$

$$;x \geq 0, (\alpha,\beta,\lambda) > 0$$

Hazard rate function(HRF) of PSG distribution

$$h(x) = \frac{f(x)}{S(x)}$$

$$= \frac{\beta\lambda \exp(-\beta x - \alpha e^{-\beta x})\{1+\alpha(1-e^{-\beta x})\}\exp[-\lambda\{1-(1-e^{-\beta x})\exp(-\alpha\, e^{-\beta x}\}]}{\exp[-\lambda\{1-(1-e^{-\beta x})\exp(-\alpha e^{-\beta x})\}] - \mathrm{e}^{-\lambda}} \tag{2.4}$$

Figure 1 illustrates the PSG distribution's curves of the PDF and HRF where PDF's curve displayed variety of shapes. The PSG model's hazard rate function (HRF) is observed to be flexible as it exhibited different shapes like reverse bathtub, increasing–decreasing, increasing for different values of parameters.

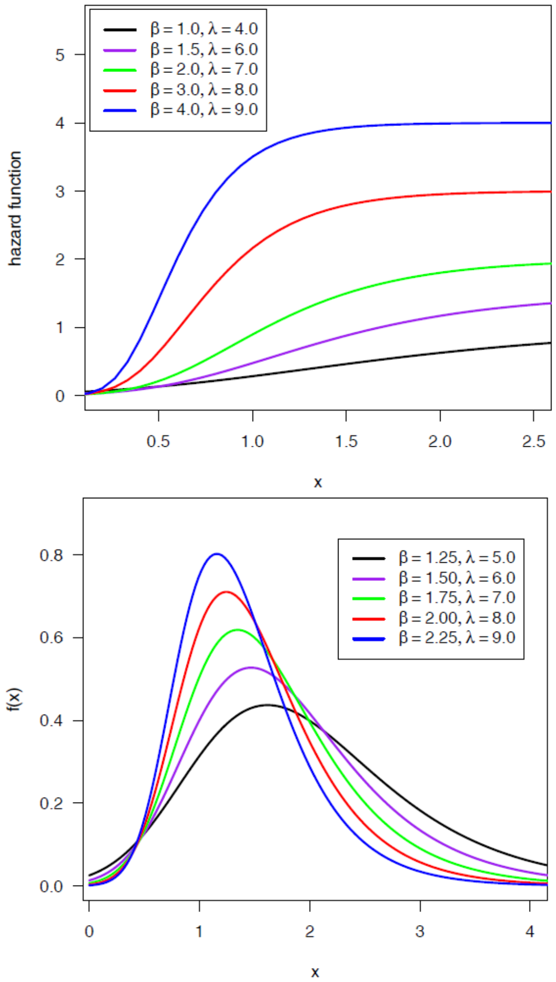


Fig. 1. For fixed α and various values of λ and β, graph of hazard function (upper panel) and PDF (lower panel)

PSG distribution's quantile function

$$Q(p) = F^{-1}(p)$$

$$\alpha e^{-\beta x} - \ln(1-e^{-\beta x}) + \ln b = 0$$

$$where\ b \ = 1 + \frac{1}{\lambda}\ln\{1-(1-e^{-\lambda})(1-p)\}; 0 < p < 1 \tag{2.5}$$

Simulating values of X with the CDF (2.1) gives random numbers of the PSG distribution. Let D denote a uniform random variable in (0,1), then the simulated values of X are calculated as,

$$\alpha e^{-\beta x} - \ln(1-e^{-\beta x}) + \ln b = 0$$

$$where\ b \ = 1 + \frac{1}{\lambda}\ln\{1-(1-e^{-\lambda})(1-d)\}; 0 < d < 1 \tag{2.6}$$

Skewness and Kurtosis:

The coefficient of skewness of PSG distribution can be obtained as

$$S_k(B) = \frac{Q(3/4)+Q(1/4)-2Q(1/2)}{Q(3/4)-Q(1/4)}$$ and

The coefficient of kurtosis given by (Moors, 1988) is

$$K_u(M) = \frac{Q(0.875)-Q(0.625)+Q(0.375)-Q(0.125)}{Q(0.75)-Q(0.25)}$$

III. METHODS OF PARAMETER ESTIMATION

Here parameter estimation of the unknown parameter is done with brief explanation of the estimation method

