

# Double Domination Number of Some Families of Graph

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**Abstract:** In a graph  $G = (V, E)$  each vertex is said to dominate every vertex in its closed neighborhood. In a graph  $G$ , if  $S$  is a subset of  $V$  then  $S$  is a double dominating set of  $G$  if every vertex in  $V$  is dominated by at least two vertices in  $S$ . The smallest cardinality of a double dominating set is called the double domination number  $\gamma_{x2}(G)$ . [4]. In this paper, we computed some relations between double domination number, domination number, number of vertices ( $n$ ) and maximum degree ( $\Delta$ ) of Helm graph, Friendship graph, Ladder graph, Circular Ladder graph, Barbell graph, Gear graph and Firecracker graph.

**Keywords:** dominating set, domination number, double dominating set, double domination number. We denote  $n, \Delta$  respectively by number of vertices, maximum degree of a graph  $G$ .

## I. INTRODUCTION

Frank Harary and T. W. Haynes defined and studied the concept of double domination in graphs. Domination and double domination numbers are defined only for graphs with non- isolated vertices. Every graph with no-isolated vertex has a double dominating set. In a graph  $G = (V, E)$ , a subset  $S \subseteq V$  is a dominating set of  $G$  if every vertex  $v$  of  $V - S$  has a neighbor in  $S$ . The domination number  $\gamma(G)$  is the minimum size of a dominating set of vertices in  $G$ . In a graph  $G$  with vertex set  $V$  and edge set  $E$ , a subset  $S$  of  $V$  is a double dominating set of  $G$  if every vertex  $v \in V$ , is in  $V - S$  and has at least two neighbors in  $S$ . The smallest cardinality of a double dominating set of  $G$  is known as the double domination number  $\gamma_{x2}$ . [6] [4] [8].

## II. DEFINITIONS

**2.1. Dominating set:** A non-empty set  $D \subseteq V$  of a graph  $G$  is a “dominating set” of  $G$  if every vertex in  $V - D$  is adjacent to some vertex in  $D$ . [1]

**2.2. Domination number:** The “domination number”  $\gamma(G)$  is the number of vertices in a smallest dominating set for  $G$ . [7]

**2.3. Double dominating set:** A subset  $S \subseteq V(G)$  is a “double dominating set” for  $G$  if  $S$  dominates every vertex of  $G$  at least twice. [4]

**2.4. Double domination number:** The “double domination number”  $\gamma_{x2}(G)$  is the smallest cardinality of a double dominating set of  $G$ . [4]

Example:

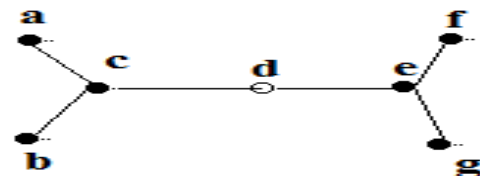


Fig.1

Now consider the graph Fig.1. The set  $S = \{a, b, c, e, f, g\}$  forms a double dominating set of  $G$  and double domination number  $\gamma_{x2} = 6$ .

## III. DEFINITIONS OF SOME FAMILIES OF GRAPHS

**3.1. “Helm Graph”:** It is designed from a wheel graph with  $n$  vertices by adjoining a pendant edge at each node of the cycle and denoted by  $H_n$ . [8]

**3.2. “Friendship Graph”:** It is constructed by joining  $n$  copies of the cycle graph  $C_3$  with a common vertex and represented by  $F_n$ . It is a planar undirected graph with  $2n+1$  vertices and  $3n$  edges. [7]

**3.3. Ladder Graph:** The “Ladder graph”  $L_n$  can be obtained as the Cartesian product of two path graphs  $P_2$  and  $P_n$ , where  $P_n$  is a path graph. The Ladder graph  $L_n$  is a planar undirected graph with  $2n$  vertices and  $3n-2$  edges. [7]

