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Performance of Joint Quality Monitoring Schemes under Gaussian distribution

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Abstract Jointly monitoring the process mean and variance has become a well-known topic in statistical quality control literature after it is considered as a bivariate problem. Many joint monitoring schemes have been proposed by using the Shewhart, cumulative sum and exponentially weighted moving average techniques. In this paper, best performing schemes from each technique has been selected and compared for their performance using average run length properties. It was found that selection of better joint monitoring scheme based on the shift in mean and variance to be detected quickly. In particular, the Shewhart distance joint monitoring scheme performs well when there is larger shifts in mean, variance or in both. In addition, the Shewhart distance joint monitoring scheme performs specific when there is no shift in mean and decrease in variance. For the smaller shifts in mean, variance or in both, cumulative sum and exponentially weighted moving average joint monitoring schemes can be recommended. At this scenario exponentially weighted moving average joint monitoring scheme performs marginally better than the cumulative sum scheme.

Keywords: Average run length, Control chart, Cumulative sum, Exponentially weighted moving average, Joint monitoring scheme, Shewhart scheme

I. INTRODUCTION

In quality monitoring, control charts are commonly used to monitor the process mean and variance separately by plotting them on two separate charts. But the process monitoring was identified as a bi-variate problem by Gan because a change in the variance can affect the control limits of the mean chart [1,2]. In industry, a special cause can change both the mean and variance. A new machine in the packing industry may be the reason for some simultaneous shift in mean and variance. Another example for the simultaneous shift was given by Gan *et al.* in 2004 in the circuit manufacturing, that a defect in stencil caused shifts in both the mean and variance of the thickness of the solder paste printed onto circuit boards [3]. Subsequently many authors emphasized the importance of jointly monitoring the mean and variance simultaneously [4,5 & 6]. Therefore joint monitoring (JM) of process mean and variance has become famous and many joint monitoring schemes were introduced using Shewhart, Cumulative sum (CUSUM) and Exponentially weighted moving average (EWMA) techniques. In this paper three schemes developed under the JM techniques have been evaluated for their performances in detecting the shifts in mean and variance using average run length (ARL) properties.

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II. SCHEMES UNDER STUDY

One JM scheme from each technique - Shewhart, CUSUM and EWMA has been selected for the comparative study and the selected schemes are discussed in this section.

A. Shewhart Distance JM Scheme

Gan proposed two Shewhart JM schemes one with rectangular control region and the other with elliptical control region [1]. A Shewhart distance JM scheme (SS_d) was proposed in 2010 by standardizing the mean and variance as the variables U_t and V_t respectively, for the independently and identically normally distributed process characteristic X_{tj} where t is the sample number and j is the j^{th} unit of the sample and $j = 1, 2, \dots, n$ [7].

$$U_t = \frac{\bar{X}_t - \mu_0}{\sigma_0/\sqrt{n}} \quad \dots\dots\dots\dots\dots\dots\dots\dots \quad (1)$$

and

$$V_t = \Phi^{-1}\left[H\left(\frac{(n-1)S_t^2}{\sigma_0}; n-1\right)\right] \quad \dots\dots (2)$$

where,

$$H\left(\frac{(n-1)S_t^2}{\sigma_0}; n-1\right) = H(w; v) = P(W \le w) \quad for\ W \sim \chi_v^2 \dots . (3)$$

the chi-square distribution with v degrees of freedom. Here μ_0 is the process mean, σ_0 is the process standard deviation, $\bar{X}_t$ is the t^{th} sample mean and S_t^2 is the t^{th} sample variance. The SS_d scheme was set up by plotting the statistics D_t against the sample number t where,

$$D_t = \sqrt{U_t^2 + V_t^2} \dots\dots\dots\dots\dots\dots .. (4)$$

D_t^2 is distributed chi-squared with degrees of freedom 2 when the process is in-control. The upper control limit for this scheme is defined as

