

Fatigue Lifetime Prediction of Laminated Composites, a Review



Aidy Ali, M. F. Abdullah, M. K. Faidzi, Kannan Rassiah

Abstract. It is evident that laminated reinforced composite are successfully prolonging the life of composites compared to particle and fiber type of reinforced composites method. The question on how these laminated composites take up the fatigue loading is crucial in order to give sound confident to industry replacing their design from metal to composites based. The lack of confident and uncertainties' life of composites components become an issue to designer to shift from metal based to composites based especially when the design required to be done in short time. This review gives a clear picture the state of fatigue life modelling and the life prediction of laminated composites structures. The types of model are favorable when it is accurate, simple and required less input parameters. In the end, this review gives clear pictures on mechanism that involve and the fundamental of formula that available at present.

Keywords: Fatigue of Composite, Laminated Composites, Modelling of Fatigue Composite, Life Prediction.

I. INTRODUCTION

Composites materials were formed from two or more dissimilar materials, where each of the materials contributes to the properties of the composites [1]. Composites is a combination of two or more chemically distinct and insoluble phases of materials. After combination, the properties and structural performance are superior than those material which make up the composite. However, in composite, the components which make up the composite remain separate and distinct. There is a recognizable interface between the materials [2]. Generally, composite referred to any multiphase material that exhibits significant proportion of the properties of all constituent materials such that better properties is obtained through combination of them [3]. Composites consist of matrix and filler [1].

Matrix is the component which supports the filler in place and protect them against physical damage.

The examples of materials that can be used as matrix are epoxy, phenolic, unsaturated polyester, aluminum, magnesium, silicon carbide, silicon nitride, and etc. Filler is the component which lend it properties, such as strength, to the composites. The examples of materials that can be used as fillers are carbon, graphene, glass, aramids, tungsten, and etc. [4]. In recent years, natural fibers were also used as fillers for composite reinforcement. [5]. The types of natural fibers used as fillers included bamboo, Kenaf, Flax and Sisal [6].

Composite can be further classify into three categories depending upon the materials used as matrix. As it can be seen in the examples of material given above, the materials used was metal, polymer, and ceramic. By classifying composite According to the materials of matrix, three categories are form, which are metal-matrix composite, polymer-matrix composite and ceramic-matrix composite [2]. For metal-matrix composite, the matrix materials used is metal, such as aluminum, titanium, magnesium, copper, and etc. Typical applications include satellite, helicopter structures and compressor blade. For polymer-matrix composite, the matrix materials used is polymer, such as epoxy, unsaturated polyester, phenolic, silicon, and etc. Typical application included military aircraft, automobile bodies, helmets, and pressure vessels. For ceramic-matrix composites, the matrix materials use is ceramic, such as silicon carbide, silicon nitride, aluminum oxide, mullite, and etc. Typical applications include cutting tools, dies for extrusion and drawing of metals, deep-sea mining equipment, and automotive engine components [2].

Other than the types of matrix, composites can also be classified according to the types of filler. Generally, it can be classified into four types, as in Figure 1 namely:

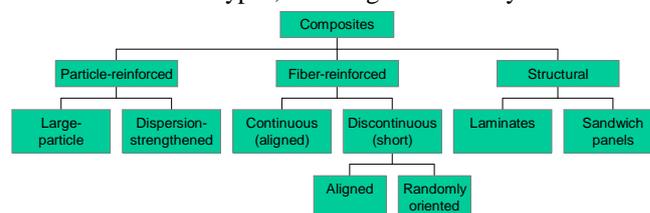


Figure 1: Commonly agreed type of composites [3].

a) Particles Reinforced

Particles composite is form when the filler use is roughly round in shape and the size is very small. Schematic illustration is shown in Figure 2.

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Classification: Particle-Reinforced

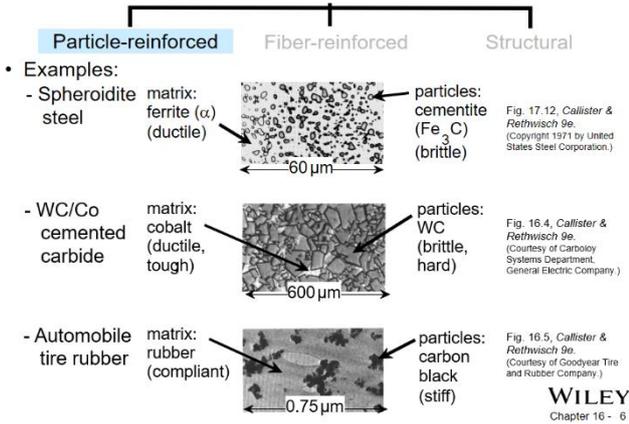


Figure 2: Particle Reinforced Composites [3].

b) Fiber Reinforced

Short or long fibers composite is form when the filler use have length to diameter ratio. The length to diameter ratio of short fibers is roughly 100 while long fibers is roughly infinity. Schematic illustration is shown in Figure 3.

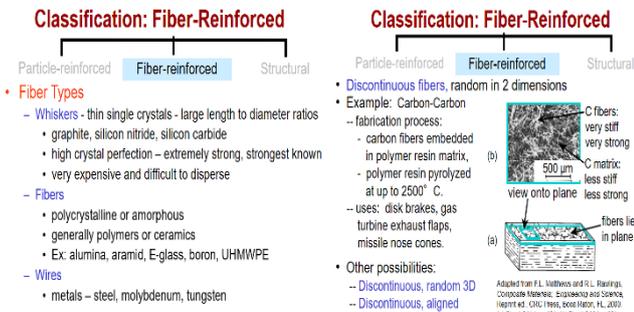


Figure 3: Fiber Reinforced Composites [3].

c) Structural Reinforced

Could be sandwich form or laminated form of composite. In laminated cases the filler use is in the form of sheet. In sandwich case, the filler use is in the form of foam, thick and cover by other layer of laminate fiber. Schematic illustration is shown in Figure 4.



Figure 4: Structural Reinforced Composites [3].

