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An Inventory Model based on Imperfect Production with Shortage Backorder

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Abstract: In this paper, we discussed about the imperfect items. In practice items may get damaged due to production or transportation conditions. Each lot receives some imperfect items. This model also considers the effects of business strategies such as optimal order size of raw materials, production rates and unit production costs, and idle time in different areas on the cooperation of marketing systems. The model can be used in industries such as textiles and footwear, chemicals, food. We develop an inventory model based on imperfect products and shortages. We consider demand is constant and continuous. Purpose of this study is not only to find the retailer`s optimal replenishment policies but also to minimize the total average cost. Finally, a numerical example is presented to illustrate the proposed model and sensitivity analysis of the optimal solution concerning parameters is carried out using the Mathematica 10.0 software.

Keywords: Demand, Deterioration, Imperfect items, Inventory, Shortage Backordering.

I. INTRODUCTION

In the past few years, when creating an inventory policy, many inventory researchers assumed that 100% of the right quality items were present in each ordered item. But, this is not the case in real life. The presence of incomplete details in ordered lots is due to natural disasters, infection damage or breakage, and many other reasons. Therefore, the manufacturer received may include some defective items. Nowadays, there is a trend to develop inventory policy for real-life situations. Therefore, when developing an inventory control policy, one should develop a model assuming that each lot does not contain 100% correct items. [1]proposed a model based on the exponentially decaying inventory. [2] discussed a method to find minimum cost allocations of screening inspection efforts to justify both quality requirements and a direct cost of outgoing defectives. [3]discussed an inventory model in which the percentage of defective items of the lot is a random variable with known probability distributions. [4] developed two mathematical models with constant demand and deterioration but different replenishment interval in the first model. They assume replenishment orders place at the same range and in second model order placed at a different interval. [5] developed a model with deterioration and shortage for the situation of fixed cycle time and different levels of order quantities for the time proportional demand. [6] discussed an EOQ model by considering the permissible delay in payment. [7]proposed a model on products with imperfect quality. They assumed two policies Blind purchase and selective purchase. In blind purchase, the merchandise purchased without inspection but in particular purchase products with good quality are used to buy. An inventory model with an economical production cycle in which they consider the imperfect production process is proposed by [8]. [9] discussed a model based on defective items in this model. They believed that defective units could not use, and faulty units replace these units. In this direction, so many researchers work on deteriorating items and imperfect quality items [10]–[14].[15] derived an inventory model with an optimal replenishment policy that minimizes the total cost per unit time. [16]discussed a four-layer green supply chain inventory model composed of a raw- materials supplier, a manufacturer, and a retailer in a fixed time horizon of one year where total profit performed in some cycles of the same lengths. They also considered a two-level credit period. [17]developed two inventory models for two different operational policies. In the first policy, rework is done, but shortages are not allowed. On the other hand, in another system, revise is done with shortages. The present paper provides an approach to study the problem of a retailer dealing with imperfect quality deteriorating items with the lack. Shortage fully backlogged. It assumed that the demand rate is less than the screening rate. The rest of the paper is structured as follows: Section 2, we summarize the assumptions and notations required to state the problem. In Section 3, we present a general mathematical formulation for the challenge, while in Section 4, we formulate some solution methodologies to solve the model. In section 5, we have illustrated the model by a numerical example, and sensitivity analysis has carried out concerning some associated parameters. Finally, section 6 discusses the conclusion.

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II. ASSUMPTIONS AND NOTATIONS

A. Assumptions

In developing the mathematical models of the inventory system the following assumptions are used

- The demand rate is deterministic and constant.
- Lead time is zero.
- Infinite number of replenishment.
- Screening rate is greater then demand rate.
- Shortage is allowed and is completely backlogged.
- Imperfect items are free from deterioration.
- Each lot received has a random ration of defective items, *x*, with a known probability density function f(*x*).

B. Notations

The following notations are used to develop the model:

Q	Ordering Cost
C	Retailer`s purchasing Cost
H	Holding cost per unit time
α	Deterioration rate
λ	Screening rate in units/ unit time
b	Unit screening cost
t_s	Screening time
t_q	time for fulfill shortages, tq = y/D
K	Order quantity
T	Length of replenishment cycle
D	Demand rate / unit time
y	Maximum backorder level
p	Selling price per unit
$f(x)$	Probability density function of x
E(x)	Expected value of $x, E(x)\ \int_a^b xf(x)dx$, 0 < a < b<1
C_2	Shortage cost / unit / unit time
S_c	Salvage cost / defective unit
$I_1(t)$	Inventory level $(0, t_s)$
$I_2(t)$	Inventory level in (t_s, t_p)

III. MATHEMATICAL MODEL

We consider the *K* lot size issue dispatched to the retailer instantaneously with a purchasing and ordering cost, *C* / units, and *Q* /units, respectively. It assumed that whatever quantity received contains some amount of defective products *x* with a probability density function, *f*(*x*). All the faulty products found are kept in stock and sold when the cycle finished, with some discount price of C_s/units, C_s< C. Let α proportion of inventory lost/ time due to declination of items. It assumed that as screening completed, the backlogged demand gets starts. In (0, t_s), because of demand and deterioration in items, inventory level decreased. At times t_s, with the screening process's completion, the inventory level falls by the expected quantity of damaged items. Again in the interval (t_s, t_p), the inventory level tends to zero due to the effect of demand and deterioration. When inventory level reaches 0, the shortage starts till the new lot size received. With a new lot, the lack gets fulfilled, and the new process of production begins.