$$P(D^2 \le UCL_D^2) = \frac{1}{ARL}, for\ D^2 \sim \chi_2^2 \dots\dots (5).$$

A Shewhart Max JM scheme was proposed in 2004 by plotting the variable M_t against the sample number t where,

$$M_t = max[|U_t|, |V_t|] \dots\dots .. (6).$$

Razmy and Peiris evaluated all the available Shewhart JM schemes for their performance using ARL properties under different scenarios and found that in overall, the SS_d schemes perform best [8]. McCracken *et. al.* evaluated the control charts for simultaneous monitoring of unknown mean and variance of normally distributed processes and concluded that the SS_d schemes has the better ARL properties in detecting the shifts in mean and variance [9]. Therefore, SS_d scheme has been selected for comparative study under the Shewhart scheme in this paper.

B. Combined CUSUM JM Scheme

A combined CUSUM JM scheme with a rectangular control region (CC) was developed by Chang and Gan in 1995 [10]. This combined scheme consists of four CUSUM charts namely, upper and lower sided CUSUM mean charts, and upper and lower sided CUSUM variance charts. Each chart is obtained by plotting the desired statistic against the sample number t. Table 1 summarizes for each CUSUM chart what statistic is to be plotted against sample number. K_m, K_l and K_p are positive constant often called the reference value and $B_0 = b_0 = F_0 = f_0 = 0$. An out-of-control signal is issued when any of the upper-sided chart statistics exceed the UCL or any of the lower-sided chart statistics is less than the lower control limit (LCL). The optimum chart parameters of this scheme can be found in Chang and Gan [10].

C. Combined EWMA JM Scheme

A combined EWMA JM scheme with a rectangular control region (EE) was developed by Gan in 1995 [11]. This combined scheme consists of two EWMA charts namely, EWMA mean chart and EWMA variance chart. Each chart is obtained by plotting the desired statistic against the sample number t. Table 2 summarizes for each EWMA chart what statistic is to be plotted against sample number. E_0 is usually set at μ_0 and λ_m is a positive constant such that $0 < \lambda_m \le 1$ and it is selected based on the shift in the mean to be detected quickly. The optimum values for λ_m is discussed by Crowder [12]. e_0 is usually set at $E[log(S_t^2)]$ and λ_v is a positive constant such that $0 < \lambda_v \le 1$ and it is selected based on the shift in the variance to be detected quickly. The optimum values for λ_v is discussed by Chang and Gan [13]. An out-of-control signal is issued when any of the statistics E_t or e_t plots outside the rectangular control region bounded by the UCLs and LCLs of the EWMA mean and variance charts.

Table 1. Control Charts for the CUSUM JM Scheme

CUSUM	Chart	Statistics
Sample Mean	$Upper-sided$	$C_t = max[0, C_{t-1} + \bar{X}_t - K_m]$
Sample Mean	$Lower-sided$	$c_t = max[0, c_{t-1} + \bar{X}_t + K_m]$
Sample Variance	$Upper-sided$	$G_t = max[0, G_{t-1} + log(S_t^2) - K_l]$
Sample Variance	$Lower-sided$	$g_t = max[0, g_{t-1} + log(S_t^2) + K_p]$

Table 2. Control Charts for the CUSUM JM Scheme

EWMA	Statistics
Sample Mean	$E_t = (1-\lambda_m)E_{t-1} + \lambda_m \bar{X}_t$
Sample Variance	$e_t = (1-\lambda_v)e_{t-1} + \lambda_v log(S_t^2)$

III. METHODOLOGY

Some known normally distributed processes to be developed to compare the performance of these three JM schemes based on their out of control ARLs when there is shift in mean, variance or both. A process with in-control mean $\mu_0 = 0$ and variance $\sigma_0^2 = 1$ with sample size $n = 5$ was simulated for the study for easy understanding and comparison purpose.