II. LITERATURE REVIEW

Hybrid composite

To improve the properties of composite mentioned above, hybrid composites were developed by carrying out research [7]. Composite that has two or more types of fillers in the same matrix result in hybrid composite [8]. When two or more types of fibers which belong to different categories of materials were incorporate into a matrix, hybrid composite is formed. Hybrids have better all-around combination of

properties than composites containing only one fiber as it combined advantages properties of various incorporate fibers and improving the performance of composites [3][9]. The behavior of it would be weighed sum of properties of individual components of fibers in which advantages and disadvantages of each fiber were balanced. The use of two or more types of fibers could complement the disadvantages of each fiber with the advantages of inherit properties. In hybrid composite, selection of each fiber has been done carefully to offset the poor quality of the other fiber [8]. Generally, hybrid composites refer to use of various combination of fibers incorporate into polymer matrix to form hybrid composite which have unique features and excellent properties [7].

There were several types of arrangement of fibers of hybrid composite, which were:

Tow-by-tow

Tows of the two or more constituent phase of fibers are mixed in regular or random manner.

1) Sandwich

Also known as core-shell. One material is sandwiched between two layers of another material.

2) Laminated

Alternate layers of the two or more fibers are stacked in a regular manner.

3) Intimately mixed

Constituent fibers are mixed randomly to ensure no over-concentration of any one type of fiber is present in the composite.

4) Others

Composites that are reinforced with ribs, pultruded wires, thin veils of fiber or combinations of the arrangement mentioned above.

The major reinforcing fiber incorporate into thermosetting resins to produce hybrid composite are E-glass as reinforcement of E-glass in polyester resin produced composite with impact strength comparable to reinforced thermoplastic [8].

Hybrid laminated composites

The term, laminated composite, is best described as composite made up of two dimensional sheets or plies which have been stacked together. The two dimensional sheets or panels were made up of unidirectional continuous fibers or woven fabric fibers which would be used as reinforcement for the matrix [3]. The building block of laminate composite is lamina, which consist of unidirectional fibers or woven fabric fibers [10]. For laminate which consist of unidirectional fibers, laminate would be stacked according to their high strength direction in different orientation across each layer to produce high strength in number of directions for two dimensional plane [3]. For laminate which consist of woven fabric fibers, laminate of the same types of fibers could be stacked randomly as it did not affect their strength. Figure 5 showed stacking of unidirectional continuous fibers in different direction to form laminated composite.

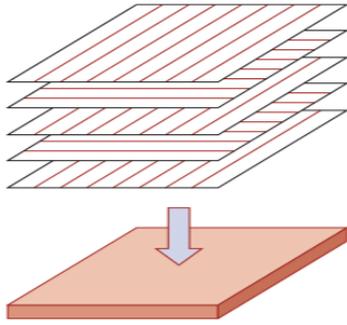


Figure 5: Stacking of unidirectional continuous fibers [3].

Unidirectional fibers have to be stacked in different orientation while woven fabric fibers do not were due to anisotropic properties of materials [1]. Composites with continuous fibers are orthotropic in nature, at which the properties of composite are different for direction along the orientation of fibers and direction perpendicular to the orientation of fibers. The strength and stiffness of laminated composites were influenced by fiber orientation and sequence of stacking of fiber [11]. For woven fabric lamina fibers, the sequence of stacking of different types of fibers did effect the strength and stiffness of laminated composite [10]. Symmetric laminated composite normally processes better mechanical properties than asymmetric laminated composite [12].

Classification of reinforcement

Fundamentally, fatigue failures begin initiating, propagating and failures occur below the yielding of the composites, the load must be below yielding to avoid plastic deformation and the materials must give up under cyclic repetitive loading with very low load. Therefore, the elastic modulus become very important to model the fatigue behavior of composite. This section covers the measurement and calculation of elastic modulus composite with respective it reinforced classification. In the case of particle reinforced, the composites are assuming to be uniformly distributed and isotropic. Equation that govern the strength in particle reinforced composite are using the rule of mixture equation suggested that the elastic modulus fall between upper bound (Eq. 1.) and lower bound limit (Eq. 2),

$$E_c(u) = E_m V_m + E_p V_p \tag{1}$$

$$E_c(l) = \frac{E_m E_p}{E_m V_m + E_p V_p} \tag{2}$$

In this expression, $E_c(u)$ denoted the upper limit of elastic modulus, $E_c(l)$ denoted the lower limit of elastic modulus, V denoted the volume fraction, and subscripts c , m and p represent composites, matrix and particulate phases respectively. In the case of fiber reinforced, the composites are assuming to follow the direction of the fiber whether it is longitudinal (Eq. 3) or transverse (E. 4) direction to the loading direction.

$$E_c l = E_m V_m + E_f V_f \tag{3}$$

$$E_c t = \frac{E_m E_f}{E_f V_m + E_m V_f} \tag{4}$$

In this expression, $E_c l$ denoted the longitudinal elastic modulus, $E_c t$ denoted the transvers elastic modulus, V denoted the volume fraction, and subscripts f and m represent, fiber and matrix phases respectively. In the case of structural reinforced, the composites are multi-layered and normally low density composite used in applications

requiring high structural integrity, high tensile, compressive, flexural and torsional strength. The properties of these type of composites depend not only on the properties of the constituent materials, but also on the geometrical design, number of layer, the direction of the layer, or even the layer in the form of woven. Laminated composites and sandwich panels composites are two of the most common structural composites.

Laminated composites strength often measured based on number of layer, unidirectional, cross-ply, angle-ply, multidirectional and even in consideration of woven, the parameters of wrap and weft. To begin understand the anisotropic materials, the common isotropic linear elastic materials behavior have to be first understand and should be modified, in the case of anisotropic, the materials are normally laminated or in the form of combination of modulus of several fiber, when this so call dissimilar modulus, $E_1 \neq E_2 \neq E_3$, the materials are said to be *orthotropic* (See Figure 6).

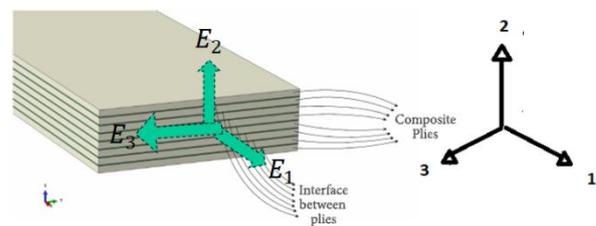


Figure 6: An orthotropic material.