3.1. Maximum Likelihood Estimation (MLE) method

Consider a random sample denoted by $\underline{x} = (x_1,\ldots,x_n)$ 'n' sample size from $PSG(\alpha,\ \beta,\ \lambda)$ then the log likelihood function is,

$$l = n\ln\alpha + n\ln\beta - \beta\sum_{i=1}^{n} x_i - \alpha\sum_{i=1}^{n} e^{-\beta x_i} + \sum_{i=1}^{n}\ln\{1+\alpha(1-e^{-\beta x_i})\}$$
$$-\sum_{i=1}^{n}\lambda\{1-(1-e^{-\beta x_i})\exp(-\alpha e^{-\beta x_i})\} \tag{3.1.1}$$

By differentiating (3.1.1) w.r.t. β, λ and α ,we obtain

$$\frac{\partial l}{\partial\alpha} = \frac{n}{\alpha} + \sum_{i=1}^{n}\log(1+\beta/x_i) - \lambda\sum_{i=1}^{n}(1+\beta/x_i)^{-\alpha}\ln(1+\beta/x_i)$$

$$\frac{\partial l}{\partial\beta} = \frac{n}{\beta} + \sum_{i=1}^{n}\frac{1}{x(1+\beta x_i)} + \lambda\alpha\sum_{i=1}^{n}\frac{1}{x(1+\beta x_i)^{\alpha+1}}$$

$$\frac{\partial l}{\partial\lambda} = \frac{n}{\lambda} + \frac{ne^{-\lambda}}{1-e^{-\lambda}} - \sum_{i=1}^{n}(1+\beta/x_i)$$

Solving these non-linear functions for (α, β, λ) by equating to zero we will obtain the ML estimators of the PSG distribution. The computer softwares like R, Mathmatica, Matlab etc can be used to solve them manually. Let $\underline{\varphi} = (\alpha,\beta,\lambda)$ parameter vector and consider MLE of $\underline{\varphi}$ as $\hat{\underline{\varphi}} = (\hat{\alpha},\hat{\beta},\hat{\lambda})$, then normal distribution is followed by $(\hat{\underline{\varphi}}-\underline{\varphi}) \rightarrow N_3\left[0,\left(A(\underline{\varphi})\right)^{-1}\right]$, where $A(\underline{\varphi})$ is the information matrix of Fisher obtained by,

$$A = \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix}$$

where

$$A_{11} = \frac{\partial^2 l}{\partial\alpha^2},\ A_{12} = \frac{\partial^2 l}{\partial\alpha\partial\beta},\ A_{13} = \frac{\partial^2 l}{\partial\alpha\partial\lambda}$$

$$A_{21} = \frac{\partial^2 l}{\partial\beta\partial\alpha},\ A_{22} = \frac{\partial^2 l}{\partial\beta^2},\ A_{23} = \frac{\partial^2 l}{\partial\beta\partial\lambda}$$

$$A_{31} = \frac{\partial^2 l}{\partial\lambda\partial\alpha},\ A_{32} = \frac{\partial^2 l}{\partial\beta\partial\lambda},\ A_{33} = \frac{\partial^2 l}{\partial\lambda^2}$$

MLE having an asymptotic variance $\left(A(\underline{\varphi})\right)^{-1}$ is practically useless with $\underline{\varphi}$ unknown so putting estimated parameters value we approximate asymptotic variance. Via the algorithm of Newton-Raphson, maximization of likelihood gives the observed information matrix and the var-cov matrix is,

$$\left(A(\underline{\varphi})\right)^{-1} = \begin{pmatrix} V(\hat{\alpha}) & \mathrm{cov}(\hat{\alpha},\hat{\beta}) & \mathrm{cov}(\hat{\alpha},\hat{\lambda}) \\ \mathrm{cov}(\hat{\alpha},\hat{\beta}) & V(\hat{\beta}) & \mathrm{cov}(\hat{\beta},\hat{\lambda}) \\ \mathrm{cov}(\hat{\lambda},\hat{\alpha}) & \mathrm{cov}(\hat{\lambda},\hat{\beta}) & V(\hat{\lambda}) \end{pmatrix} \tag{3.2.1}$$

