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Collaboration Graph on Abel's Prize Winners -II

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Abstract: ***Construction of Collaboration graph is an interesting task. Here we take up a Problem for Abel's Prize Winners centered at Paul Erdős. The number of Abel's prize winners as on 2019 is 20 , But the collaboration Graph G has 47 vertices and 87 edges and gives some properties. We constructed the collaboration graph G of Abel's Prize Winners in [11]. In this paper, we analyzed the some properties of G like Distance, Diameter, Eccentricity, Chromatic number, Chromatic index and etc.***

Keywords : Distance , Diameter, Eccentricity, Chromatic number, Chromatic index.

I. INTRODUCTION

The collaboration graph is a graph model where the vertices are researchers (dead or alive) from all academic disciplines and where two distinct researches are joined by an edge whenever they published an article or book. The notation d (u ,v) is the distance between two vertices u and v which is equal to the number of edges in the shortest path between u and v . Clearly d (u ,u) = 0. We now consider the collaboration sub graph centered at Paul Erdős (1913-1996). For a researcher v, the number d(Erdős ,v) is called the Erdős number of v. That is, Paul Erdős himself has Erdős number 0 and his coauthors have Erdős number 1. People not having Erdős number 0 or 1 but who has published with someone with Erdős number 1 have Erdős number 2, and so on. Those who are not linked in this way to Paul Erdős have Erdős number ∞. 511 people have Erdős number 1, and over 11000 have Erdős number 2. For more details see [1,5,6,8].

II. ABOUT ABEL'S PRIZE

The Abel Prize was established on 1 January 2002. The purpose is to award the Abel Prize for outstanding scientific work in the field of mathematics. The prize amount is 7.5 million Norwegian Kroner and was awarded for the first time on 3 June 2003. For more details refer [9].

III. CONSTRUCTION OF THE GRAPH G

Construction of Abel's prize winners of Collaboration Graph G is given in [11]. G has 47 vertices. In this vertices, only 16 members (V_2-V_{17}) are directly connected to Paul Erdős by path of length 1(ie. Erdős number 1), Erdős number 2 members are V_{18}- $V_{36,}$ the remaining members with Erdős number 3 namely V_{37}- $V_{47.}$ If there is a coauthor relationship between any 2 co authors , then there is a path between that 2 co-authors. The vertex v_1 is the Paul Erdős with Erdős number 0. For details refer [1-5,7].

The forty seven vertices and eighty seven edges of G are given below.

V(G)={ $v_{1,}$ $v_{2...}$ $v_{47\}}$ where v_1 = Paul Erdős, v_2 = Sarvadaman Chowla ,v_3 = FanChung , v_4 = Irving Kaplansky ,v_5 = Vilmos Totik ,v_6 = Kai Lai Chung, v_7= Béla Bollobás ,v_8 = Harold Davenport, v_9 = János Pach,v_{10} = Hugh L. Montgomery, v_{11} = Noga Alon ,v_{12} = Endre Szemerédi, v_{13} = Peter C. Fishburn, v_{14}= Alan J. Hoffman, v_{15} = AndrewM.Odlyzko , v_{16} = Stanisław Hartman ,v_{17}=László Babai,v_{18}= Jean-Pierre Serre ,v_{19}= Armand Borel,$v_{20=}$ Shlomo sternberg,v_{21}=Richard Friederich Arens, v_{22}=Lennart Axel Edvard Carleson,v_{23}=Srinivasa R S. Varadhan,v_{24}= John Griggs Thompson,v_{25}= Enrico Bombieri ,v_{26}= Mikhael Gromov ,v_{27}= John Torrence Tate,v_{28}= Shmuel Friedland,v_{29}= Daniel S.Freed, $v_{30=}$ Jean Bourgain,v_{31}= Harold W. Kuhn ,v_{32} = Christopher M. Skinner , v_{33}= Yves F. Meyer,v_{34} = Robert P. Langlands ,v_{35}= Micha Sharir ,v_{36}= William M. Kantor,v_{37}= Michael Francis Atiyah ,v_{38}= Isadore Manuel Singer,v_{39}= Peter David Lax ,v_{40} = Jacques Tits,v_{41}= John Willard Milnor,v_{42} = Pierre Deligne, $v_{43=}$Yakov Grigor′evich Sinai , v_{44}=Louis Nirenberg , v_{45}= Andrew J. Wiles , v_{46}= John Forbes Nash Jr.,v_{47}= Karen Keskulla Uhlenbeck.

