

# Tensor Decomposition of KUKA Industrial Robot (KR16-2) in Rotational System



Alok Mishra, P. K. Dwivedi, Kamlesh Singh

**Abstract-** This paper refers to study of industrial robot (KUKA KR16-2), in which we have considered the matrix decomposition and tensor decomposition model in rotational motion. We have considered robotic matrix & Tensor and defined a modal product between robot rotation matrix and a tensor. Further we have proposed the third order tensor for the motion of Industrial robot and tried to find out the useful result. At last we have shown that the tensor model is providing alternate way to find the solution.

This third order tensor will be denoted by upper case script letters (A, B, C, .....).  $A_{ijk}$  refers to the number in row  $i$  and column  $j$  of matrix  $k$  of  $A$  as per Robotic motion structure. The set of 3rd-order tensors for industrial robot with real number entries is denoted  $R^{n_1 \times n_2 \times n_3}$

## I. INTRODUCTION

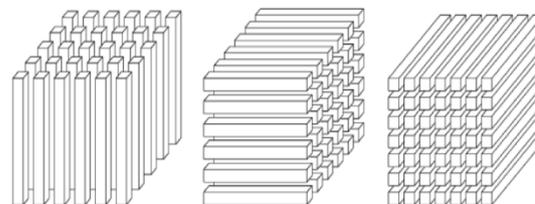
A robot is a component of an Advance Industrial Manipulator with the help of integration of advance motion system and Critical Function Transformation in motion control. Other components of this system i.e. machine tools, control devices, different auxiliary elements, and transport machines. The KUKA robotic arms are heavy and it is powerful machines capable of inflicting serious harm by crushing or laceration. The machines each weight around 250kg and draw a peak power of around 10 kilo-Watts. The robotic cell is gauzed for protection and safety depending upon safety standards employed by various organizations. The gauzing also serves to trigger an emergency stop if anyone enters into the cell. In literature review, the Artificial Intelligence based robotic motion path implementation by advance integration software like RTOS VxWorks v8.2.2. [6][7]. We are trying to attempt a less complex, easy to implement and both critical solution for KUKA Industrial Robot



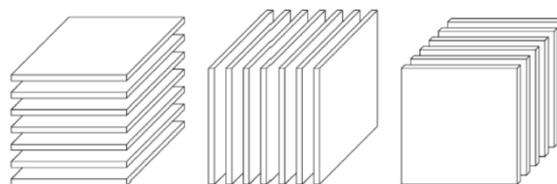
According to Industrial Robot KUKA KR16-2, rotation motion is governed by higher order tensors. Analogous to rows or columns of a matrix, 3rd-order tensors have robot implementation route. Robot route is implemented by 3rd order tensor. These robotics route areas are units which are equal to vector parts of matrix. There are rows, columns, and robot routes, the area units visualized in Figure 1. Under Robotic motion a 3rd-order tensor is sliced in three dimensions having horizontal, lateral, and frontal robotic slices. For industrial robot the area units are visualized in Figure 2. [3],[5],[7],[8]

## II. INDUSTRIAL ROBOT & TENSOR

A tensor is a multidimensional array, in which the order of tensor denotes the dimension of the array. As an example, a scalar is just associate order-0 tensor, a vector order-1, a matrix order-2, and any tensor with order-3 or larger is delineated as a better order tensor. For this paper we'll be specialize in the only higher-order tensor for industrial robot, the order-3 tensor, which might be envisioned as a form of Rubik's cube according to robotic manipulator KUKA KR16-2. [1],[2], [3]. An  $n_1 \times n_2 \times n_3$  third order tensor for KUKA Industrial Robot is a  $n_3$  array of  $n_1 \times n_2$  matrices.