Thus for α, β and λ, using MLEs' asymptotic normality, approximate 100(1-α) % confidence intervals is given as $\hat{\alpha} \pm Z_{\alpha/2}SE(\hat{\alpha})$, $\hat{\beta} \pm Z_{\alpha/2}SE(\hat{\beta})$ and, $\hat{\lambda} \pm Z_{\alpha/2}SE(\hat{\lambda})$

3.2. LSE method

The another estimation method we have used is least-square estimation to estimate PSG distribution's α, β and λ which is calculated with minimization of

$$B(X;\alpha,\beta,\lambda) = \sum_{i=1}^{n}\left[F(X_i) - \frac{i}{n+1}\right]^2 \tag{3.2.1}$$

w.r.t λ, α and β.

From a distribution function F(.), suppose $F(X_i)$ represents ordered random variables ($X_{(1)} < X_{(2)} < \ldots < X_{(n)}$)'s CDF and random sample is denoted as $\{X_1, X_2,\ \ldots, X_n\}$ with "n" size. The LSEs of the unknown parameters ($\hat{\alpha},\hat{\beta}, and\ \hat{\lambda}$) is acquired with minimization of

$$B(X;\alpha,\beta,\lambda) = \sum_{i=1}^{n}\left[\frac{1}{(1-e^{-\lambda})}\{1-\exp[-\lambda\{1-(1-e^{-\beta x_i})\exp(-\alpha e^{-\beta x_i})\}]\} - \frac{i}{n+1}\right]^2 \tag{3.2.2}$$

w.r.t β, λ and α.

Differentiation of (3.2.2) w.r.t the unknown parameters (β, λ and α) we get

$$\frac{\partial B}{\partial\alpha} = -\frac{2\lambda}{(1-e^{-\lambda})}\sum_{i=1}^{n}\left[\frac{1}{(1-e^{-\lambda})}\{1-Z(x_i)\} - \frac{i}{n+1}\right]$$
$$Z(x_i)\exp(-\alpha e^{-\beta x_i} - \beta x_i)\{1-(1-e^{-\beta x_i})$$

$$\frac{\partial B}{\partial\beta}$$
$$= 2\alpha\lambda\sum_{i=1}^{n}\left[\frac{1}{(1-e^{-\lambda})}\{1-Z(x_i)\} - \frac{i}{n+1}\right]$$
$$Z(x_i)x_i\exp(-\alpha e^{-\beta x_i} - \beta x_i)\left[-\alpha(1-e^{-\beta x_i})+1\right]$$

$$\frac{\partial B}{\partial \lambda}=2\sum_{i=1}^{n}\left[\frac{1}{(1-e^{-\lambda})}\{1-Z(x_i)\}-\frac{i}{n+1}\right]\left[\begin{array}{l}\frac{1}{(1-e^{-\lambda})}Z(x_i)\{1-(1-e^{-\beta x_i})\exp(-\alpha e^{-\beta x_i})\}\\+\frac{e^{-\lambda}}{(1-e^{-\lambda})^2}\{1-Z(x_i)\}\end{array}\right]$$

Where

$$Z(x_i)=\exp[-\lambda\{1-(1-e^{-\beta x_i})\exp(-\alpha e^{-\beta x_i})\}]$$

Likewise weighted LSEs is given with minimization of

$$B(X;\alpha,\beta,\lambda)=\sum_{i=1}^{n}w_i\left[F(X_{(i)})-\frac{i}{n+1}\right]$$

w.r.t β, λ and α.