E(G)={ e_1, $e_{2,...}$ e_{87} } where e_1=(v_1,v_2) , e_2 =(v_1,v_3) , e_3 = (v_1,v_4), e_4= (v_1,v_5), e_5= (v_1,v_6), e_6= (v_1,v_7), e_7 = (v_1,v_8), e_8= (v_1,v_9), e_9= (v_1,v_{10}) , e_{10} = (v_1,v_{11}), e_{11} = (v_1,v_{12}), e_{12} = (v_1,v_{13}) , e_{13} = (v_1,v_{14}), e_{14} = (v_1,v_{15}), e_{15} = (v_1,v_{16}), e_{16}= (v_1,v_{17}), e_{17} = (v_2,v_8) , e_{18} = (v_2,v_{18}) , e_{19} = (v_2,v_{19}), e_{20} = (v_3,v_7), e_{21}= (v_3,v_{11}), e_{22}= (v_3,v_{12}) , e_{23}= (v_3,v_{13}) , e_{24}= (v_3,v_{15}), e_{25} = (v_3,v_{17}), e_{26}= (v_3,v_{20}), e_{27} = (v_3,v_{34}), e_{28}= (v_4,v_{21}) , e_{29}= (v_5,v_{22}) , e_{30}= (v_6,v_{23}), e_{31}=

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(v_7,v_{11}), $e_{32}=(v_7,v_{12})$, $e_{33}=(v_7,v_{24})$,$e_{34}=(v_8,v_{25})$, $e_{35}=(v_9,v_{11})$, $e_{36}=(v_9,v_{12})$, $e_{37}=(v_9,v_{26})$, $e_{38}=(v_9,v_{35})$, $e_{39}=(v_{10},v_{15})$, $e_{40}=(v_{10},v_{24})$, $e_{41}=(v_{10},v_{25})$, $e_{42}=(v_{10},v_{27})$, $e_{43}=(v_{11},v_{12})$, $e_{44}=(v_{11},v_{15})$, $e_{45}=(v_{11},v_{17})$, $e_{46}=(v_{11},v_{28})$, $e_{47}=(v_{11},v_{30})$, $e_{48}=(v_{11},v_{35})$, $e_{49}=(v_{12},v_{17})$, $e_{50}=(v_{13},v_{15})$, $e_{51}=(v_{13},v_{29})$, $e_{52}=(v_{14},v_{31})$, $e_{53}=(v_{15},v_{25})$,$e_{54}=(v_{15},v_{29})$, $e_{55}=(v_{15},v_{32})$, $e_{56}=(v_{16},v_{33})$,
$e_{57}=(v_{17},v_{36})$, $e_{58}=(v_{18},v_{19})$, $e_{59}=(v_{18},v_{25})$, $e_{60}=(v_{18},v_{27})$, $e_{61}=(v_{18},v_{37})$, $e_{62}=(v_{18},v_{40})$, $e_{63}=(v_{18},v_{41})$, $e_{64}=(v_{18},v_{42})$, $e_{65}=(v_{19},v_{37})$, $e_{66}=(v_{19},v_{39})$, $e_{67}=(v_{19},v_{40})$, $e_{68}=(v_{20},v_{38})$, $e_{69}=(v_{21},v_{38})$, $e_{70}=(v_{22},v_{39})$, $e_{71}=(v_{23},v_{44})$, $e_{72}=(v_{25},v_{30})$, $e_{73}=(v_{25},v_{40})$, $e_{74}=(v_{25},v_{44})$, $e_{75}=(v_{26},v_{30})$, $e_{76}=(v_{28},v_{41})$, $e_{77}=(v_{29},v_{42})$, $e_{78}=(v_{30},v_{43})$, $e_{79}=(v_{31},v_{46})$, $e_{80}=(v_{32},v_{45})$, $e_{81}=(v_{35},v_{39})$, $e_{82}=(v_{35},v_{44})$, $e_{83}=(v_{36},v_{40})$, $e_{84}=(v_{37},v_{38})$, e_{85},$=(v_{39},v_{44})$, $e_{86}=(v_{41},v_{46})$, $e_{87}=(v_{47},v_{19})$.