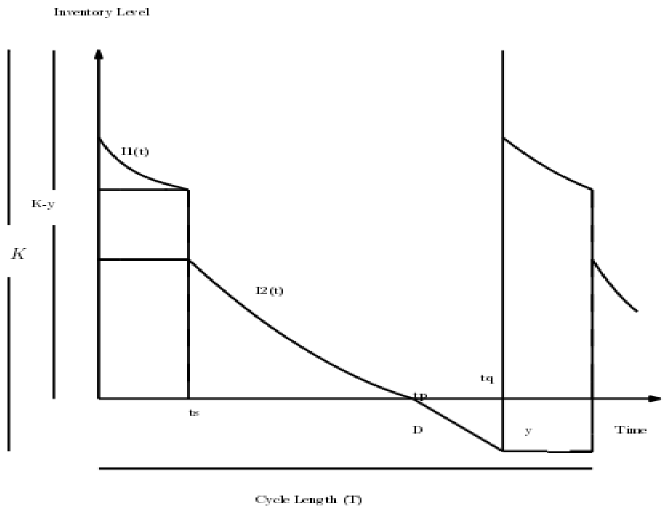

Fig.1. Graphical representation of cycle length T.

Let I_1 (t) be the inventory level during period ($0 \le t \le t_s$)

Differential equation for this period is

$$\frac{dI_1(t)}{dt} + \alpha I_1(t) = -D;\ 0 \le t \le t_s \quad (1)$$

With boundary condition at t = 0, $I_1(0) = K$

Let I_2 (t) be the inventory level during period ($t_s \le t \le t_p$)

Differential equation for this period is

$$\frac{dI_2(t)}{dt} + \alpha I_2(t) = -D;\ t_s \le t \le t_p \quad (2)$$

With boundary condition at t = t_s, $I_2(t_s) = I_{eff}(t_s)$

The amount of backlogged shortage during the interval ($t_p \le t \le t_q$) satisfies the differential equation

$$\frac{dI_3(t)}{dt} = -yD \quad (3)$$

With boundary condition at t = t_p, $I_3(t_p) = 0$

Solution of equation (1) by using boundary condition at t = 0, $I_1(0) = K$.

$$I_1(t) = \frac{D}{\alpha}(e^{-\alpha t} - 1) + Ke^{-\alpha t} \quad (4)$$

Once the screening process finished, the number of imperfect product at time t_s and backorder are xK and y respectively.

Further, the effective inventory level at t =t_s, after removing the defective items and backorders is given by

$$I_{eff}(t_s) = \frac{D}{\alpha}(e^{-\alpha t_s} - 1) + Ke^{-\alpha t_s} - xK - y \quad (5)$$

Solution of equation (2) by using boundary condition at t = t_s, $I_1(t_s) = I_{eff}(t_s)$

$$I_2(t) = \frac{D}{\alpha}(e^{-\alpha t} - 1) + Ke^{-\alpha t} - (xK + y)e^{\alpha(t_s - t)} \quad (6)$$

Solution of (3) is given by

$$I_3(t) = yD(t_p - t) \quad (7)$$

Solve the equation (5) for t = t_p such as $I_2(t_p) = 0$.

$t_p = \frac{1}{\alpha}[\log\left(\frac{D}{\alpha} + K - xKe^{\alpha t_s} - ye^{\alpha t_s}\right) - \log\left(\frac{D}{\alpha}\right)]$ (8)

Now total time for entire cycle to complete is T = $t_p + t_q$, where $t_q = \frac{y}{D}$

- Total sales revenue is the sum of the revenue generated at (0, T) and sale of the imperfect products times is p$(1-x)K + C_s xK$ (9)
- Ordering cost = Q (10)
- Screening cost = bK (11)
- Deteriorating cost = C$(K - DT)$ (12)
- Shortage cost = $S_c y \int_{t_p}^{t_q} -I_3(t)\ dt$

  $= \frac{S_c y D}{2}(t_p - t_q)^2$ (13)
- Holding cost = h$[\int_0^{t_s} I_1(t)dt + \int_{t_s}^{t_p} I_2(t)dt]$ (14)

Retailer`s total profit = Sales Revenue – Ordering Cost – Deterioration Cost - Holding Cost – Shortage Cost (15)

Total expected profit per unit time is calculated with the help of known probability density function $f(x)$, x is a random variable as

E[$\frac{(K,y)}{T}$] = p($1 - E[x]$) $K + C_s\ E[x]K - Q - bK - \frac{S_c y D}{2}$ ($t_p - t_q)^2 -$ C$(K - DT) -$ h[$\frac{D}{\alpha}\left(-\frac{1}{\alpha}(e^{-\alpha t_s} - 1\right) -$ $ts{-}K1ae{-}ats{-}1+$ $Da{-}1ae{-}atp{-}e{-}ats{-}tp{-}ts{-}$ $\frac{(xK+y)}{\alpha}\left(1 - e^{\alpha(t_s - t_p)}\right)$] (16)

and E[T] = $\frac{(1-E[x])K}{D}$

The main purpose is to find the optimal values of K and y which minimize the optimal cost function. The necessary conditions to be optimal are $\frac{\partial E[K,y]}{\partial K} = 0$ and $\frac{\partial E[K,y]}{\partial y} = 0$.