**3.4. “Circular Ladder graph”  $CL_n$ :** It is a connected planar and Hamiltonian graph which is constructible by connecting the four 2-degree vertices in a straight way or by the Cartesian product of a cycle of length  $n \geq 3$  and an edge. In symbols,  $CL_n = C_n \times P_2$ . It has  $2n$  nodes and  $3n$  edges. They are the polyhedral graphs of prisms, so they are more commonly called prism graphs. [7]

**3.5. “n-Barbell graph”  $B_n$ :** It is obtained by connecting two copies of a complete graph  $K_n$  by a cut edge. It has  $2n$  vertices and  $2n+1$  edges. [8]

**3.6. “Gear graph”  $G_n$ :** It is a wheel graph  $W_n$  with a vertex added between every pair of adjacency vertices of the  $n$ -cycle. It has  $2n+1$  vertices and  $3n$  edges. [3]

**3.7. “Firecracker Graph”  $F(n,k)$ :** It is framed by the concatenation of  $nk$ -stars by linking one leaf from each. It has order  $nk$  and size  $nk-1$ . [2]

Manuscript received on July 12, 2020.  
Revised Manuscript received on July 22, 2020.  
Manuscript published on July 30, 2020.

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**IV RESULTS ON DOUBLE DOMINATION OF SOME FAMILIES OF GRAPHS**

**Theorem 4.1:**

For any Helm graph  $H_n$  ( $n \geq 3$ ), double domination number

$$\gamma_{\times 2}(H_n) = \begin{cases} \Delta, & \text{if } n = 3 \\ \Delta + 1, & \text{if } n \geq 4 \end{cases}$$

**Proof:**

Case (i) for  $n=3$ ,

The Helm graph  $H_n$  has a single universal vertex  $v$  connecting all the vertices of the cycle with vertex set  $\{x_1, x_2, x_3\}$  and each node of the cycle adjoining a pendant edge. Let  $y_1, y_2, y_3$  be the pendant vertices. Here vertex  $v$  and pendant vertices are enough to dominate all the vertices of graph  $H_n$  twice. Here,  $\Delta = n+1$ . Let us consider the double dominating set of  $H_n = \{v, y_1, y_2, y_3\}$ . Thus double domination number  $\gamma_{\times 2}(H_n)$  is  $\{v, y_1, y_2, y_3\}$ . Hence  $\gamma_{\times 2}(H_n) = \Delta$ .

Case (ii) for  $n \geq 4$ ,

The Helm graph  $H_n$  has a universal vertex  $v$  connecting the vertices of the cycle having vertex set  $\{x_1, x_2, x_3, \dots, x_n\}$  and each node of the cycle adjoining a pendant edge. Let  $\{y_1, y_2, y_3, \dots, y_n\}$  be the pendant vertices. Here vertex  $v$  and pendant vertices will dominate the remaining vertices of graph  $H_n$  twice. Here  $\Delta = n$ . Let us consider the double dominating set of  $H_n = \{v, y_1, y_2, y_3, \dots, y_n\}$ . Thus double domination number is  $\gamma_{\times 2}(H_n) = \{v, y_1, y_2, y_3, \dots, y_n\}$ .

Hence  $\gamma_{\times 2}(H_n) = \Delta + 1$ .

**Theorem 4.2:**

For any Friendship graph  $F_n$  ( $n \geq 2$ ), double domination number  $\gamma_{\times 2}(F_n) = 1 + \frac{\Delta}{2}$ .

**Proof:** The Friendship graph  $F_n$  has  $2n+1$  vertices and  $3n$  edges. Let the vertex set be  $\{v, v_1, v_2, v_3, \dots, v_{2n}\}$  and  $\Delta = 2n \Rightarrow n = \frac{\Delta}{2}$ . Here vertex  $v$  and  $n$  vertices dominates all the vertices of the graph  $F_n$  twice. Let us consider the double dominating set of  $F_n = \{v, v_1, v_3, v_5, \dots, v_n\}$ . Thus double domination number is

$$\gamma_{\times 2}(F_n) = \{v, v_1, v_3, v_5, \dots, v_n\}. \text{ Hence } \gamma_{\times 2}(F_n) = 1 + \frac{\Delta}{2}.$$

**Theorem 4.3:**

For any Ladder graph  $L_n$  ( $n \geq 2$ ), double domination number  $\gamma_{\times 2}(L_n) = n$ .