Here weights is given as $w_i=\frac{1}{Var(X_{(i)})}=\frac{(n+1)^2(n+2)}{i(n-i+1)}$

Minimization of the following equation

$$B(X;\alpha,\beta,\lambda)$$
$$=\sum_{i=1}^{n}\frac{(n+1)^2(n+2)}{i(n-i+1)}\left[\frac{1}{(1-e^{-\lambda})}\{1-\exp[-\lambda\{1-(1-e^{-\beta x_i})\exp(-\alpha e^{-\beta x_i})\}]\}-\frac{i}{n+1}\right]^2$$

(3.2.3)

w.r.t. α, β and λ gives the unknown parameter's weighted least square estimators

3.3. CVME estimation method

The Cramer-Von-Mises estimators of unknown parameter can be attained with minimization of

$$C(X;\alpha,\beta,\lambda)=\frac{1}{12n}+\sum_{i=1}^{n}\left[F(x_{i:n}|\alpha,\beta,\lambda)-\frac{2i-1}{2n}\right]^2$$

$$=\frac{1}{12n}+\sum_{i=1}^{n}\left[\frac{1}{(1-e^{-\lambda})}\{1-\exp[-\lambda\{1-(1-e^{-\beta x_i})\exp(-\alpha e^{-\beta x_i})\}]\}-\frac{2i-1}{2n}\right]^2$$

(3.3.1)

Differentiation of (3.3.1) w.r.t λ, β and α , following can be obtained

$$\frac{\partial C}{\partial \alpha}=\frac{2\lambda}{(1-e^{-\lambda})}\sum_{i=1}^{n}\left[\frac{1}{(1-e^{-\lambda})}\{1-Z(x_i)\}-\frac{2i-1}{2n}\right]$$
$$Z(x_i)\exp(-\alpha e^{-\beta x_i}-\beta x_i)\{1-(1-e^{-\beta x_i})$$

$$\frac{\partial C}{\partial \beta}=2\alpha\lambda\sum_{i=1}^{n}\left[\frac{1}{(1-e^{-\lambda})}\{1-Z(x_i)\}-\frac{2i-1}{2n}\right]$$
$$Z(x_i)x_i\exp(-\alpha e^{-\beta x_i}-\beta x_i)\left[-\alpha(1-e^{-\beta x_i})+1\right]$$

$$\frac{\partial C}{\partial \lambda}=2\sum_{i=1}^{n}\left[\frac{1}{(1-e^{-\lambda})}\{1-Z(x_i)\}-\frac{2i-1}{2n}\right]$$
$$\left[\frac{1}{(1-e^{-\lambda})}Z(x_i)\{1-(1-e^{-\beta x_i})\exp(-\alpha e^{-\beta x_i})\}+\frac{e^{-\lambda}}{(1-e^{-\lambda})^2}\{1-Z(x_i)\}\right]$$

Where $Z(x_i)=\exp[-\lambda\{1-(1-e^{-\beta x_i})\exp(-\alpha e^{-\beta x_i})\}]$

After solving non-linear equations $\frac{\partial C}{\partial \alpha}=0,\frac{\partial C}{\partial \beta}=0\ and\ \frac{\partial C}{\partial \lambda}=0$ CVM estimators can be obtained

IV. Application To A Real Dataset

Here, we illustrated the PSG distribution's applicability and suitability using a real set of data (Bader & Priest, 1982). Following data is composed of single carbon fibers' tensile strength (GPA) where the fibers' gauge lengths is 10mm and size of sample is 63.

2.614, 2.616, 2.624, 2.659, 3.030, 3.139, 3.145, 3.220, 3.223, 3.235, 2.996, 3.243, 3.264, 3.272, 3.294, 3.332, 2.675, 2.937, 2.937, 1.901, 2.132, 2.203, 2.228, 4.225, 4.395, 5.0202.738, 2.740, 2.856, 2.575, 2.917, 2.928, 4.024, 2.350, 2.257, 2.361, 2.977, 2.397, 2.396, 2.454, 2.445, 2.518, 2.474, 2.618, 2.522, 3.346, 2.525, 3.408, 3.377, 2.532, 3.493, 3.435, 3.501, 3.554, 3.537, 3.125, 3.628, 3.562, 3.871, 3.852, 3.886, 3.971, 4.027,

By utilizing R software (R Core Team, 2020) of the optim() function, we have calculated the MLEs of PSG distribution by maximizing the likelihood function (3.1.1) (Mailund, 2017) where Log-Likelihood's values was obtained as l = -56.5282. For α, β, and λ MLE's with their standard errors (SE) has been illustrated in Table 1.