Using Pajek program ,we constructed the graph G. The following graph is the visualization of Abel's Prize Winners Collaboration Graph G. For more details refer [10].

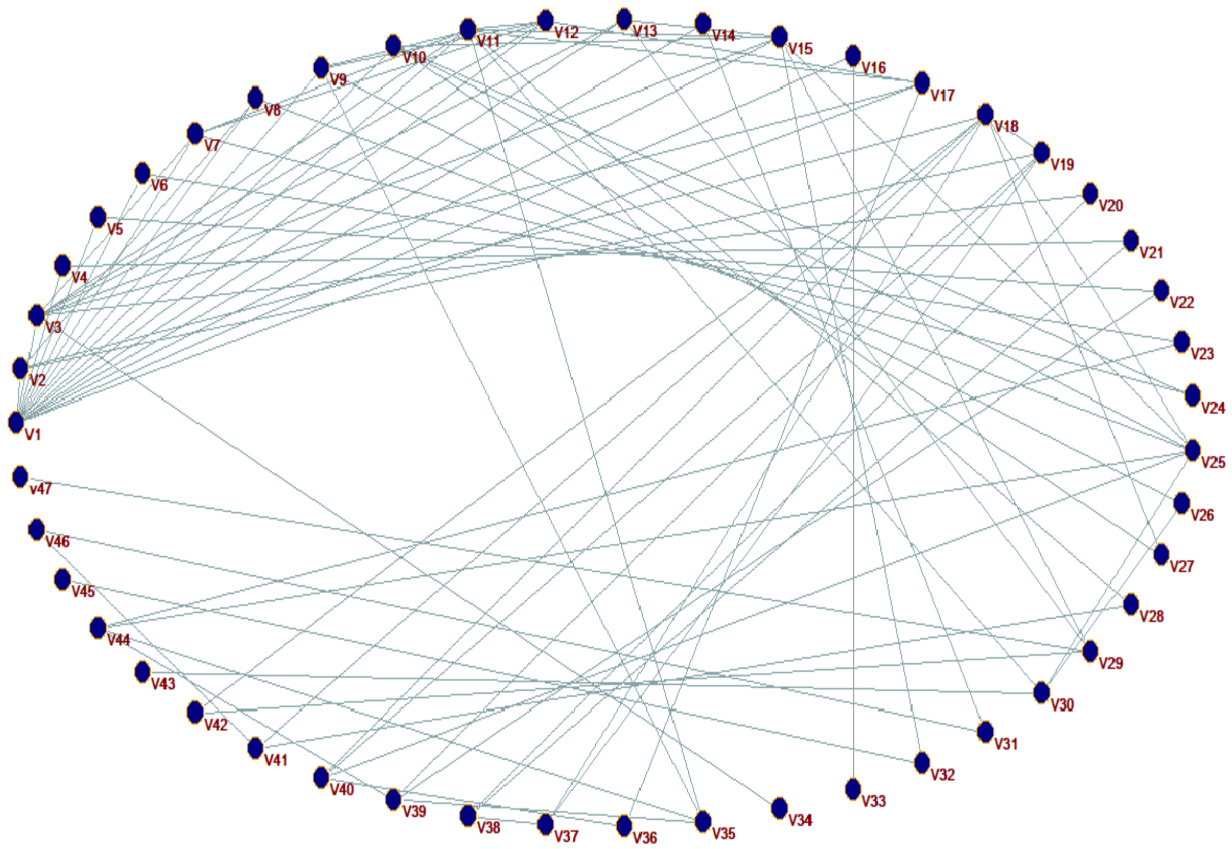

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Figure.1 Graph G

IV. DIAMETER , RADIUS, ECCENTRICITY OF THE GRAPH G

The maximum distance between a vertex v and any other vertex v_i of a graph G is called the ***Eccentricity e(v)*** of a vertex v for all i.