**Proof:** The Ladder graph  $L_n$  has  $2n$  nodes and  $(3n-2)$  edges. Let the vertex set be  $\{x_1, x_2, x_3, \dots, x_n, y_1, y_2, y_3, \dots, y_n\}$  and the edge set be  $\{x_i y_i, x_i x_{i+1}, y_i y_{i+1}\}$  for  $1 \leq i \leq n$ . Let us consider the double dominating set of  $L_n = \{x_1, x_3, x_5, \dots, x_{n-1}, y_2, y_4, y_6, \dots, y_n\}$ . Thus double domination number is

$$\gamma_{\times 2}(L_n) = \{x_1, x_3, x_5, \dots, x_{n-1}, y_2, y_4, y_6, \dots, y_n\}.$$

$$\text{Hence } \gamma_{\times 2}(L_n) = \frac{n}{2} + \frac{n}{2} = n.$$

**Theorem 4.4:**

For any Circular Ladder graph  $CL_n$  ( $n \geq 3$ ), double domination number  $\gamma_{\times 2}(CL_n) = n$ .

**Proof:** Circular Ladder graph  $CL_n$  has  $2n$  nodes and  $3n$  edges with vertex set  $\{v_1, v_2, v_3, \dots, v_{2n}\}$ . Here  $n$  number of nodes is enough to dominate all the vertices of the graph  $CL_n$  twice. Let us consider the double dominating set of  $CL_n = \{v_1, v_2, v_3, \dots, v_n\}$ . Thus double domination number is  $\gamma_{\times 2}(CL_n) = \{v_1, v_2, v_3, \dots, v_n\}$ . Hence  $\gamma_{\times 2}(CL_n) = n$ .

**Theorem 4.5:**

For any Barbell graph  $B_n$  ( $n \geq 3$ ), double domination number  $\gamma_{\times 2}(B_n) = 4$ .

**Proof :** The Barbell graph  $B_n$  has  $2n$  vertices and  $2n+1$  edges with vertex set  $\{x_1, x_2, x_3, \dots, x_n, y_1, y_2, y_3, \dots, y_n\}$ . Since  $B_n$  is obtained by connecting two copies of complete graph. So, any two vertices say  $x_1$  and  $x_2$  dominates all the vertices of single copy of complete graph twice and since it has two copies. The double dominating set of  $B_n = \{x_1, x_2, y_1, y_2\}$ . Thus double domination number is  $\gamma_{\times 2}(B_n) = \{x_1, x_2, y_1, y_2\}$ .

Hence  $\gamma_{\times 2}(B_n) = 4$ .

**Theorem 4.6:**

For any Gear graph  $G_n$  ( $n \geq 3$ ), double domination number  $\gamma_{\times 2}(G_n) = \Delta$ .

**Proof :** The Gear graph has  $2n+1$  vertices with vertex set  $\{v, v_1, v_2, v_3, \dots, v_n, u_1, u_2, u_3, \dots, u_n\}$  and  $3n$  edges with  $\Delta = n$ . As  $G_n$  is constructed from the wheel graph  $W_n$  by adding a vertex between every pair of adjacent vertices of the  $n$ -cycle. Here the vertices of the cycle  $\{v_1, v_2, v_3, \dots, v_n\}$  dominates all the vertices of  $G_n$  twice. Let us consider the double dominating set of  $G_n = \{v_1, v_2, v_3, \dots, v_n\}$ . Thus double domination number is  $\gamma_{\times 2}(G_n) = \{v_1, v_2, v_3, \dots, v_n\}$ . Hence  $\gamma_{\times 2}(G_n) = \Delta$ .