Table 1

MLE with SE for α, β, and λ of PSG

Parameter	MLE	SE
alpha	13.5877	7.2056
beta	2.0139	0.1749
theta	18.8875	4.4254

Log-likelihood function's plot for α, β and λ has been illustrated in Figure 2 and found that the ML estimates can be calculated uniquely.

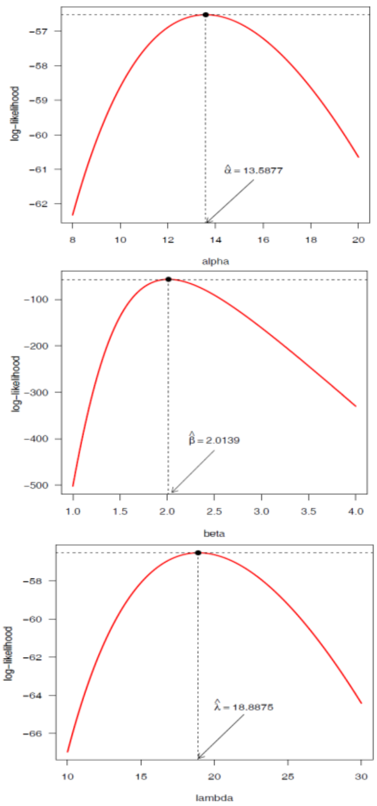


Figure 2. For the parameters α, β and λ, plots of log-likelihood function

We have presented the graph of K-S plot and Q-Q plot in Figure 3 and where PSG distribution is observed to fit the data very well.

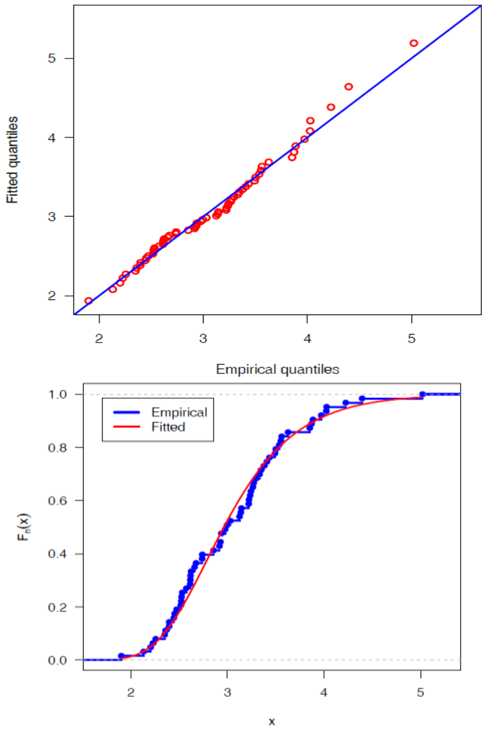


Figure 3. The plots of Q-Q (upper panel) and K-S (lower panel) of the PSG distribution.

With the help of MLE, LSE and CVE method estimated parameters values and their negative log-likelihood, and AIC criterion in Table 2.

Table 2

Estimated parameters, log-likelihood, and AIC

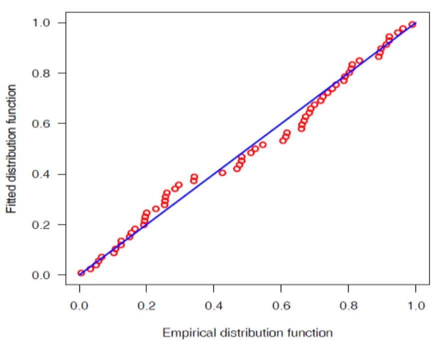
Method of Estimation	$\hat{\alpha}$	$\hat{\beta}$	$\hat{\lambda}$	-LL	AIC
MLE	13.5877	2.0139	18.8875	56.5282	119.0565
LSE	15.9309	1.8403	10.2438	57.3261	120.6522
CVE	16.7643	1.8753	10.7990	57.0725	120.1449

The KS, W and A^2 statistic with their corresponding p-value of MLE, LSE and CVE estimates we have presented in Table 3.