Some cases, the short transverse and long transvers or the plane transverse modulus have good approximation ($E_2 = E_3$); such a material is called *transversely isotropic*. The elastic constitutive is now can be modified to account for this anisotropy, and the following form is the modified usual equations of isotropic to transversely isotropic materials:

$$\begin{Bmatrix} \epsilon_1 \\ \epsilon_2 \\ \gamma_{12} \end{Bmatrix} = \begin{bmatrix} 1/E_1 & -\nu_{21}/E_2 & 0 \\ -\nu_{12}/E_1 & 1/E_2 & 0 \\ 0 & 0 & 1/G_{12} \end{bmatrix} \begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{Bmatrix} \tag{5}$$

The parameter ν_{12} is the principal Poisson’s ratio; it is the ratio of the strain induced in the 2-direction by a strain applied in the 1-direction. This parameter is not limited to values less than 0.5 as in isotropic materials. Conversely, ν_{21} gives the strain induced in the 1-direction by 2. Figure 6 shows an orthotropic material. A strain applied in the 2-direction. Since the 2-direction (transverse to the fibers) usually has much less stiffness than the 1-direction, a given strain in the 1-direction will usually develop a much larger strain in the 2-direction than will the same strain in the 2-direction induce a strain in the 1-direction. Hence we will usually have $\nu_{12} > \nu_{21}$. There are five constants in the above equation (E_1 , E_2 , ν_{12} , ν_{21} and G_{12}). However, only four of them are independent; since the S matrix is symmetric, we have $\nu_{21}/E_2 = \nu_{12}/E_1$.

Mechanisms of fatigue in metals versus mechanism of fatigue in composites

Fatigue is a description of the progressive, localized and permanent damage, in the form of a crack and a plastic zone, which results into catastrophic failure.

Fatigue failure can occur at stress levels much lower than the monotonic yield strength. Fatigue, as a subject, describes the process of the nucleation of a crack from a defect, and its propagation until failure occurs. This process is influenced by mechanical, micro structural and environmental factors. Before we understand and model the fatigue in composites, it is better to be cleared the fatigue in metals that is more stable to be analyzed and it is homogenous.

Dislocations within Metallic Grains

Dislocations are areas of localized lattice distortion. There are two types of dislocation – edge and screw. An edge dislocation is effectively an extra half plane of atoms in the crystal lattice. It is moved by shear stresses perpendicular to its plane. If the shear stresses acting upon the dislocation are of sufficient magnitude, interatomic bonds within the crystal are broken and reformed so that the half plane of atoms moves through the crystal. This process is shown in Figure 7. For plastic deformation to occur, the dislocations must flow or move. The process in which dislocation motion produces plastic deformation is called ‘slip’.

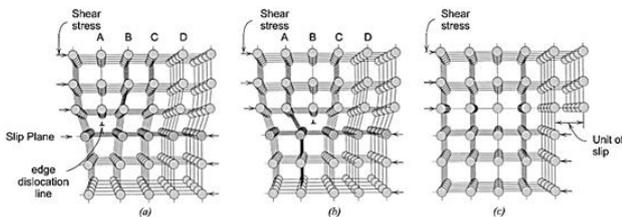


Figure 7: Atomic rearrangements that accompany the motion of an Edge dislocation as it moves in response to a shear stress

Crack Initiation

During cyclic loading, dislocations move forward and backward along slip planes within the metallic crystals. Repeated movements of dislocations along slip planes result in the formation of intrusion and extrusion lips on the surface of the metal. These “lips” or slip steps cause local stress concentration. Therefore, micro cracks can nucleate from these surface discontinuities, which are a direct result of the cyclic loading. Micro cracks can also form where there is a local stress concentration or areas of low local strength. Example of surface crack initiation in Figure 8.

Crack Propagation

Once the micro crack has been formed it will initially propagate in stage I (shear mode) growth until the governing micro structural feature, i.e. grain boundary, ceases to affect its propagation. In general, grain boundaries resist the propagation of micro cracks because plastic deformation is discontinuous across them [13]. Stage I cracks propagate often along slip planes in crystals. The crack will grow by shear along the primary slip systems. This creates a “zig-zag” path or wiggly path shown below in Figure 8.

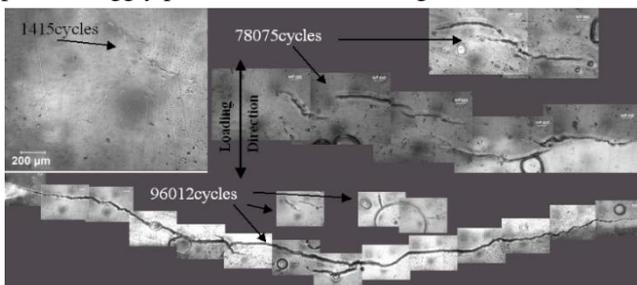


Figure 8: Replica Images for LCF of 2024-T351 Alloy, Initiation, Crack Coalescence and Long Crack [14-15].

When the propagation of the micro crack becomes independent to the presence of microstructural features, the rate is controlled by the stress intensity factor range, ΔK ,

$$\Delta K = Y\Delta\sigma\sqrt{\pi a} \quad (6)$$

where: ΔK = Stress intensity factor range,
 a = Crack length in m,
 Y = Geometry factor,
 $\Delta\sigma$ = Nominal stress

A typical ΔK controlled propagation is shown in Figure 9. Equation (6) models the stress intensity factor with respect to the crack length and the stress range applied to the crack. In stage II, the K values are higher so the plastic zone grows and becomes larger than the grain size. Therefore, crack growth is largely independent of the microstructure. One proposal for the growth mechanism is Laird’s model (1982) [16]. As a component containing the crack is loaded in tension, the crack tip opens. At the crack tip the stress concentration factor, K , is very large. Therefore, the local stress at the tip is higher than the yield point resulting in plastic flow along two slip planes at 45 degrees to the loading direction. The material ahead of the crack tip cannot support any load and tears. In stage II, crack growth can be approximated by the Paris Linearity Law 1950s, (see Figure 9). We can integrate the Paris between the initial crack length and the critical crack length to find the remaining life of the component, N , where a_c and a_f are the initial and final crack lengths respectively.

$$\frac{da}{dN} = C\Delta K^m = C(Y\Delta\sigma\sqrt{\pi a})^m \quad (7)$$

$$N_f = \frac{1}{CY^m\Delta\sigma^m\pi^{m/2}\left(\frac{m}{2}-1\right)} \left[\frac{1}{a_0^{(m/2-1)}} - \frac{1}{a_f^{(m/2-1)}} \right] \quad (8)$$

The term m and C are experimentally determined constants. The constant m , must be greater than two for this solution.