The minimum Eccentricity of any vertex of a graph G is Called the R***adius r(G)*** of G.

The maximum Eccentricity of any vertex of a graph G is called the D***iameter dm(G)*** of G.

Proposition:4.1 Median of the graph G is the complete graph with single vertex .ie. $M(G)=K_1$.

Proof: First Calculate the distance of each vertex of G

$d(v_1)=\sum_{v\in G} d(v_1,v)=d(v_1,v_2)+d(v_1,v_3)+d(v_1,v_4)+d(v_1,v_5)+d(v_1,v_6)+d(v_1,v_7)+d(v_1,v_8)+d(v_1,v_9)+d(v_1,v_{10})+d(v_1,v_{11})+d(v_1,v_{12})+d(v_1,v_{13})+d(v_1,v_{14})+d(v_1,v_{15})+d(v_1,v_{16})+d(v_1,v_{17})+d(v_1,v_{18})+d(v_1,v_{19})+d(v_1,v_{20})+d(v_1,v_{21})+d(v_1,v_{22})+d(v_1,v_{23})+d(v_1,v_{24})+d(v_1,v_{25})+d(v_1,v_{26})+d(v_1,v_{27})+d(v_1,v_{28})+d(v_1,v_{29})+d(v_1,v_{30})+d(v_1,v_{31})+d(v_1,v_{32})+d(v_1,v_{33})+d(v_1,v_{34})+d(v_1,v_{35})+d(v_1,v_{36})+d(v_1,v_{37})+d(v_1,v_{38})+d(v_1,v_{39})+d(v_1,v_{40})+d(v_1,v_{41})+d(v_1,v_{42})+d(v_1,v_{43})+d(v_1,v_{44})+d(v_1,v_{45})+d(v_1,v_{46})+d(v_1,v_{47})=1+1+1+1+1+1+1+1+1+1+1+1+1+1+1+1+2+2+2+2+2+2+2+2+2+2+2+2+2+2+2+2+2+2+2+3+3+3+3+3+3+3+3+3+3+3=87$, $d(v_2)=116$, $d(v_3)=116$, $d(v_4)=126$, $d(v_5)=127$,$d(v_6)=127$, $d(v_7)=124$, $d(v_8)=117$, $d(v_9)=119$, $d(v_{10})=109$, $d(v_{11})=111$, $d(v_{12})=119$,$d(v_{13})=120$, $d(v_{14})=127$,$d(v_{15})=110$, $d(v_{16})=130$, $d(v_{17})=119$, $d(v_{18})=127$,$d(v_{19})=142$, $d(v_{20})=153$, $d(v_{21})=160$, $d(v_{22})=162$,$d(v_{23})=161$, $d(v_{24})=162$, $d(v_{25})=138$,$d(v_{26})=150$, $d(v_{27})=153$, $d(v_{28})=157$, $d(v_{29})=152$, $d(v_{30})=154$, $d(v_{31})=154$, $d(v_{32})=155$,$d(v_{33})=174$, $d(v_{34})=155$, $d(v_{35})=152$, $d(v_{36})=153$, $d(v_{37})=165$,$d(v_{38})=201$, $d(v_{39})=162$, $d(v_{40})=157$, $d(v_{41})=155$, $d(v_{42})=166$, $d(v_{43})=186$, $d(v_{44})=162$,

$d(v_{45})=213$,$d(v_{46})=202$, $d(v_{47})=195$.
From the above , it is clear that v_1 is the median vertex as $d(v_1)=\min \{ d(v_i), 1 \le i \le 47\}$
Since the median vertex of G is unique, the median of the graph M (G) = K_1.