Table 3

The KS, W and A^2 statistic with a p-value

Method of Estimation	A^2(p-value)	W(p-value)	KS(p-value)
MLE	0.3632(0.8839)	0.0704(0.7509)	0.0883(0.7102)
LSE	0.3878(0.8603)	0.0522(0.8647)	0.0645(0.9560)
CVE	0.3521(0.8942)	0.0512(0.8710)	0.0674(0.9370)



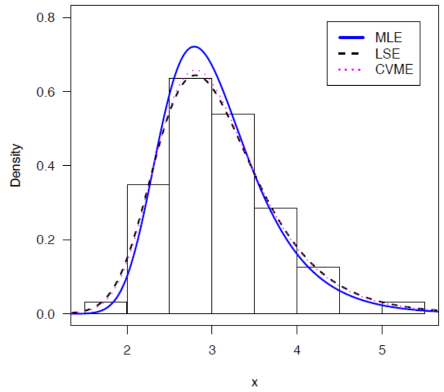


Figure 4. PSG distribution's P-P plot (upper panel) and The Histogram and the PDF via MLE, LSE and CVM (lower panel)

Here we have presented Poisson shifted Gompertz model's applicability using a real dataset used by earlier researchers. To compare the potentiality of the proposed distribution, following distribution models are taken.

I. Exponentiated Exponential Poisson (EEP):

The probability density function of EEP (Ristić & Nadarajah, 2014) is

$$f(x)=\frac{\alpha\beta\lambda}{\left(1-e^{-\lambda}\right)}e^{-\beta x}\left(1-e^{-\beta x}\right)^{\alpha-1}\exp\left\{-\lambda\left(1-e^{-\beta x}\right)^{\alpha}\right\}$$

$; x>0, \alpha>0, \lambda>0$

II. Poisson–exponential distribution (PE)

The PE distribution's PDF was defined by (Louzada-Neto et al., 2011) also it was used by (Rodrigues et al., 2018) is

$$f(x)=\frac{\beta\lambda}{\left(1-e^{-\lambda}\right)}e^{-\beta x}\exp\left(-\lambda e^{-\beta x}\right);\quad \beta>0,\lambda>0,x>0$$

III. Exponential power (EP) distribution:

EP distribution's PDF (Smith & Bain, 1975) is

$$f_{EP}(x)=\alpha\,\lambda^{\alpha}\,x^{\alpha-1}\,e^{(\lambda x)^{\alpha}}\,\exp\left\{1-e^{(\lambda x)^{\alpha}}\right\}$$

$;(\alpha,\lambda)>0,\quad x\geq 0$

.

IV. LINDLEY-EXPONENTIAL (LE) DISTRIBUTION:

LE (Bhati, 2015)'s PDF is

$$f_{LE}(x)=\lambda\left(\frac{\theta^2}{1+\theta}\right)e^{-\lambda x}\left(1-e^{-\lambda x}\right)^{\theta-1}\left\{1-\ln\left(1-e^{-\lambda x}\right)\right\}$$

$;\lambda,\theta>0, x>0$

V. Weibull Extension Model:

The PDF of WE model (Tang et al., 2003) with three parameters (α,β,λ) is

$$f_{WE}(x;\alpha,\beta,\lambda)=\lambda\beta\left(\frac{x}{\alpha}\right)^{\beta-1}\exp\left(\frac{x}{\alpha}\right)^{\beta}\exp\left\{-\lambda\alpha\left(\exp\left(\frac{x}{\alpha}\right)^{\beta}-1\right)\right\}$$

$;\ x>0$

$\alpha>0,\beta>0$ and $\lambda>0$

We have illustrated the Bayesian information criterion (BIC), Akaike information criterion (AIC), Hannan-Quinn information criterion (HQIC) and Corrected Akaike information criterion (CAIC) for the evaluation of the applicability of the PSG distribution in Table 4.

Table 4

AIC, Log-likelihood (LL), CAIC BIC, and HQIC

Model	AIC	-LL	CAIC	BIC	HQIC
PSG	119.0565	56.5282	119.4633	125.4859	121.5852
EEP	120.1261	57.0630	120.5328	126.5555	122.6548
PE	118.4105	57.2052	118.6105	122.6967	120.0963
LE	119.9929	57.9964	120.1929	124.2792	121.6787
WE	129.9731	61.9865	130.3798	136.4025	132.5018
EP	142.6598	69.3299	142.8533	146.9461	144.3456

We have presented the plot of goodness-of-fit of PSG distribution and some selected distributions are in Figure 4.