The control region for the SS_d scheme is circular and for the CC and EE schemes are rectangular. The control limits for the simulated process were found for selected in-control ARLs of 250 and 370 for each scheme through derivation or simulations and given in Table 3. Simulations were performed in SAS using the normal random number generator RANNOR.

Various shifts in mean, variance or both were applied to the original simulated process. The investigated shift in mean is given in standard deviation unit as $\Delta\frac{\sigma_0}{\sqrt{n}}$ where the new process mean after shift is

$$\mu = \mu_0 + \Delta\frac{\sigma_0}{\sqrt{n}} \ldots\ldots\ldots\ldots\ldots (7).$$

The investigated shift in variance is given in standard deviation unit $\delta\sigma_0$ where the new process standard deviation after shift is

$$\sigma_1 = \delta\sigma_0 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (8).$$

These Δ and δ are the number of standard deviation shifts in mean and variance respectively with the investigated values of

$$\Delta = 0.0, 0.2, 0.4, 0.6, 1.0, 1.5, 3.0 \; and$$

$$\delta = 0.50, 0.75, 0.95, 1.00, 1.10, 1.25, 1.50, 3.00.$$

For each scheme under comparison, all Δ and δ combinations were applied and the run lengths were calculated. For each combination of Δ and δ, 1,000,000 runs were performed to estimate the out-of-control ARLs. The standard deviations of run length values were less than 1% of the estimated ARL. When there is a shift, the scheme that gives the lowest out-of-control ARL detects the shift quickly and it is the best scheme to use under that particular shift [14].

Table 3: Control limits of the Charting Schemes with In-Control ARLs of 250 and 370

Scheme	Control Region	Control Chart Parameters ARL =250	Control Chart Parameters ARL =370
SS_d	Circular	$UCL = 3.323$	$UCL = 3.439$
CC	Rectangular	$K_m = 0.224$ $K_l = 0.005$ $K_p = 0.666$ $UCL_{mean\ chart} = 2.268$ $LCL_{mean\ chart} = -2.268$ $UCL_{variance\ chart} = 4.006$ $LCL_{variance\ chart} = -5.054$	$K_m = 0.224$ $K_l = 0.005$ $K_p = 0.666$ $UCL_{mean\ chart} = 2.4221$ $LCL_{mean\ chart} = -2.4221$ $UCL_{variance\ chart} = 4.3241$ $LCL_{variance\ chart} = -5.4735$
EE	Rectangular	$\lambda_m = 0.134$ $\lambda_v = 0.106$ $e_0 = -0.270$ $UCL_{mean\ chart} = 0.345$ $LCL_{mean\ chart} = -0.345$ $UCL_{variance\ chart} = 0.215$ $LCL_{variance\ chart} = -0.867$	$\lambda_m = 0.120$ $\lambda_v = 0.100$ $e_0 = -0.270$ $UCL_{mean\ chart} = 0.3385$ $LCL_{mean\ chart} = -0.3385$ $UCL_{variance\ chart} = 0.2205$ $LCL_{variance\ chart} = -0.8772$

IV. RESULTS AND DISCUSSION

The ARL for different shifts for the in-control ARLs of 250 and 370 are given in Tables 4 and 5. A decrease in variance when there is no shift in mean $(\Delta = 0, \delta < 1.0)$ is a better state, a best scheme should provide maximum ARL at this state. The SS_d scheme performs exceptionally better than the CC and EE Schemes. On the other hand, the CC scheme performs marginally better than the EE scheme.

When there is a small increase in variance and there is no shift in mean $(\Delta = 0, \delta = 1.05, 1.1, 1.25)$, a best scheme should provide minimum ARL. In such cases, the EE and CC schemes perform exceptionally better than the SS_d Scheme. Of the EE and CC schemes, the EE scheme performs marginally better than the CC scheme. The same type of performance was observed when there is a decrease in variance and small shift in mean.