Proposition: 4.2 Centre of the graph G is the complete graph with single vertex.ie. C (G)=K_1.

Proof: Calculate the Eccentricity of each vertex of G, $e(v_1)=3$, $e(v_2)=4$, $e(v_3)=4$, $e(v_4)=4$, $e(v_5)=4$,$e(v_6)=4$, $e(v_7)=4$,$e(v_8)=4$, $e(v_9)=4$, $e(v_{10})=4$,$e(v_{11})=4$, $e(v_{12})=4$,$e(v_{13})=4$, $e(v_{14})=4$,$e(v_{15})=4$, $e(v_{16})=4$, $e(v_{17})=4$,$e(v_{18})=5$,$e(v_{19})=5$, $e(v_{20})=5$, $e(v_{21})=5$,$e(v_{22})=5$, $e(v_{23})=5$, $e(v_{24})=5$, $e(v_{25})=5$,$e(v_{26})=5$, $e(v_{27})=5$, $e(v_{28})=5$, $e(v_{29})=5$, $e(v_{30})=5$, $e(v_{31})=5$, $e(v_{32})=5$,$e(v_{33})=5$, $e(v_{34})=5$, $e(v_{35})=5$, $e(v_{36})=5$, $e(v_{37})=6$,$e(v_{38})=6$, $e(v_{39})=6$, $e(v_{40})=6$, $e(v_{41})=6$,$e(v_{42})=6$, $e(v_{43})=6$, $e(v_{44})=6$, $e(v_{45})=6$,$e(v_{46})=6$, $e(v_{47})=6$.From this , we can see that v_1 is the only vertex having minimum eccentricity. Hence v_1 is centre of G. Therefore ,C(G)=K_1.
Corollary: 4.3 *dm(G)* = 6, where *dm(G)* is the diameter of G.
Proof: Using definition of Eccentricity of G and Proposition 4.2, *dm*(*G*) = 6.
Corollary :4.4 *r(G)* = 3,where r(G) is the radius of G.
Proof: Using definition of Eccentricity of G and Proposition 4.2, *r(G)* = 3
Proposition:4.5 G is not Geodesic.
Proof: Since if every cycle of G is odd, then G is Geodesic. Here G contains an even cycle C_4 (v_1-v_{17}-v_{11}-v_3-v_1), G is not Geodesic.
Observation:4.6 For any graph G, $r(G) \le dm(G) \le 2r(G)$
It satisfies from Corollary 4. 3 & 4.4, that is, $3 \le 6 \le 2(3)$=6.
Observation:4.7 avec(G) = $\frac{1}{n}\sum_{v\in G} e(v)$
avec(G)=$\frac{1}{47}$[3+4+4+4+4+4+4+4+4+4+4+4+4+4+4+4+4+5+5+5+5+5+5+5+5+5+5+5+5+5+5+5+5+5+5+5+6+6+6+6+6+6+6+6+6+6+6]=$\frac{288}{47}$=4.85 and it lies between the radius and diameter of G.ie. $r(G) = 3 \le 4.85 \le 6 = dm(G)$.
Similarly the average distance = $\frac{1}{|A||B|}\sum_{a\in A b\in B} d(A,B)$
= $\frac{1}{47\times 46}$ [87+116+116+126+127+127+124 +117+119+109+111+119+120+127+110+130+119+127+142+153+160+162+161+162+138+150+153+157+152+154+154+155+174+155+152+153+165+ 201+ 162+157+155+166+186+162+213+202+195]=$\frac{6882}{2162}$ =3.1831

V. CONNECTIVITY PROPERTIES OF G.

Proposition 5.1 G is not self complementary.
Proof: We call a graph G is called a self complementary graph if $G \cong \bar{G}$. From Clapham [12] ,"Every self-complementary graph has a Hamiltonian cycle". As G has a pendent vertices v_{33}, v_{34},v_{43},v_{45},v_{47}, G has no Hamiltonian cycle. Hence G is not self complementary.