Unstable Propagation, Rupture and Fracture

In stage III growth, the plastic zone ahead of the crack tip has dimensions exceeding the size predicted by LEFM. Voids, discontinuities, defects and regions where there is a local stress concentration begin to develop their own micro cracks inside of the plastic zone. These cracks have their own plastic zones at their crack tips and begin to propagate rapidly in this low strength region. As a result, the yield strength of the material begins to decrease. The continued propagation of these smaller cracks results in their joining to the main crack, causing its growth to a critical crack length. At this point the stress intensity factor reaches the fracture toughness of the material, K_{IC} and the structure will fail.

Crack Propagation Regimes

Figure 9 shows the different regimes of crack growth. The threshold stress intensity range, ΔK_{th} , is the first value where, according to Linear Elastic

Fracture Mechanic(LEFM), the crack growth behavior can be approximated by the Paris equation. In stage III the crack propagates rapidly until the fracture toughness of the material (K_{IC}) is reached, where the component will fail. Stages I and III are very sensitive to the microstructure, mean stress and residual stress.

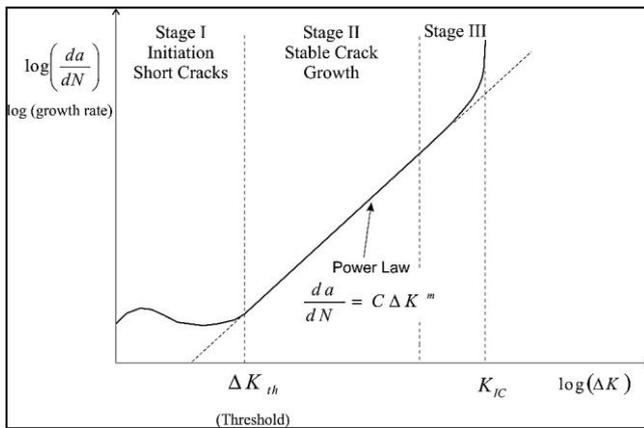


Figure 9: Crack propagation regimes relative to stress intensity range.

The fatigue modelling theories are mostly originated originate from metal-based theories. It must be noted that, at the nanometre length scale, composite materials are inhomogeneous and anisotropic, and their behaviour is therefore much more complex than that of metals, which are generally homogeneous and isotropic materials. This complexity is mainly associated with the fact that a variety of damage phenomena each with their specific growth rates and laws of interaction can occur in the case of composites, namely fiber fracture, fiber bucking, matrix cracking, matrix crazing, fiber matrix interface failure, fiber pull out, matrix aging, microbiology matrix attack, and delamination shown in Figure 10, Figure 11 and Figure 12.

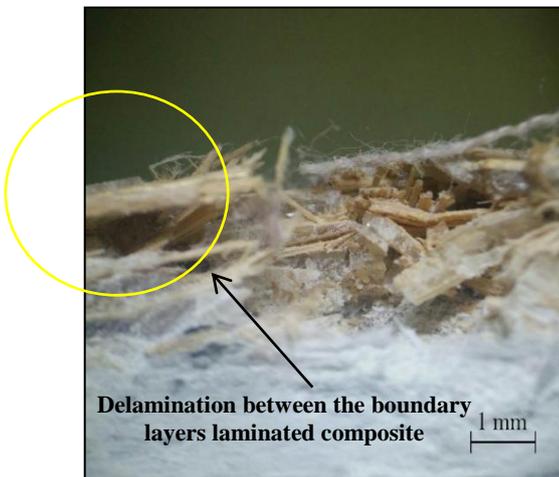


Figure 10: Delamination between Layers in Bamboo Laminated Composites [17-18].



Figure 11: Delamination between Layers in Bamboo Laminated Composites [17-18].

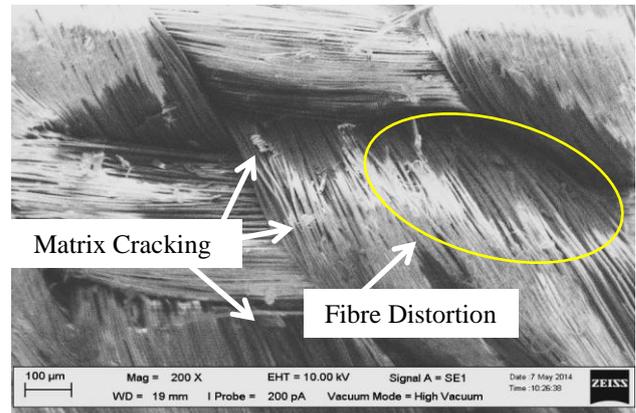


Figure 12: Delamination between Layers in Bamboo Laminated Composites [33].

The difference in fatigue behavior between fiber-reinforced composites and metals lies in several points as shown in Figure 13. It's schematizes the difference in damage evolution response between composites and metals. In the case of metals, gradual and invisible material deterioration occurs almost over the entire material lifetime and thus no or little degradation of material properties is observed during the course of fatigue progress. Namely, stiffness remains quasi-constant over the lifetime of the material. Toward the end of the material's life, macroscopically observable small cracks develop across the material and, before long, coalesce in the run up to final fracture. With constant stiffness, the linear relation between stress and strain remains constant.

To better compare Figure 14 and Figure 15 show the mechanisms and lifetime of these two group of materials. As commonly favor in applying the linear elastic analysis and linear fracture mechanics are largely for case of metals, one should be very critical to apply them for composite. This is not the case for particle-reinforced, fiber-reinforced and structural reinforced composites, where damage starts very early and the extent of damage zones grows steadily, while the type of damage in these zones can change: small matrix cracks can lead to comparatively larger delamination's, for example. Gradual deterioration, in stiffness and strength, leads to a continuous redistribution of stress and a relative reduction of the amplitude of stress concentrations within the component studied under displacement controlled situations.