When there is smaller shift in mean $(\Delta \leq 1.5)$ and smaller increase in variance $(\delta \leq 1.25)$, the EE and CC schemes perform exceptionally better than the SS_d scheme. Just as before, the EE scheme performs marginally better than the CC scheme.

For any larger shift in variance $(\delta \geq 1.5)$, the SS_d scheme performs exceptionally better than the CC and EE Schemes. Among the CC and EE schemes, the CC scheme performs marginally better than the EE scheme.

For any larger shift in mean $(\Delta \geq 3)$, the SS_d scheme performs better than the CC and EE Schemes. As before, the CC scheme performs marginally better than the EE scheme.

V. CONCLUSION

The use of JM scheme is case specific and it has to be decided based on the shift in mean and variance to be detected. Table 6 provides a guide to use appropriate schemes based on the shifts to be detected quickly.

Table 4. Average Run Lengths of JM Schemes with Respect to the different shifts in Mean (Δ) and Variance (δ) for the in-control ARL of 250

Δ	δ	SS_d	CC	EE	Δ	δ	SS_d	CC	EE
0	0.5	128.9	5.88	5.79	1	0.5	64	5.79	5.7
0	0.75	451.9	24.6	21.91	1	0.75	128.6	10.02	9.66
0	0.95	370.2	276.1	236.01	1	0.95	65.1	10.53	10.18
0	1	249.3	250.07	250.28	1	1	49.8	10.48	10.16
0	1.05	156.8	138.99	136.57	1	1.05	37.3	10.36	10.01
0	1.1	97	71.07	67.87	1	1.1	28.7	10.17	9.87
0	1.25	28.1	19.26	18.95	1	1.25	13.6	8.87	8.71
0	1.5	7.4	8.05	8.15	1	1.5	5.4	6.36	6.38
0	3	1.2	2.46	2.57	1	3	1.2	2.42	2.52
0.2	0.5	124.7	5.89	5.79	1.5	0.5	27.9	4.94	4.89
0.2	0.75	424.9	24.56	21.75	1.5	0.75	39.2	5.72	5.65
0.2	0.95	331.5	169.8	133.86	1.5	0.95	21.9	5.8	5.76
0.2	1	223.9	147.02	129.51	1.5	1	18.2	5.81	5.76
0.2	1.05	143	96.76	88.46	1.5	1.05	15.2	5.83	5.7
0.2	1.1	89.7	58.11	53.89	1.5	1.1	12.6	5.82	5.75
0.2	1.25	27.1	18.36	18	1.5	1.25	7.7	5.6	5.66
0.2	1.5	7.3	8	8.07	1.5	1.5	4	5	5.04
0.2	3	1.2	2.47	2.57	1.5	3	1.2	2.36	2.46
0.4	0.5	114	5.88	5.79	3	0.5	1.8	2.54	2.59
0.4	0.75	364.5	22.23	20.46	3	0.75	2.3	2.56	2.6
0.4	0.95	244.8	62.6	51.58	3	0.95	2.3	2.59	2.63
0.4	1	170.2	56.36	48.83	3	1	2.3	2.6	2.64
0.4	1.05	111.3	46.71	41.5	3	1.05	2.2	2.61	2.64
0.4	1.1	73.4	35.91	32.71	3	1.1	2.2	2.62	2.65
0.4	1.25	24.3	16.16	15.84	3	1.25	2	2.65	2.68
0.4	1.5	7	7.73	7.82	3	1.5	1.8	2.68	2.72
0.4	3	1.2	2.46	2.57	3	3	1.1	2.1	2.16
0.6	0.5	99.6	5.88	5.79					
0.6	0.75	279.3	18.7	16.93					
0.6	0.95	162.8	27.27	24.45					
0.6	1	115.9	26.05	23.67					
0.6	1.05	79.9	24.21	22.28					

0.6	1.1	55.4	21.46		20.06						
0.6	1.25	20.6	13.43		13.14						
0.6	1.5	6.5	7.34		7.41						
0.6	3	1.2	2.45		2.55						