**Theorem 4.7:**

For any Firecracker graph  $F(n, k)$  ( $n = 2, k \geq 3$ ), double domination number  $\gamma_{\times 2}[F(n, k)] = n \Delta$ .

**Proof:** The Firecracker graph  $F(n, k)$  ( $n = 2, k \geq 3$ ), has  $nk$  vertices and  $nk-1$  edges. The graph  $F(n, k)$  consists of  $(k-2)$  pendant vertices in each star. Let the vertex set of  $F(n, k)$  be  $\{v_1, v_2, v_3, \dots, v_k, u_1, u_2, u_3, \dots, u_k\}$  and  $\Delta = (k-1)$ . Here all pendant vertices and one of the remaining vertex dominates all the vertices of the graph  $F(n, k)$  twice, for single star. Since the graph  $F(n, k)$  has two stars, let us consider the double dominating set of  $F(n, k) = \{v_1, v_2, v_3, \dots, v_{k-2}, v_{k-1}, u_1, u_2, u_3, \dots, u_{k-2}, u_{k-1}\}$ . Thus double domination number is

$$\gamma_{\times 2}[F(n, k)] = \{v_1, v_2, v_3, \dots, v_{k-2}, v_{k-1}, u_1, u_2, u_3, \dots, u_{k-2}, u_{k-1}\}.$$

Hence  $\gamma_{\times 2}[F(n, k)] = 2(k-1) = n\Delta$ .



**Theorem 4.8:**

For any Firecracker graph  $F(n,k)$  with odd values of  $n$  ( $n \geq 3, k \geq 4$ ), double domination number

$$\gamma_{x2} [F(n,k)] = \left\lceil nk - \left(\frac{3n-1}{2}\right) \right\rceil.$$

**Proof:** The Firecracker graph  $F(n,k)$  ( $n \geq 3, k \geq 4$ ), has  $nk$  vertices and  $nk-1$  edges. The graph  $F(n,k)$  consists of  $(k-2)$  pendant vertices in each star. Let the vertex set of  $F(n,k)$  be  $\{u_1, u_2, u_3, \dots, u_k, v_1, v_2, v_3, \dots, v_k, w_1, w_2, w_3, \dots, w_k, x_1, x_2, x_3, \dots, x_k, y_1, y_2, y_3, \dots, y_k, k_1, k_2, k_3, \dots, k_{nk}\}$ . Here, all pendant vertices and some of the other remaining vertices will dominate all the vertices of the graph  $F(n,k)$  twice. Let us consider double dominating set of  $F(n,k)$  as  $\{u_1, u_2, u_3, \dots, u_{k-2}, u_{k-1}, v_1, v_2, v_3, \dots, v_{k-2}, w_1, w_2, w_3, \dots, w_{k-2}, w_{k-1}, x_1, x_2, x_3, \dots, x_{k-2}, y_1, y_2, y_3, \dots, y_{k-2}, y_{k-1}, k_1, k_2, k_3, \dots, k_{nk-2}, k_{nk-1}\}$ . The double domination number is  $\gamma_{x2}[F(n,k)] = \{u_1, u_2, u_3, \dots, u_{k-2}, u_{k-1}, v_1, v_2, v_3, \dots, v_{k-2}, w_1, w_2, w_3, \dots, w_{k-2}, w_{k-1}, x_1, x_2, x_3, \dots, x_{k-2}, y_1, y_2, y_3, \dots, y_{k-2}, y_{k-1}, k_1, k_2, k_3, \dots, k_{nk-2}, k_{nk-1}\}$ . Thus we have

$$\gamma_{x2}[F(n,k)] = \{nk - 4, nk - 7, nk - 10, nk - 13, \dots\}.$$

Hence,  $\gamma_{x2}[F(n,k)] = \left\lceil nk - \left(\frac{3n-1}{2}\right) \right\rceil$ .