Robot Column Robot Row Robot route  
**Figure 1**



Horizontal slices Lateral slices Frontal slices  
**Figure 2**

## III. ROBOTIC MATRIX & TENSOR

Consider  $A$  is a 3rd-order tensor and  $A \in R^{n_1 \times n_2 \times n_3}$ . Then the modal unfolding  $A_{(k)}$  is the matrix  $A_{(k)} \in R^{n_k \times N}$  where  $N$  is the product of the two dimensions unequal to  $n_k$ . [2] [6], [8]

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For a 3<sup>rd</sup>-order tensor,  $A \in \mathbb{R}^{n_1 \times n_2 \times n_3}$ , there are 3 modal unfolding and in general a 3<sup>rd</sup> order tensor has 3<sup>rd</sup> modal unfolding. As a simple example the modal unfolding for  $A \in \mathbb{R}^{3 \times 4 \times 2}$  are shown below, where  $A_1$  and  $A_2$  are the frontal slices of  $A$ .

$$A_1 = \begin{bmatrix} X_1 & Y_1 & Z_1 & P_1 \\ X_2 & Y_2 & Z_2 & P_2 \\ X_3 & Y_3 & Z_3 & P_3 \end{bmatrix} \quad A_2 = \begin{bmatrix} R_1 & S_1 & T_1 & U_1 \\ R_2 & S_2 & T_2 & U_2 \\ R_3 & S_3 & T_3 & U_3 \end{bmatrix}$$

$$A_{(1)} = \begin{bmatrix} X_1 & Y_1 & Z_1 & P_1 & R_1 & S_1 & T_1 & U_1 \\ X_2 & Y_2 & Z_2 & P_2 & R_2 & S_2 & T_2 & U_2 \\ X_3 & Y_3 & Z_3 & P_3 & R_3 & S_3 & T_3 & U_3 \end{bmatrix}$$

$$A_{(2)} = \begin{bmatrix} X_1 & X_2 & X_3 & R_1 & R_2 & R_3 \\ Y_1 & Y_2 & Y_3 & S_1 & S_2 & S_3 \\ Z_1 & Z_2 & Z_3 & T_1 & T_2 & T_3 \\ P_1 & P_2 & P_3 & U_1 & U_2 & U_3 \end{bmatrix}$$

$$A_{(3)} = \begin{bmatrix} X_1 & Y_1 & Z_1 & P_1 & X_2 & Y_2 & Z_2 & P_2 & X_3 & Y_3 & Z_3 & P_3 \\ R_1 & S_1 & T_1 & U_1 & R_2 & S_2 & T_2 & U_2 & R_3 & S_3 & T_3 & U_3 \end{bmatrix}$$

Using the definition of modal unfolding we can now define a modal product between a Robot Rotation matrix and a tensor.

The Modal product, denoted  $A_{(k)}$ , of a 3<sup>rd</sup> -order tensor  $A \in \mathbb{R}^{n_1 \times n_2 \times n_3}$  and a matrix  $U \in \mathbb{R}^{J \times n_k}$ , where  $J$  is any integer, is the product of modal unfolding  $A_{(k)}$  with  $U$ ,

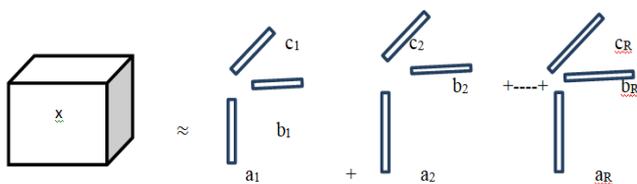
Such that  
 $B = UA_{(k)} = A_{(k)}U$ ----- (1)

These two operations will be used to generalize the standard singular value decomposition for rotation of robot.

A 3<sup>rd</sup> - order Tensor,  $A_{ijk}$ , is defined as a sum of vector outer product, denoted  $\circ$ , that equal or approximately equal  $A$ ,

For  $R = \text{rank}(A)$   
 $A = \sum_{r=1}^R a_r \circ b_r \circ c_r$ ----- (2)

And for  $R < \text{rank}(A)$   
 $A \approx \sum_{r=1}^R a_r \circ b_r \circ c_r$ ----- (3)



The vectors of composition  $a_r, b_r,$  and  $c_r$  for  $1 < r \leq R$ , are often compiled into the columns of the matrices  $A, B$  and  $C$  respectively;

$$A = [A, B, C]$$
----- (4)

This type of matrix multiplication is often denoted by the  $\odot$  symbol.