Proposition :5.2 κ(G) = κ′ (G) = 1, where κ(G), κ′ (G) are the vertex and the edge connectivity of G .
Proof: As G has pendent vertices v_{33}, v_{34}, v_{43} , v_{45}, v_{47}. we have κ(G) = κ′ (G) ≤ 1. Now κ(G) = 1 as ω(G−v_{33}) ≠ ω(G) and κ′ (G) = 1 as ω(G−(v_{33}, v_{34},v_{43},v_{45},v_{47})) ≠ ω(G).

VI. A BOUNDED FRAGMENTATION OF G

Let *G* =(V,E) where V is the vertex set and E is the edge set. Then the graph G is a (m,f(m))-***bounded fragmentation graph*** if |C(G[V−S])|≤|f(m)| for every *S*⊆*V* of size at most m, where f is a function of m and C(G[V−S]) is the number of components of *G*[*V*−*S*] which is constant if *S* has at most m vertices. The graph *G* is a ***totally g(m)-bounded fragmentation graph*** if it is a (m,f(m))-bounded fragmentation graph for all 0≤m≤*n*. For more details refer [15].
Proposition: 6.1 G is a totally 17m bounded fragmentation graph.
Proof: We know that Δ(G) = 17 , is a constant. Let S be a set of m vertices, 0 ≤ m ≤ 47. If we remove S from G, then |C(G[V−S])|≤17m. Using the above definition , G is a totally 17m bounded fragmentation graph.
Proposition :6.2 G is totally 17 bounded fragmentation graph.
Proof: Let S be a set of m vertices, 0 ≤ m ≤ 47.We know that at least one vertex from each connected component of G[V − S] is contained in any maximum independent set. Since Δ(G) = 17, the number of terms in C(G[V−S] is bounded above by 17. Hence ,by definition , G is totally 17-bounded fragmentation graph.
Proposition : 6.3 G is a totally (m + 8)-bounded fragmentation graphs.

Proof: We see that G has 8 disjoint paths viz. [v_{34},v_3,v_{20},v_{38},v_{21},v_4],[v_{45},v_{32},v_{15},v_{13},v_{29},v_{47}], [v_5,v_{22},v_{39},v_{19},v_{37}], [v_6,v_{23},v_{44},v_{35},v_{11},v_9,v_{26},v_{30},v_{43}], [v_{24},v_7,v_{12},v_{17},v_{36}], [v_{33},v_{16},v_1,v_{10},v_{27}], [v_{28},v_{41},v_{46},v_{31},v_{14}], [v_{42},v_{18},v_2,v_8,v_{25},v_{40}]. If we remove a vertex from a path then the path is split into at most two sub paths .ie at most two connected components. Therefore , a removal any m vertices, 1 ≤ m ≤47 can add at most m connected components. Here we have 8 disjoint paths, So the number of connected components is at most (m + 8). Hence G is totally (m +8)-bounded fragmentation graph.

VII. COLORING PROPERTIES OF G

Assigning all the vertices of a graph G with colors such that any two adjacent vertices are not colored by same color is called ***Proper Coloring of a graph*** G.
A graph G that requires k different colors for its proper coloring of vertices and not less than k, is called a ***k -Chromatic graph*** and the number k is called ***Chromatic number*** χ of G.
If a graph G that requires k different colors for its proper coloring of edges and not less than k then the number k is called ***Chromatic index*** χ' of G.