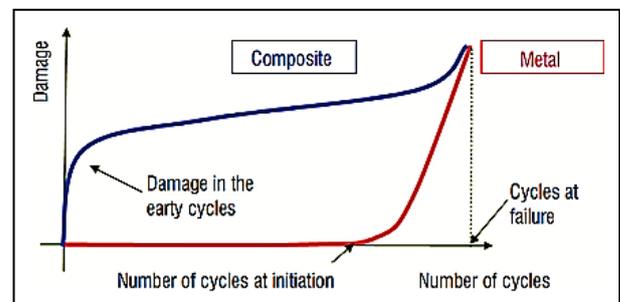


Figure 13: Evolution of Damage with the Number of Cycles, For Composites and Metals [19].

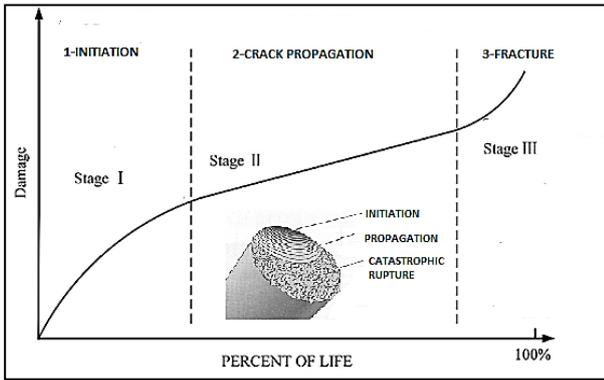


Figure 14: Fatigue Damage Mechanisms of Metals [19].

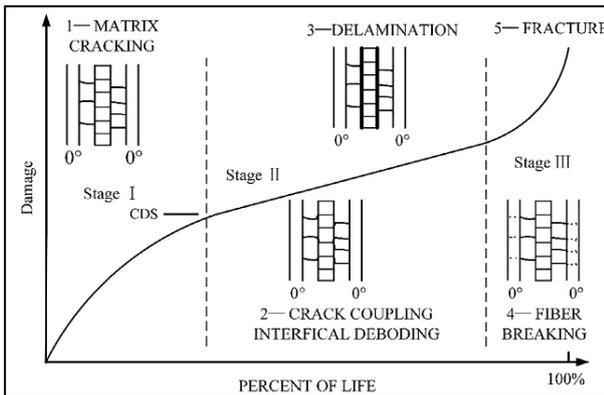


Figure 15: Fatigue Damage Mechanisms of Composites [20].

The modelling the fatigue process in a fiber reinforced composite requires the simulation of the complete sequence of damage states and growth of damage inside the component. Consequently, methodologies derived for metals are mostly unsuitable for composites. The complexity of composites requires incorporation of fiber architecture types whether it is unidirectional, woven, random and ply angle. Figure 16 represent the fatigue life of several fiber architecture effect. Having understand the different mechanism of fatigue in metallic materials compared with composites, once can now look back what have been developed in term of modelling the fatigue lifetime of engineering materials. The revolution of the models are bases on Paris Law, Miner’s rule, continuum micro and miso mechanical, degradation model, residual strength and progressive damage model. The Paris fitting power law, Newtonian stress method and the energy method are the main pillar of the models.

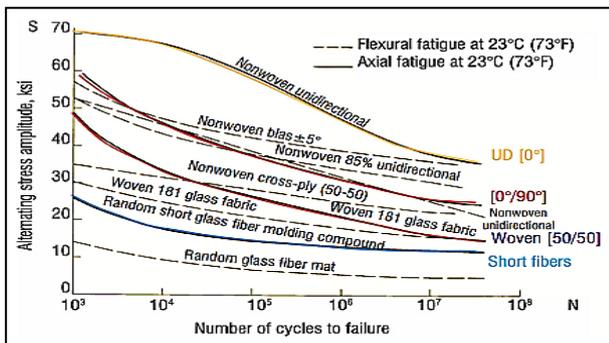


Figure 16. Comparative fatigue strength of a same resin/glass composition with various fibre architectures [21]

Constant amplitude fatigue model of laminated composited

In other to predict the cycle life time of the laminated composited, the literature review Figure 17 must be conduct to analyse the previous research in empirical modelling of predicting life time laminate composited. To develop the model of fatigue loading, all mechanical properties and parameter data of laminated composited must be recorded to implement in the model and validate the developed model with well published experiment data.

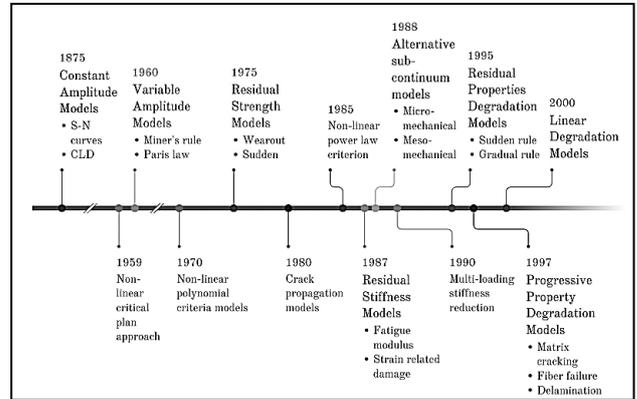


Figure 17: Timeline of Fatigue Model’s Elaboration [22]. Kinetic Theory of Fracture (KTF)

One of the model of fatigue loading is by using kinetic theory fracture. The researcher is predicting the delamination fatigue and matrix in polymer composited by using the model kinetic theory of fracture. In this study, a matrix failure courses by fatigue methodology based on the kinetic theory of fracture all can write as (KTF). Kinetic theory of fracture was developing that used the physics of the matrix constitute on the crack damage in both interlaminar and lamina region. The model was calibrating from either laminar or plus – minus 45 laminar fatigue test. Three different laminates were comprised subjected to tension-tension fatigue loading in the finite element simulation by open hole coupon. The result is when the damage accumulative inside the plies due to matrix failure with fatigue cycle were benchmark against the previous publisher experiment data [23].