For instance,

$$A \odot B = [a_1 | a_2 | a_3 | \dots | a_n] \odot [b_1 | b_2 | b_3 | \dots | b_n] = [a_1 \circ b_1 | a_2 \circ b_2 | a_3 \circ b_3 | \dots | a_n \circ b_n]$$
----- (5)

The Rotation of Industrial Robot (KUKA KR 16)

A robotic matrix  $M$ , we would like to approximate it as well as possible with another robotic matrix.  $M$  of a lower rank robotic matrix, minimizing the norm of the difference between the two robotics matrices;

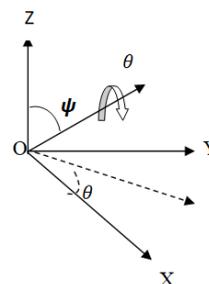
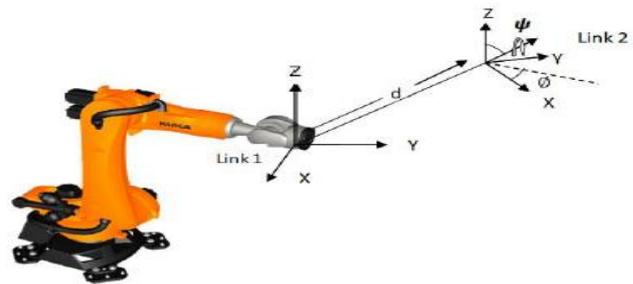
$$\text{Min } \|M - \hat{M}\| \text{ with } \hat{M} = AB^T$$

Interesting an invertible robotic rotation  $R$  together with its inverse  $R^{-1}$  between robotic function  $A$  and robotic function  $B^T$  and absorbing robotic  $R$  on the left with  $A$  and  $R^{-1}$  on the right with robotic function  $B^T$  we can again construct two matrix  $\tilde{A}$  and  $\tilde{B}^T$

$$\hat{M} = AB^T = ARR^{-1}B^T = (AR)(R^{-1}B^T) = (AR)(BR^{-T})^T = \tilde{A}\tilde{B}^T$$
----- (6)

It means that the rank decomposition of a robotic matrix is generally highly non-unique.

A rotation matrix for robot is used to rotate a robotic motion properties this rotation matrix represented by a 3<sup>rd</sup> order tensor of KUKA Robot, about an axis or axes. This process is done to orientation ally average robotic motion over axes that are known to allow robotic free rotation. The robotic rotation matrix commonly used to do this is below for KUKA Robot and the corresponding axes of robotic rotation of KR16-2.[3] [4] [6] [8]



It is immediately evident that the robotic rotation matrix gets complex quickly when robotic rotation an object in 3 dimensions around 3 rotational axes for KUKA Robot.[3][5][7]

If final matrix of Robotic Rotation Function (RRF) in Rotation in three dimensions is given by,

$$[R = [R_1 \odot R_2 \odot R_3]]$$
----- (7)
$$\begin{bmatrix} \cos(\psi) \cos(\theta) \cos(\phi) - \cos(\psi) \sin(\theta) \sin(\phi) & \cos(\psi) \cos(\theta) \sin(\phi) + \cos(\psi) \sin(\theta) \cos(\phi) & \cos(\psi) \sin(\theta) \\ \sin(\psi) \cos(\theta) \cos(\phi) - \sin(\psi) \sin(\theta) \sin(\phi) & \sin(\psi) \cos(\theta) \sin(\phi) + \sin(\psi) \sin(\theta) \cos(\phi) & \sin(\psi) \sin(\theta) \\ -\sin(\psi) \sin(\theta) \cos(\phi) & \sin(\psi) \sin(\theta) \sin(\phi) & \cos(\psi) \end{bmatrix}$$

$$R = \begin{bmatrix} \cos(\psi) \cos(\theta) \cos(\phi) - \cos(\psi) \sin(\theta) \sin(\phi) & \cos(\psi) \cos(\theta) \sin(\phi) + \cos(\psi) \sin(\theta) \cos(\phi) & \cos(\psi) \sin(\theta) \\ \sin(\psi) \cos(\theta) \cos(\phi) - \sin(\psi) \sin(\theta) \sin(\phi) & \sin(\psi) \cos(\theta) \sin(\phi) + \sin(\psi) \sin(\theta) \cos(\phi) & \sin(\psi) \sin(\theta) \\ -\sin(\psi) \sin(\theta) \cos(\phi) & \sin(\psi) \sin(\theta) \sin(\phi) & \cos(\psi) \end{bmatrix}$$