**Theorem 4.9:**

For any Firecracker graph  $F(n,k)$  with even values of  $n$  ( $n \geq 4, k \geq 4$ ), double domination number

$$\gamma_{x2} [F(n, k)] = \left\lceil nk - \left(\frac{3n-2}{2}\right) \right\rceil$$

**Proof:** The Firecracker graph  $F(n,k)$  ( $n \geq 4, k \geq 4$ ), has  $nk$  vertices and  $nk-1$  edges. The graph  $F(n,k)$  consists of  $(k-2)$  pendant vertices in each star.

Let the vertex set of  $F(n,k)$  be  $\{u_1, u_2, u_3, \dots, u_k, v_1, v_2, v_3, \dots, v_k, w_1, w_2, w_3, \dots, w_k, x_1, x_2, x_3, \dots, x_k, y_1, y_2, y_3, \dots, y_k, k_1, k_2, k_3, \dots, k_{nk}\}$ . Here all pendant vertices along with some of the rest of vertices will dominate all the vertices of the graph  $F(n,k)$  twice.

Let us consider double dominating set of  $F(n,k)$  as  $\{u_1, u_2, u_3, \dots, u_{k-1}, v_1, v_2, v_3, \dots, v_{k-2}, w_1, w_2, w_3, \dots, w_{k-1}, x_1, x_2, x_3, \dots, x_{k-2}, y_1, y_2, y_3, \dots, y_{k-1}, k_1, k_2, k_3, \dots, k_{nk-1}\}$ .

The double domination number is

$$\gamma_{x2}[F(n,k)] = \{u_1, u_2, u_3, \dots, u_{k-1}, v_1, v_2, v_3, \dots, v_{k-2}, w_1, w_2, w_3, \dots, w_{k-1}, x_1, x_2, x_3, \dots, x_{k-2}, y_1, y_2, y_3, \dots, y_{k-1}, k_1, k_2, k_3, \dots, k_{nk-1}\}.$$

Thus we have

$$\gamma_{x2} [F(n,k)] = \{nk - 5, nk - 8, nk - 11, nk - 14, \dots\}.$$

Hence,  $\gamma_{x2}[F(n,k)] = \left\lceil nk - \left(\frac{3n-2}{2}\right) \right\rceil$ .

**V. RELATION BETWEEN DOUBLE DOMINATION AND DOMINATION NUMBER.**

**Theorem 5.1:**

For any Helm graph  $H_n$  ( $n \geq 3$ ), double domination number  $\gamma_{x2}(H_n) = \gamma(H_n) + 1$ .

**Proof :** The Helm graph  $H_n$  has  $2n+1$  vertices and  $3n$  edges which consists of single universal vertex  $v$  connecting all the vertices of the cycle and each node of the cycle adjoining a pendant edge with vertex set  $\{v, x_1, x_2, x_3, x_4, \dots, x_{2n}\}$ . Here, vertex  $v$  and pendant vertices dominate all the other vertices of graph  $H_n$  twice. Let us consider the double dominating set of  $H_n = \{v, x_1, x_2, x_3, x_4, \dots, x_n\}$ . Thus double domination number is  $\gamma_{x2}(H_n) = \{v, x_1, x_2, x_3, x_4, \dots, x_n\}$ . Hence  $\gamma_{x2}(H_n) = n+1$ -----equation (1). Whereas, in a dominating set of  $H_n$  all the vertices of the cycle are enough to dominate all the vertices of the graph. Let us consider the dominating set of  $H_n = \{x_1, x_2, x_3, x_4, \dots, x_n\}$ . Thus domination number is  $\gamma(H_n) = \{x_1, x_2, x_3, x_4, \dots, x_n\}$ .

Hence  $\gamma(H_n) = n$ ----- equation (2). From equations (1) and (2),  $\gamma_{x2}(H_n) = \gamma(H_n) + 1$ .

**Theorem 5.2:**

For any Friendship graph  $F_n$  ( $n \geq 2$ ), double domination number  $\gamma_{x2}(F_n) = \gamma(F_n) + n$ .