Proposition : 7.1 $\chi(G)$ = 5, where $\chi(G)$ is the Chromatic number of G.
Proof: Consider the Collaboration Graph G which is a simple and connected graph with vertices V={v_1, v_2... v_{47} }and the edges E={e_1, e_2... e_{87}}.Let S_1⊂S is the set of non-adjacent vertices where S_1={v_1,v_{18}, v_{20}, v_{21}, v_{22}, v_{23}, v_{24}, v_{26}, v_{29}, v_{34}, v_{35}, v_{43}, v_{45}}.Color S_1 by color 1.and then similarly consider the another set of non-adjacent vertices S_2={v_2,v_3, v_4, v_5, v_6,v_9,

$v_{10}, v_{14}, v_{16}, v_{28}, v_{30}, v_{38}, v_{40}, v_{42}, v_{46}, v_{47}\}$.Color S_2 by color 2. $S_3=\{v_7, v_8, v_{13}, v_{17}, v_{19}, v_{27}, v_{31}, v_{32}, v_{33}, v_{41}, v_{44}\}$. Color S_3 by color 3. $S_4=\{v_{12}, v_{15}, v_{39}\}$,color it by color 4. $S_5=\{v_{11}, v_{25}, v_{36}, v_{37}\}$,color it by color 5. Now all the vertices of G are colored and any two adjacent vertices are not colored with the same color. Hence G is 5-Chromatic Graph. ie . $\chi(G)$ = 5.Refer Figure.2