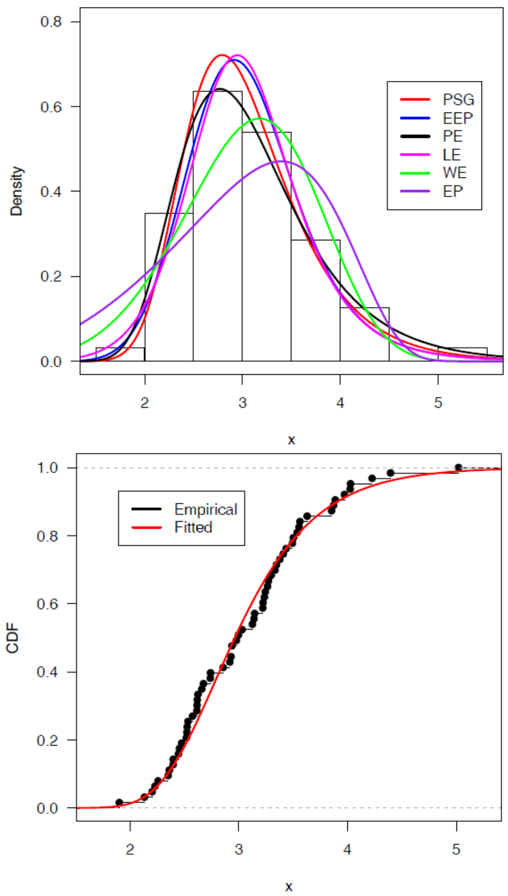


Figure 4. Empirical distribution function with estimated distribution function (upper panel) and The PDF along with histogram for distribution models taken (lower panel) of PSG distribution.

To compare the PSG distribution's goodness of fit among different models, different values of goodness of fit statistics are presented in Table 5 where the test statistics for model purposed was observed to have low value also the p-value was higher. Thus conclusion that PSG distribution shows better fit with more reliability and consistency in results among others taken for comparison

Table 5

The goodness-of-fit statistics and their corresponding p-value

Model	*KS(p-value)*	*AD(p-value)*	*CVM(p-value)*
PSG	0.0883(0.7102)	0.0704(0.7509)	0.3632(0.8839)
EEP	0.0907(0.6784)	0.0714(0.7451)	0.4002(0.8480)
PE	0.0635(0.9613)	0.0543(0.8516)	0.4044(0.8438)
LE	0.0919(0.6613)	0.0764(0.7144)	0.4613(0.7859)
WE	0.0879(0.7148)	0.12498(0.4771)	0.9381(0.3911)
EP	0.1443(0.1452)	0.3504(0.0978)	2.3516(0.0595)

V. CONCLUSION

A new distribution named Poisson shifted Gompertz distribution is introduced. A comprehensive study of some distributional characteristics of the new distribution is presented build a clearer picture for the distribution purposed. Parameter estimation is carried out with MLE along with CVME and LSE. The curves of the PDF of the proposed distribution have shown that its shape is increasing-decreasing and right skewed and flexible for modeling real-life data. Also, the graph of the hazard function is monotonically increasing or constant or reverse j-shaped according to model parameters' value. The proposed distribution's applicability and suitability has been evaluated by considering a real set of data and the results exposed that the proposed distribution is much flexible as compared to some other fitted distributions.