**Proof:** The Friendship graph  $F_n$  has  $2n+1$  vertices and  $3n$  edges. Let the vertex set be  $\{v, v_1, v_2, v_3, \dots, v_{2n}\}$ . Here, vertex  $v$  and  $n$  vertices dominates all the vertices of the graph  $F_n$  twice. Let us consider the double dominating set of  $F_n = \{v, v_1, v_3, v_5, \dots, v_n\}$ . The double domination number is  $\gamma_{x2}(F_n) = \{v, v_1, v_3, v_5, \dots, v_n\}$ . Hence  $\gamma_{x2}(F_n) = n + 1$ ----- equation (1). Whereas, in a dominating set of  $F_n$  common vertex will dominate all the vertices of the graph. Let us consider the domination set of  $F_n = \{v\}$ . Thus domination number is  $\gamma(F_n) = \{v\}$ .

Hence  $\gamma(F_n) = 1$ ----- equation (2). From equations (1) and (2)  $\gamma_{x2}(F_n) = \gamma(F_n) + n$ .

**Theorem 5.3:**

For any Barbell graph  $B_n$  ( $n \geq 3$ ), double domination number  $\gamma_{x2}(B_n) = 2 \gamma(B_n)$ .

**Proof :** The Barbell graph  $B_n$  has  $2n$  vertices and  $2n+1$  edges with vertex set  $\{x_1, x_2, x_3, x_4, \dots, x_{2n}\}$ . Since  $B_n$  is obtained by connecting two copies of complete graph. In dominating set of  $B_n$  only two vertices dominate the remaining vertices of the graph. The dominating set of  $B_n = \{x_1, x_2\}$ . Thus domination number is  $\gamma(B_n) = \{x_1, x_2\}$ . Hence  $\gamma(B_n) = 2$ ----- equation (1). From theorem 4.5  $\gamma_{x2}(B_n) = 4$ ----- equation (2). From equations (1) and (2)  $\gamma_{x2}(B_n) = 2 \gamma(B_n)$ .

**Theorem 5.4 :**

For any Firecracker graph  $F(n, k)$  ( $n = 2, k \geq 3$ ), double domination number  $\gamma_{x2} [F(n,k)] = \gamma [F(n,k)] + 1$ .

**Proof :**  $F(n, k)$  ( $n = 2, k \geq 3$ ), has  $nk$  vertices and  $nk-1$  edges. It consists of  $(k-2)$  pendant vertices in each star. Let the vertex set of  $F(n,k)$  be  $\{v_1, v_2, v_3, \dots, v_k, u_1, u_2, u_3, \dots, u_k\}$  and  $\Delta = (k-1)$ . Here two vertices (i.e., one from each star) are enough to dominate all the vertices of the graph.



## Double Domination Number of Some Families of Graph

The dominating set of  $F(n,k) = \{v_1, u_1\}$ . Thus domination number is  $\gamma [F(n,k)] = \{v_1, u_1\}$ . Hence  $\gamma [F(n,k)] = 2 = n - \dots$  equation (1). From theorem 4.7

$\gamma_{x2} [F(n,k)] = n \Delta$ -----equation (2). From equations (1) and (2)  $\gamma_{x2} [F(n,k)] = \gamma [F(n,k)] \Delta$ .

### Theorem 5.5:

For any Firecracker graph  $F(n, k)$  where  $n$  is odd ( $n \geq 3, k \geq 4$ ), the double domination number is

$$\gamma_{x2}[F(n, k)] = \left[ k\gamma(F) - \left( \frac{3n-1}{2} \right) \right].$$

**Proof :** The Firecracker graph  $F(n,k)$  ( $n \geq 3, k \geq 4$ ), has  $nk$  vertices and  $nk-1$  edges. The graph  $F(n, k)$  consists of  $(k-2)$  pendant vertices in each star. Let the vertex set of  $F(n,k)$  be  $\{u_1, u_2, u_3, \dots, u_k, v_1, v_2, v_3, \dots, v_k, w_1, w_2, w_3, \dots, w_k, \dots, k_1, k_2, k_3, \dots, k_{nk}\}$ . Here  $n$  vertices (i.e., central vertex of each star) are enough to dominate all the vertices of the graph. The dominating set of  $F(n,k)$  is  $\{u_1, v_1, w_1, \dots, k_1\}$ . Thus the domination number is  $\gamma = \{u_1, v_1, w_1, \dots, k_1\}$ . Hence  $\gamma [F(n,k)] = n$  -----equation (1). From theorem 4.8