The sufficient condition to prove the concavity of the expected profit function must be satisfied:

$$\left(\frac{\partial^2 E[K,y]}{\partial K \partial y}\right)^2 - \left(\frac{\partial^2 E[K,y]}{\partial K^2}\right)\left(\frac{\partial^2 E[K,y]}{\partial b^2}\right) \le 0, \quad (17)$$

$$\frac{\partial^2 E[K,y]}{\partial b^2} \le 0, \frac{\partial^2 E[K,y]}{\partial K^2} \le 0 \quad (18)$$

Since it is difficult to find the derivatives and prove concavity by hand. By using software Mathematica, all the profit functions have been originated graphically with the help of numerical values and graph for different examples are shown in figures below.

IV. NUMERICAL EXAMPLE

Example 1. D = 50000 units per year, , x = 0.01, Q = \$100 per cycle, C = \$25/unit, h = \$10/unit/year, α = 0.05, b = \$ 0.5/units, C_s = \$20/unit and S_c = \$15/year, p = \$50/ unit.

Optimal order level K = 830 units, y = 20/units and the expected total profit is \$ 68187.82.

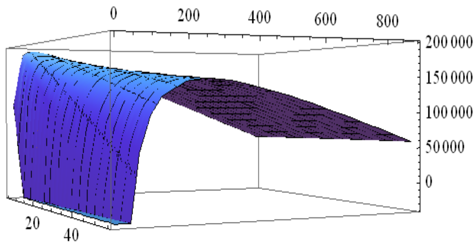

Fig. 2. Graphical representation for example 1.

Example 2. D = 50000 units per year, h = \$15/unit/year, α = 0.08, Q = \$150 per cycle, C = \$28/units, p = \$45/ unit, x = 0.01, b = \$ 0.4/units, C_s = \$15/unit and S_c = \$28/year.

Optimal order level K = 805 units, y = 15/units and the expected total profit is 108081.035.

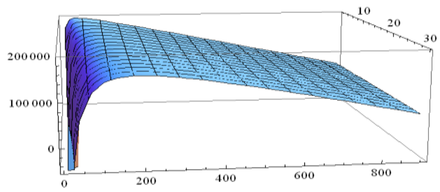

Fig.3. Graphical representation for example 2.

V. SENSITIVITY ANALYSIS

Sensitivity analysis has been discussed to study the impact of various factor on lot size and retailer's expected factors. Factors are like holding cost (h), shortage cost (S_c), expected number of imperfect quality ($E[x]$).

Table 1: Effect of change f defectives on retailer`s expected profit, backorder level and optimal order level.

X	*K*	*y*	Profit
0.01	687	96	95089.93
0.02	691	93	94497.56
0.03	695	91	93895.11
0.04	699	88	93265.73
0.05	704	86	92482.26
0.06	708	83	91814.35
0.07	716	78	90448.65

Table 2: Effect of changing the shortage cost and holding cost on the optimal order level, backorder level and retailer`s expected profit.

S_c	h	K	y	Profit
5	5	213	23	83881.42
	10	176	60	139440.56
	15	137	97	162860.26
8	5	188	34	79812.7
	10	151	71	130537.68
	15	114	92	169340.15
11	5	178	0	92020.49
	10	141	43	152751.68
	15	104	98	161556.23

From table 1 it is evident that as defective items increases the optimal order quantity increases but the profit decreases significantly. As the defectives are increasing in the ordered lot, more items are salvaged at a discounted price, resulting in profits. Table 2 establishes a trade-off between holding and shortage cost. As the holding cost increases, the backorder level increases and optimal order-level decreases resulting in higher profit. Holding cost and backorder cost are directly proportional to the total cost. As holding cost and backorder cost increases profit increases vice versa. These observations entail that an inventory model that jointly incorporates shortages performs better than others that incorporate them separately.

VI. CONCLUSION

The significant role of the study introduced in this paper is the formation of the imperfect items. In practice items may get damaged due to production or transportation conditions. The presence of items of imperfect quality has a significant effect on order quantity. This study should motivate production and operations managers to focus on quality checks and ensure that the percentage of commodities with imperfect quality is kept to a minimum. This study is to determine the retailer`s optimal replenishment policies that minimize the total cost. The model has authenticated with the help of a numerical example and Sensitivity analysis of key parameter has provided to the retailer to take proper decision under a dissimilar situation. Also, the concavity of the total cost function has been built up graphical technique with the assistance of Mathematica 10.0.

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