REFERENCES

1. Alkarni, S. and Oraby, A. (2012). A compound class of Poisson and lifetime distributions, *J. Stat. Appl. Pro*., 1(1), 45-51.
2. Bader, M. G., & Priest, A. M. (1982). Statistical aspects of fiber and bundle strength in hybrid composites. *Progress in science and engineering of composites*, 1129-1136.
3. Barreto-Souza, W. and Cribari-Neto, F. (2009). A generalization of the exponential-Poisson distribution. *Statistics and Probability Letters*, 79, 2493-2500.
4. Bemmaor, A. C. (1992). Modeling the diffusion of new durable goods: Word-of-mouth effect versus consumer heterogeneity. In *Research traditions in marketing* (pp. 201-229). Springer, Dordrecht.
5. Bereta, E. M., Louzanda, F., & Franco, M. A. (2011). The poisson-weibull distribution. *Adv. Applic. Statist*, *22*(2), 107-118.
6. Cancho, V. G., Louzada-Neto, F. and Barriga, G. D. C. (2011). The Poisson-exponential lifetime distribution. *Computational Statistics and Data Analysis*, 55, 677-686.
7. Chaudhary, A.K. & Kumar, V. (2020). A Bayesian Estimation and Prediction of Gompertz Extension Distribution Using the MCMC Method.*Nepal Journal of ScienceandTechnology(NJST)*,19(1),142-160.
8. Chaudhary, A. K. & Kumar, V. (2020). Poisson Inverse NHE Distribution with Theory and Applications. *International Journal of Science and Research (IJSR),* 9(12), 1603-1610
9. Chaudhary,A.K.& Kumar, V.(2020).Poisson NHE Distribution: Properties and Applications. *International Journal of Applied Research(IJAR),6(12),399-409.*

10. Gompertz, B. (1824). On the nature of the function expressive of the law of human mortality and on the new mode of determining the value of life contingencies, *Phil. Trans. Royal Soc. A*, 115, 513–580.
11. Joshi, R. K. & Kumar, V. (2020). Poisson Exponential Power distribution: Properties and Application. *International Journal of Mathematics & Computer Research* 8(11), 2152-2158. DOI: https://doi.org/10.47191/ijmcr/v8i11.01
12. Joshi, R. K. & Kumar, V. (2020). Lindley Gompertz distribution with properties and application. *International Journal of Statistics and Applied Mathematics*, 5(6), 28-37
13. Kaviayarasu, V. & Fawaz, P. (2017). A Reliability Sampling Plan to ensure Percentiles through Weibull Poisson Distribution, *International Journal of Pure and Applied Mathematics*, 117(13), 155-163.
14. Kus, C. (2007). A new lifetime distribution. *Computational Statistics and Data Analysis* 51, 4497-4509.
15. Kyurkchiev, V. E. S. S. E. L. I. N., Kiskinov, H. R. I. S. T. O., Rahneva, O. L. G. A., & Spasov, G. E. O. R. G. I. (2018). A Note on the Exponentiated Exponential-Poisson Software Reliability Model. *Neural, Parallel, and Scientific Computations*, *26*(3), 257-267.
16. Louzada-Neto, F., Cancho, V.G. & Barriga, G.D.C. (2011). The Poisson–exponential distribution: a Bayesian approach, *Journal of Applied Statistics*, *38*(6), 1239-1248.
17. Lu, W. & Shi, D. (2012). A new compounding life distribution: the Weibull–Poisson distribution, *Journal of Applied Statistics*, 39:1, 21-38.
18. Mailund, T. (2017). *Functional Programming in R: Advanced Statistical Programming for Data Science, Analysis and Finance*. Apress, Aarhus N, Denmark ISBN-13 (pbk): 978-1-4842-2745-9 ISBN-13 (electronic): 978-1-4842-2746-6 DOI 10.1007/978-1-4842-2746-6
19. Moors, J. J. A. (1988). A quantile alternative for kurtosis. *Journal of the Royal Statistical Society: Series D (The Statistician)*, *37*(1), 25-32.
20. R Core Team (2020). R: A language and environment for statistical computing. R Foundation for Statistical Computing, Vienna, Austria. URL https://www.R-project.org/.
21. Ristić, M. M., & Nadarajah, S. (2014). A new lifetime distribution. *Journal of Statistical Computation and Simulation*, *84*(1), 135-150.
22. Rodrigues,G.C., Louzada, F. and Ramos, P.L.(2018). Poisson–exponential distribution: different methods of estimation, *Journal of Applied Statistics*, 45(1), 128-144.
23. Smith, R.M. and Bain, L.J. (1975). An exponential power life-test distribution, *Communications in Statistics*, 4, 469-481.
24. Tang, Y., Xie, M., & Goh, T. N. (2003). Statistical analysis of a Weibull extension model. *Communications in Statistics-Theory and Methods*, *32*(5), 913-928.

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