The kinetic theory of fracture (KTF) for fatigue failure of polymers was derived respectively:

$$N_f = \frac{h}{(kT)^2} f \gamma (\sigma_{max} - \sigma_{min}) \exp\left(\frac{U}{kT}\right) \left[\exp\left(\frac{\gamma \sigma_{max}}{kT}\right) - \exp\left(\frac{\gamma \sigma_{min}}{kT}\right) \right]^{-1} \quad (9)$$

- where: N_f = Number of cycle to failure
- h = Planck’s constant
- k = Boltzmann constant
- T = Absolute temperature
- f = Frequency
- U = Activation energy
- γ = Activation volume
- σ = Stress

From the equation, the number of cycle to failure can be calculate. During the under load controlled fatigue, number of cycle to failure is independent of the shape factor, λ .



Glass Fibers Reinforced Plastics under random loading (GFRPs)

The new research of fatigue model base on stiffness degradation to life prediction of glass fiber reinforced plastic (GFRP) under random ocean current loading. In this study, the model used Weibull cumulative density function. The stress amplitude and effect of stress ration on the model parameter will analyse using result from the constant amplitude test. The fatigue life was predicting by proposed model and by conversional life predict using Palmgren. Mine’s Rule [24].

To calculate the fatigue model is using stiffness degradation model. This model method can be used in characterizing the S-N curve. Generally, the specimen retain stiffness E_f , event at failure cycle. The fatigue damage parameter was present as follow [24]:

$$D_n = \frac{E_o - E_n}{E_o - E_f} \tag{10}$$

where: D_n = Fatigue damage parameter at n^{th} cycle
 E_o = Initial stiffness
 E_n = Stiffness at n^{th} cycle
 E_f = Stiffness at failure

The proposed of stiffness degradation model can be analyse using Weibull function. There are three stages the different on rate of increase in damage parameter in fatigue damage curve. This function can be expressed by using Weibull cumulative density function. Formula of Weibull density function was present as follow [24]:

$$\frac{n}{N_f} = 1 - \exp \left[- \left(\frac{D_n}{\lambda} \right)^k \right] \tag{11}$$

where: N_f = Cycle
 k = Shape parameter
 λ = Scale parameter

The curve of Weilbull function, depends on these parameter (shape parameter and scale parameter are function of stress amplitude and stress ratio).

Damage Composite Material Subjected Fatigue Loading

To predict the progression of damage in different stress level is analyse based on the applies stress, residual stress and cycle ratio. The specimen in this study is using E-glass/vinyl ester laminate. Using the propose model, the damage progression can be predicting by three different stages levels on fatigue loading of the specimen. In order to predict the damage fatigue loading, under a given temperature, frequency, specimen geometry and moisture, the fatigue damage function of the material depend on the stress applied and number of fatigue cycle was present [25]:

$$D = D(n : \text{stress}) = D(n : N) \tag{12}$$

where: N = Number of loading cycle
 N_f = Number of cycle of failure under applied stress

The Power Law stated that during the fatigue loading, the damage progresses continuously. The damage factor is given by equation [25]:

$$\frac{dD}{dN} = \frac{A\sigma_a^C}{BD^{B-1}} \tag{13}$$

where: D = Damage factor

σ_a = Applied stress

B, C = Constant

The new model function of residual strength, stress applied, and cycle ratio was proposed to predict the damage growth on the composited material. After integrating the equation and applying the boundary condition, therefore the damage factor D and number of cycle N_f , can be calculate by using the equation below [25]:

$$D = \left[\left(\frac{\sigma_a}{\sigma_r} \right)^c \left(\frac{N}{N_f} \right) \right]^{\frac{1}{B}} \tag{14}$$

Composite Laminated Delamination Simulation and Experiment

From simulation view there are two most famous methods for delamination modelling, virtual crack closure technique (VCCT) and cohesive zone model (CZM). The technique is described, and their merits and drawback are discussed. For the experimental view, the topic range from delamination fracture toughness (DFT) test under static, dynamic, cyclic loading condition impact test that aim to obtain the impact resistance and residual stress after the impact [26].

Fatigue delamination is a combine damage mechanic and fracture mechanic approach, where the fatigue damage parameter is derived and used to reduce stiffness of strength the material. Two basic approach to obtain fatigue damage accumulative, first is proposed by Roe and Siegmund and another one is by integrating fatigue crack growth rate (FCGR). Fatigue growth rate relate the crack growth speed to the energy release rate change or stress intensity factor change at the delamination front. The fatigue crack growth rate curve usually called Paris Law, can be obtain [26]:

$$\frac{da}{dN} = D (\Delta K)^B \tag{15}$$

where: a = Crack length
 N = Number of loading cycle
 D, B = curve fitting parameter of experiment data from mode I and II
 $\Delta K = K_{max} - K_{min}$ = Maximum and Minimum stress intensity factor

The fatigue crack growth rates can also be expressed in them of energy release rate, and this is most commonly used of formulation [26]:

$$\frac{da}{dN} = C (\Delta G)^m \tag{16}$$

where: C, m = Experiment fitted data
 $\Delta G = G_{max} - G_{min}$ = Maximum and minimum strain energy release rate

Proposed a semi-empirical equation that unifies the effect of the stress ratio, mixed-mode ratio and threshold by adding the normalizing term [26 and 32]:

$$\frac{da}{dN} = C \left(\frac{\Delta K - \Delta K_{th}}{K_c - K_m} \right)^m \tag{17}$$



where: K_c = Fracture toughness
 K_m = mixed mode stress intensity factor
 ΔK_{th} = Stress intensity factor threshold

When the damage length inside the cohesive element is used to calculate damage accumulation rate [27]:

$$\frac{\partial d}{\partial N} = \frac{1-d_s-d_{f,u}}{0.5 \left(\frac{G_c}{G_c}\right) L_{CZ}} \cdot \frac{\partial a}{\partial N} \quad (18)$$

where: d_s = Static damage parameter
 $d_{f,u}$ = Unwanted fatigue damage
 L_{CZ} = cohesive length

Matrix Cracking Under Static and Fatigue Loading

This study is present the experiment to investigate the progressive matrix crack in carbon/ epoxy cross ply laminated under static and fatigue loading. Two model was proposed to evaluate the fatigue crack. In this review, the crack density is used mechanical damage variable. First model is evaluating from the most popular model is Paris Law function and the second is assume from the fatigue degradation [27].