Table 5. Average Run Lengths of JM Schemes with Respect to the different shifts in Mean (Δ) and Variance (δ) for the in-control ARL of 370

Δ	δ	SS_d	CC	EE		Δ	δ	SS_d	CC	EE
0	0.5	192.7	6.3	6.23		1	0.5	95.7	6.23	6.16
0	0.75	686.7	27.07	24.37		1	0.75	192.5	10.77	10.54
0	0.95	564	405.53	331.51		1	0.95	90.4	11.26	11.09
0	1	370.9	370.02	370.77		1	1	67.5	11.23	10.98
0	1.05	224.1	188.87	186.01		1	1.05	49.5	10.98	10.88
0	1.1	133.6	88.74	84.16		1	1.1	37	10.86	10.75
0	1.25	35.9	21.6	21.22		1	1.25	16.4	9.55	9.62
0	1.5	8.6	8.77	8.89		1	1.5	6.2	6.82	6.98
0	3	1.2	2.65	2.76		1	3	1.2	2.59	2.72
0.2	0.5	187.6	6.31	6.24		1.5	0.5	41.2	5.28	5.3
0.2	0.75	653.1	26.77	24.35		1.5	0.75	56	6.09	6.12
0.2	0.95	500.8	226.39	176.66		1.5	0.95	28.7	6.15	6.21
0.2	1	330.2	193.21	166.54		1.5	1	23.4	6.17	6.23
0.2	1.05	203.1	123.89	112.41		1.5	1.05	19	6.18	6.23
0.2	1.1	124.4	70.85	65.66		1.5	1.1	15.5	6.17	6.23
0.2	1.25	34.3	20.5	20.14		1.5	1.25	9.1	6.04	6.15
0.2	1.5	8.5	8.68	8.83		1.5	1.5	4.5	5.36	5.5
0.2	3	1.2	2.64	2.77		1.5	3	1.2	2.53	2.64
0.4	0.5	172.3	6.31	6.27		3	0.5	2.1	2.73	2.85
0.4	0.75	555.1	25.63	22.89		3	0.75	2.7	2.71	2.83
0.4	0.95	363.3	73.41	59.98		3	0.95	2.6	2.73	2.84
0.4	1	244.1	65.78	56.58		3	1	2.6	2.74	2.84
0.4	1.05	156.7	54.02	47.87		3	1.05	2.5	2.74	2.85
0.4	1.1	100.1	41.07	38.21		3	1.1	2.4	2.75	2.86
0.4	1.25	30.6	17.99	17.7		3	1.25	2.2	2.78	2.88
0.4	1.5	8.1	8.39	8.52		3	1.5	1.9	2.82	2.93
0.4	3	1.2	2.64	2.77		3	3	1.1	2.23	2.34
0.6	0.5	150.3	6.31	6.23						
0.6	0.75	424.7	23.43	18.84						
0.6	0.95	237	44.79	27.13						
0.6	1	164.6	41.73	26.29						
0.6	1.05	110.6	37.48	24.81						
0.6	1.1	74.6	30.93	22.6						
0.6	1.25	25.6	16.46	14.63						

0.6	1.5	7.6	8.2	8.12						
0.6	3	1.2	2.63	2.75						

Table 6. Recommended JM Schemes for Detecting Various Shifts in Mean and Variance

	Decrease in Variance $(\delta < 1)$	**Small increase in variance** $(1 < \delta \leq 1.25)$	**Larger Increase in variance** $(\delta \geq 1.25)$
In-Control Mean $(\Delta = 0)$	SS_d	EE_1, CC_2	SS_d
Small Shift in Mean $(0 < \Delta \leq 1.5)$	EE_1, CC_2	EE_1, CC_2	SS_d
Larger Shift in Mean $(\Delta \geq 1.5)$	SS_d	SS_d	SS_d

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