$\gamma_{x2}[F(n, k)] = \left[ nk - \left( \frac{3n-1}{2} \right) \right]$  ----- equation (2). From equations (1) and (2)

$$\gamma_{x2}[F(n, k)] = \left[ k\gamma(F) - \left( \frac{3n-1}{2} \right) \right].$$

### Theorem 5.6 :

Every Firecracker graph  $F(n,k)$  ( $n \geq 4, k \geq 4$ ), where  $n$  is even, the double domination number is

$$\gamma_{x2}[F(n, k)] = \left[ k\gamma[F] - \left( \frac{3n-2}{2} \right) \right].$$

**Proof :** The Firecracker graph  $F(n,k)$  ( $n \geq 4, k \geq 4$ ), has  $nk$  vertices and  $nk-1$  edges. The graph  $F(n, k)$  consists of  $(k-2)$  pendant vertices in each star. Let the vertex set of  $F(n,k)$  be  $\{u_1, u_2, u_3, \dots, u_k, v_1, v_2, v_3, \dots, v_k, w_1, w_2, w_3, \dots, w_k, \dots, k_1, k_2, k_3, \dots, k_{nk}\}$ . Here  $n$  vertices (i.e., central vertex of each star) are enough to dominate all the vertices of the graph. The dominating set of  $F(n,k)$  is  $\{u_1, v_1, w_1, \dots, k_1\}$ . Thus the domination number is  $\gamma = \{u_1, v_1, w_1, \dots, k_1\}$ . Hence  $\gamma (F(n,k)) = n$  ----- equation (1). From theorem 4.9

$\gamma_{x2}[F(n, k)] = \left[ nk - \left( \frac{3n-2}{2} \right) \right]$  ----- equation (2)

From equations (1) and (2)

$$\gamma_{x2}[F(n, k)] = \left[ k\gamma[F] - \left( \frac{3n-2}{2} \right) \right].$$

## VI. CONCLUSION

We have computed relation between double domination number, domination number, number of vertices and maximum degree of some graphs.

### REFERENCE:

1. K. Ameenai Bibi and R.Selvakumar, "The Inverse Split and Non-split Domination in Graphs", International Journal of Computer Applications (0975-8887), Volume 8-No 7, October 2010.
2. Ashaq Ali, Hifza Iqbal, Waqas Nazeer, Shin Min Kang, "On Topological Indices for the line graph of firecracker graph".

- International Journal of Pure and Applied Mathematics Volume 116 No. 4 2017, 1035-1042.
3. Ersin Aslan and Alpay Kirlangic, "Computing The Scattering Number and The Toughness for Gear Graphs", Bulletin of Society of Mathematicians Banja Luka, ISSN 0354-5792, ISSN 1986-521X(o) Vol. 18(2011), 5-15.
4. F. Harary and T.W.Haynes, "Double domination in graphs", Ars Combin. 55 April(2000) 201-213.
5. T.W.Haynes, S.Teresa. Hedetniemi and P.J.Slater, "Domination in Graphs": Advanced Topics (Marcel Dekker, New York, (1998).
6. Mustapha. Chellali, Abdelkader Khelladi and Frederic Maffray, "EXACT DOUBLE DOMINATION IN GRAPHS", Discussiones Mathematicae 291 Graph Theory 25 (2005), Volume :25, Issue:3, page 291-302, ISSN: 2083-5892.
7. From Wikipedia, the free encyclopedia <https://en.wikipedia.org/wiki/>.
8. Wolfram Math World, <https://mathworld.wolfram.com/>.

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