Paris law function describe the crack growth rate under fatigue loading especially for delamination crack in laminated composited. Conversional Paris Law relate the crack growth rate to a power law of the stress intensity factor (ΔK) range during one cycle. Thus the equation was present as [27]:

$$\frac{da}{dN} = \alpha [g(\Delta K, \Delta G, G_{max}, \dots)]^n \quad (19)$$

where: g = Loading function
 ΔK = Metal fatigue

Delamination Fatigue for Complex Three Dimensional Multi-Interface Case

This research proposes to improve delamination fatigue cohesive interface model formulation to purpose the fatigue damage simulation in complex 3D cases. Fatigue initiation is incorporate without changing the propagation characteristic using strength reduction method. Using fatigue damage, the cycle is counted using time progression natural of explicit finite element analysis and frequency parameter. The progression fatigue damage is modification from the original equation of Paris Law [28]:

$$\frac{da}{dN} = C \left(\frac{\Delta G}{G_c}\right)^m, \quad \Delta G = (1 - R^2)G_{max} \quad (20)$$

where: a = Crack length
 C, m = Paris parameter
 ΔG = amplitude strain energy release
 G_c = Critical strain energy release

R can be expresses as minimum stress divide maximum stress and the maximum strain energy release rate can be obtained from integration of local traction displacement history in integration point [28]:

$$G_{max}^t = \sum_{k=1}^{nsetp} (t) \left(\frac{\sigma_1^k + \sigma_1^{k-1}}{2}\right) (\delta_1^k - \delta_1^{k-1}) + \sum_{k=1}^{nsetp} (t) \left(\frac{\sigma_{11}^k + \sigma_{11}^{k-1}}{2}\right) (\delta_{11}^k - \delta_{11}^{k-1}) \quad (21)$$

where: k = Increment number
 (t) = Increment number between time zero
 T = time

Therefore the fatigue life of crack tip element can be obtain as;

$$\Delta N_e = \frac{dN}{da} L_f \quad (22)$$

where: L_f = Fatigue critical length
 ΔN_e = fatigue life

Following is the equation are proposed as a local rate approach.

$$\Delta D_f = (1 - D_s - D_f) \cdot \gamma, \quad \gamma = 1 - \left(\frac{0.01}{1 - D_s - D_f}\right)^{\frac{\Delta N}{\Delta N_e - \Delta N_t}} \quad (23)$$

where: ΔN_t = Number of cycle
 γ = Control Parameter

Delamination Crack Growth in Glass Fiber Reinforced Plastic (GFRP) Composite Laminated

Mathematic modelling and FE simulation was conduct to study the delamination crack growth in class fiber reinforced (GFRP) composite for laminated structure under fatigue loading. Allix and Ladevese propose the classic static damage model and modified to be fatigue damage model. Then the model was used in finite element [29].

For fatigue model, using mathematically formulation and assumption. The cycle fatigue load varies between maximum and minimum is constant amplitude. Hence, the predict of fatigue damage, as follow [29]:

$$\dot{d}_f = \frac{\partial d_f}{\partial t} = \begin{cases} g \left(d, \frac{Y}{Y_c}\right) \frac{\dot{Y}(t)}{Y_c} & \text{if } \dot{Y} \geq 0 \text{ and } f \geq 0 \\ 0 & \text{if } \dot{Y} < 0 \text{ or } f < 0 \end{cases} \quad (24)$$

where: f = Loading function
 Y = Threshold damage energy release rate
 g = Dimensional parameter Govern

From the sum of mathematically equation in this study, the total damage due to fatigue loading can be derive as follow [29]:

$$(N + \Delta N) = d_f(N + \Delta N) + d_s(N + \Delta N) \quad (25)$$

In addition, the proposed of fatigue model is able to produce line crack growth rate by the classical Paris Law [29]:

$$\frac{da}{dN} = B \left[\frac{\Delta G}{G_c}\right]^m \quad (26)$$

where: ΔG = Maximum and minimum energy release rate
 G_c = Fracture toughness

Modelling by Power-Law Equation and Artificial Neural Network

This study to propose of Power-Law equation and artificial neural network to conduct modelling fatigue delamination growth (FDG) in polymer based in fiber reinforce composited. Barenblatt’s principle are applied to identified either suitable expression of delamination driving force between square rooted energy release rate range and associated peak value [30].

For the delamination force, the relation between stiffness intensity factor (SIF) range and corresponding peak value express as [30]:

$$K_{L,max} = \frac{\Delta K_L}{1-R} \tag{27}$$

where: R = Stress ratio
L = I, II fracture mode

Let the mixed mode fatigue delamination growth is unidirectional composited be considers, with a prescribe waveform the cycle load at fixed frequency. Then, propose the scenario where the fatigue delamination growth at constant temperature and the moisture concentration of the material has fixed uniform value, the fatigue delamination growth rate can be expressed generally as [28]:

$$\frac{da}{dN} = f_0 [\Delta\sqrt{G}, \sqrt{G_{max}}, \Delta\sqrt{G_{th}(\phi)}, \sqrt{G_c(\phi)}, \sigma_c(\phi), l_c(\phi), R, \phi, h, a] \tag{28}$$

$$\Delta\sqrt{G_{th}} = (\sqrt{G_{max}} - \sqrt{G_{min}})_{th}$$

Here is the following dimension can be present as follow [30]:

$$[a] = [h] = [l_c] = [da/dN] = L$$

$$[\Delta\sqrt{G}] = [\sqrt{G_{max}}] = [\Delta\sqrt{G_{th}}] = [\sqrt{G_c}] = F^{1/2}L^{-1/2}$$

$$[\sigma_c] = FL^{-2}$$

$$[R] = [\phi] = F^0L^0$$

where: a = Delamination length
 $\Delta\sqrt{G_{th}(\phi)}$ = Mixed mode

Probabilistic Strength Base Matrix Crack Evolution Model in Multi Direction

Mathematically calculation model has been conduct to predict the matrix crack evolution model in multi directional polymer matrix composited lamination under in plane fatigue loading. The parameter by Smith Watson Topper has been used to for model the number of cycle to initiate the first matrix crack and log normal probability distribution has used to handle the scatter in crack initial life. Paris Law model was proposed to base on the mixed mode effective stress intensity factor. The new crack initiation has been simulated by using crack initiation curve and Palmgren Miner Rule [31].