$$\begin{bmatrix} \cos(\psi) \cos(\theta) \cos(\phi) - \cos(\psi) \sin(\theta) \sin(\phi) & \cos(\psi) \cos(\theta) \sin(\phi) + \cos(\psi) \sin(\theta) \cos(\phi) & \cos(\psi) \sin(\theta) \\ \sin(\psi) \cos(\theta) \cos(\phi) - \sin(\psi) \sin(\theta) \sin(\phi) & \sin(\psi) \cos(\theta) \sin(\phi) + \sin(\psi) \sin(\theta) \cos(\phi) & \sin(\psi) \sin(\theta) \\ -\sin(\psi) \sin(\theta) \cos(\phi) & \sin(\psi) \sin(\theta) \sin(\phi) & \cos(\psi) \end{bmatrix}$$

$$R = \begin{bmatrix} C\psi c\theta c\phi - C\psi s\theta s\phi & -C\psi c\theta s\phi - C\psi s\theta c\phi & C\psi s\theta \\ S\psi c\theta c\phi - S\psi s\theta s\phi & -S\psi c\theta s\phi - S\psi s\theta c\phi & S\psi s\theta \\ -C\psi s\theta c\phi & S\psi s\theta s\phi & C\psi \end{bmatrix}$$

$$R = \begin{bmatrix} C\psi s\theta c\phi + C\psi c\theta s\phi & -C\psi c\theta c\phi - S\psi s\theta s\phi & -C\psi s\theta \\ -C\psi s\theta s\phi & S\psi s\theta c\phi & S\psi c\theta \end{bmatrix}$$



$C\psi s\theta$

As per above mentioned content, we refer to robotic rotating a 3rd order tensor,

$$A_{ijk} = A^{qrs} R_{qi} R_{rj} R_{sk} \text{-----}(8)$$

#### IV. CONCLUSION

On the basis of definition of modal matrix and tensor, we have defined a modal product between a robot rotation matrix and a tensor and found the rank of decomposition of robotic matrix is generally highly non unique. Further on account of investigations of industrial robot and tensors, it has been obtained that the route of industrial robot is implemented by third order tensor. Hence we are in position to conclude that the tensor model of industrial robot provides the alternate way to find out the solution in comparison to other existing robotic path implementations.

#### REFERENCES

1. G.H. Golub and C.F. Van Loan, Matrix Computations, (The Johns Hopkins University Press 2013).
2. Hongfei Wang et al, "Quantitative spectral and orientational analysis in surface sum frequency generation vibrational spectroscopy (SFG-VS)", International Reviews in Physical Chemistry (2005), (24) no. 2, 191-256.
3. Kolda, T.G. and Bader, B.W., "Tensor Decompositions and Applications", SIAM Review (2009), (51) no. 3, 455-500.
4. Carroll, J. Douglas and Chang, Jih-Jie, "Analysis of individual differences in multidimensional scaling via an n-way generalization of Eckart-Young decomposition", Psychometrika (1970), (35) no. 3, 283-319.
5. P. Paatero, "A weighted non-negative least squares algorithm for three-way PARAFAC factor analysis", Chemometrics and Intelligent Laboratory Systems (1997), (38) no. 2, 223-242.
6. N. K. M. Faber and R. Bro, and P. K. Hopke, "Recent developments in CANDECOMP/PARAFAC algorithms: A critical review", Chemometrics and Intelligent Laboratory Systems (2003), (65), 119-137.
7. J. B. Kruskal, "Rank, decomposition, and uniqueness for 3-way and N-way arrays, in Multiway Data Analysis", 7-18.
8. Tyler Ueltschi, "Third-Order Tensor Decompositions and Their Application in Quantum Chemistry, in University of Puget Sound Tacoma, Washington, USA", 1-9.

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**Dr Alok Mishra** having 25 years of experience in field of academics he published more than 50 research papers in National and International journals of repute. Dr. Mishra has also presented 40 research papers in National and International conferences. He has written 4 books. He granted two Patents, handled two DST Projects and guided multiple M Tech & Ph. D Scholars. He is member of

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