Matrix cracking in the material element assume to occurred when meets Hashin’s strength criterion. The matrix creaking failure mode as follow [31]:

$$\frac{\sigma_{22}^2}{Y_t^2} + \frac{\tau_{12}^2}{s^2} \geq 0 \tag{29}$$

where: σ_{22} = Transvers stress
 τ_{12} = In plane shear stress
 Y_t = Transvers strength

S = Shear strength

Weibull distribution has successfully used to describe the variation in transvers strength, the laminated can be express as follow [31]:

$$P(Y_t) = 1 - \exp \left[- \left(\frac{Y_t}{\beta} \right)^m \right] \tag{30}$$

where: β = Scale parameter
m = Shape parameter

The Basquin’s type Power Law equation express as follow:

$$\Delta W^* = W_f (N)^b$$

The expression of K at matrix crack tip of layer composited material can be derive as:

$$K = \sigma_D \sqrt{t}$$

By principle of additive energy release rate, the effective stress intensity factor (SIF) due to first mode and second mode loading as follow [31]:

$$K_{eff} = \sqrt{K_I^2 + K_{II}^2}$$

Here is expression by Paris Law equation can be present as follow [31]:

$$\frac{da}{dn} = C_{par} (\Delta K_{eff})^{m_{par}} \tag{31}$$

where: W_f, b = Power law fitting constant
 σ_D = Average stress driving MCGR
 K_{eff} = Driving force
 ΔK_{eff} = effective stress intensity factor range
 C_{par}, m_{par} = Material parameter

Laminated composites fatigue modelling equation (case study)

Fatigue modelling equation is to develop a new model or empirical model that can predict the fatigue lifetime estimation of bamboo laminated composites. The most popular modelling equation that used to predict the life time is classic modelling equation by Paris-Law and equation by Power Law. This both equation will be subjected to refer when conduct the new modelling for predict the fatigue life time of laminated composite material. The Power Law stated that during the fatigue loading, the damage progresses continuously. The damage factor is given by equation:

$$\frac{dD}{dN} = \frac{A\sigma_a^C}{BD^{B-1}} \tag{32}$$

where: D = Damage factor
 σ_a = Number of cycle of failure
B, C = Constant

After integrating the equation and applying the boundary condition, therefore the damage factor D and number of cycle N_f , can be calculate by using the equation below;

$$D = \left[\left(\frac{\sigma_a}{\sigma_r} \right)^C \left(\frac{N}{N_f} \right) \right]^{\frac{1}{B}} \tag{33}$$



Cumulative damage model can also be defined as:

$$D = 1 - \frac{\sigma_r}{\sigma_0} \quad (34)$$

Substituting the above equation in the present damage model and rearranging given:

$$B = \frac{\log \left[\left(\frac{\sigma_a}{\sigma_r} \right)^c \frac{N}{N_f} \right]}{\log \left[1 - \frac{\sigma_r}{\sigma_0} \right]} \quad (35)$$

From the equation, the number of cycle failure N_f will be subjected and can be predict for the fatigue life time cycle.

N = Number of loading cycle

D = Damage factor

σ_a = applied stress

B, C = Constant

N_f = Number of cycle failure

B and C value were calculated by plotting both term by varying, C value until the best fit obtained. some parameter for the experiment is don't exist to use in the modelling, therefore the equation can be modified to get the best curve in the (S-N) curve and validated with the experiment fatigue curve. Another of equation of power Law is follow the Smith Watson Topper (SWT) parameter with the number of cycle for matrix crack initiation (N) have been fit with the Basquin's type.

$$\Delta W^* = W_f (N)^b \quad (36)$$

where W_f and b is Power Law fitting constant and N is matrix crack initial life SWT parameter base matrix crack initial life for orientation (0/90) and (0/45/0/-45) laminated as shown in Figure 18. Example of predicted curve (S-N) result from Fatigue Modelling equation and (S-N) result from fatigue experimental of laminated bamboo composites. The result is accurately or nearest with the small 4% of error. As shown in Figure 19.

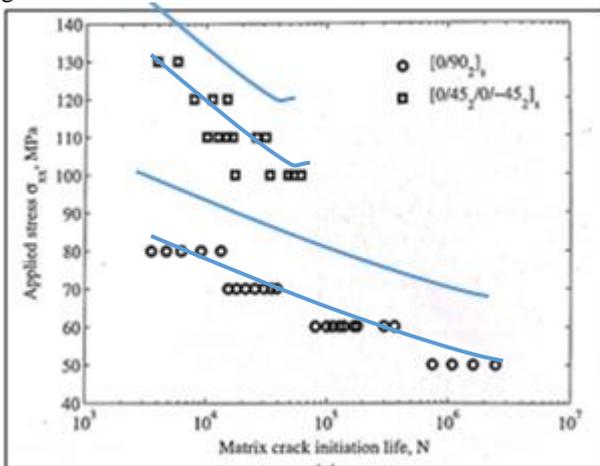


Figure 18: Graph Curve of Life Cycle, N versus Stress Applied, σ

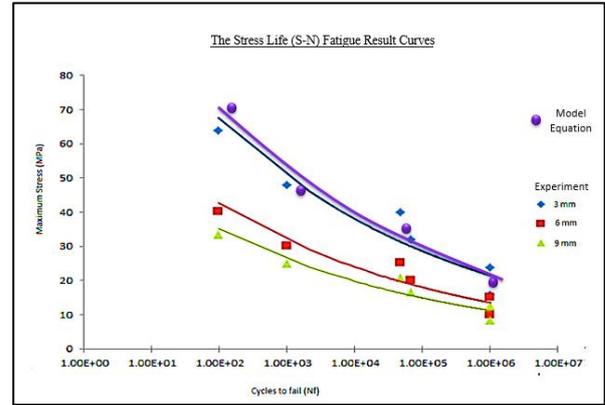


Figure 19: Predicted fatigue life of laminated bamboo composite using damage factor in Paris law.

III. CONCLUSION

Having review model that available to predict the fatigue life of laminated composites, the following can be summarized:

- a) In order to model the fatigue in laminated composites, one should have the fracture toughness of their composite, the fatigue curve S-N of their composites, and the mechanism of damage factor that normally measured using curve fitting techniques.
- b) The most popular model used at present is modified the Paris Law Equation that is widely used for metal and adopting them. It is used because its simplicity and less error.
- c) The energy and probabilistic model also introduced by many researcher and the results are comparable.

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