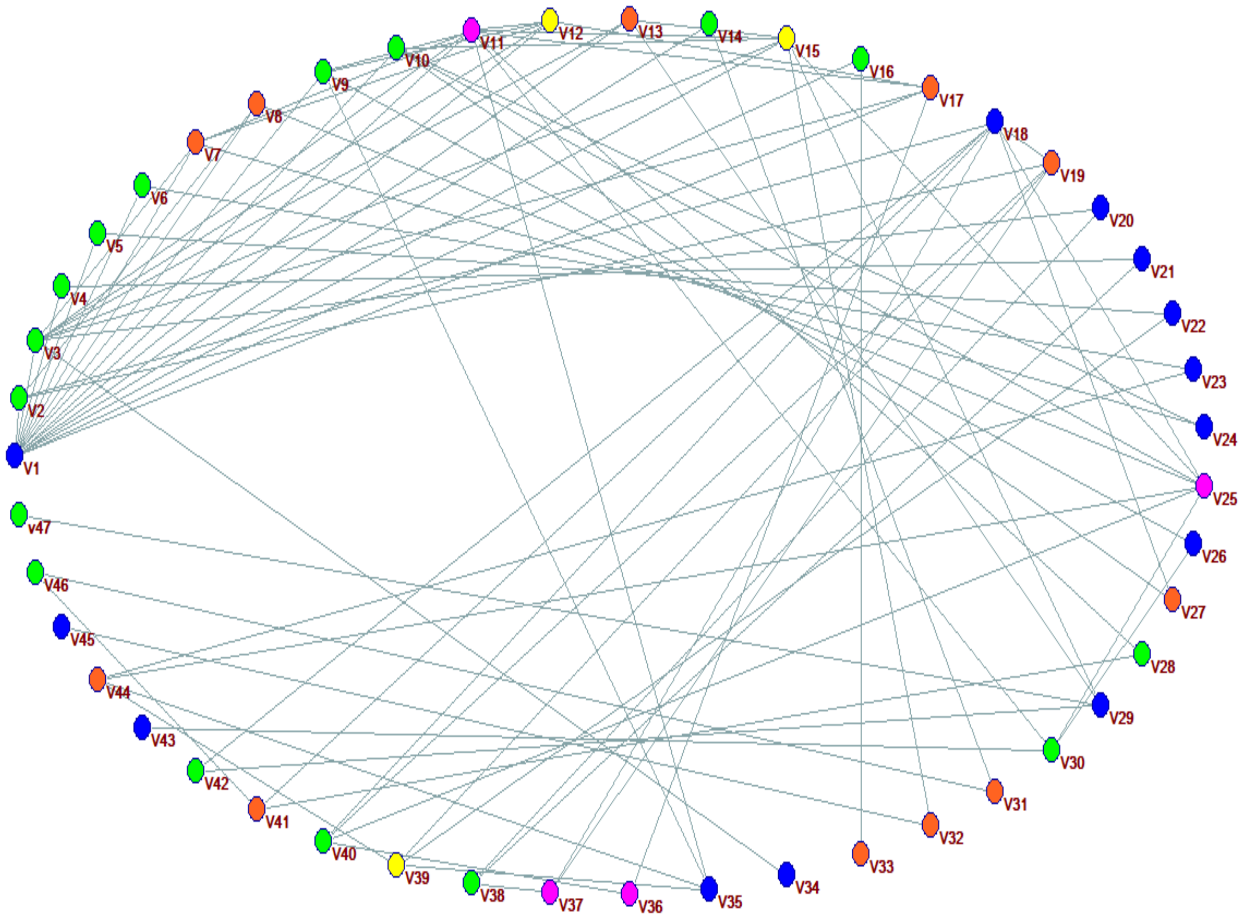

Figure.2

Observation: 7.2

(BROOK'S THEOREM). Let G be a connected graph. Then $\chi(G) \leq \Delta(G)$ unless G is either a complete graph or an odd cycle.

Clearly G is not a complete graph ,so it satisfies $5=\chi(G) \leq \Delta(G)=16$.by Proposition :7.1

Proposition:7.3 $\chi'(G)$= 16, $\chi'(G)$ is the Chromatic index of G.

Proof: Consider the Collaboration Graph G which is a simple and connected graph with vertices $V=\{v_1, v_{2\ldots}\ v_{47}\}$and the edges $E=\{e_1, e_{2\ldots}\ e_{87}\}$.Let $E_1 \subset E$ is the set of non-adjacent edges where $E_1=[e_1,e_{22},e_{37},e_{47},e_{59},e_{65},e_{77},e_{82}]$.color E_1 by 1.and then similarly consider the another set of non-adjacent edges $E_2=[e_2,e_{43},e_{53},e_{60},e_{66}]$. Color E_2 by color 2. $E_3=[e_3,e_{25},e_{44},e_{61},e_{74}]$. Color E_3 by color 3. $E_4=[e_4,e_{24},e_{63},e_{72}]$.color it by color 4.$E_5$ = $[e_{39},e_{73}, e_5,e_{18},e_{23},e_{46}]$.Color E_5 by color 5.E_6 $=[e_6,e_{21}]$.Color it by color 6. $E_7 = [e_7,e_{40},e_{57}]$.Color it by color 7.$E_8= [e_8]$. Color it by 8. $E_9=[e_9,e_{68},e_{19}, e_{27},e_{28},e_{29},e_{30},e_{31},e_{34}, e_{38},e_{49}, e_{51},e_{55}, e_{56},e_{62},e_{76}, e_{78}, e_{79},e_{85})]$.Color it by color 9. $E_{10}=[e_{10}]$. Color it by color 10. $E_{11} = [e_{11}]$.Color it by color 11.$E_{12}=[e_{12}]$. Color it by color 12. $E_{13} = [e_{13}, e_{32}, e_{45}]$. Color it by color 13. $E_{14} =[e_{83},e_{14})]$. Color it by color14. $E_{15}=[e_{15},e_{42},e_{26}, e_{33},e_{36},e_{48}, e_{50},e_{52}, e_{58},e_{69},e_{70} ,e_{75},e_{87},e_{80},e_{86},e_{71}]$.Color it by color15. $E_{16}=[e_{41}, e_{16},e_{17}, e_{20}, e_{35},e_{54},e_{64},e_{67},e_{81},e_{84}]$.Color it by color16. Now all the edges of G are colored and any two adjacent edges are not colored with the same color. Hence G is 16 Edge _Colorable. ie. $\chi'(G)$= 16. Refer Figure 3.

Observation:7.4 (Gupta [14] ,Vizing [13]). If G is a graph, then $\chi'(G) \leq \Delta(G) +1$.

In G, $\Delta(G)$= 16, From Proposition :7.3 , $\chi'(G)$= 16 . Hence $16=\chi'(G) \leq \Delta(G) +1=16+1=17$.

VIII. CONCLUSION

In this paper, we have analyzed the some properties of the graph G like Distance, Diameter, Eccentricity and Chromatic number and Chromatic index.

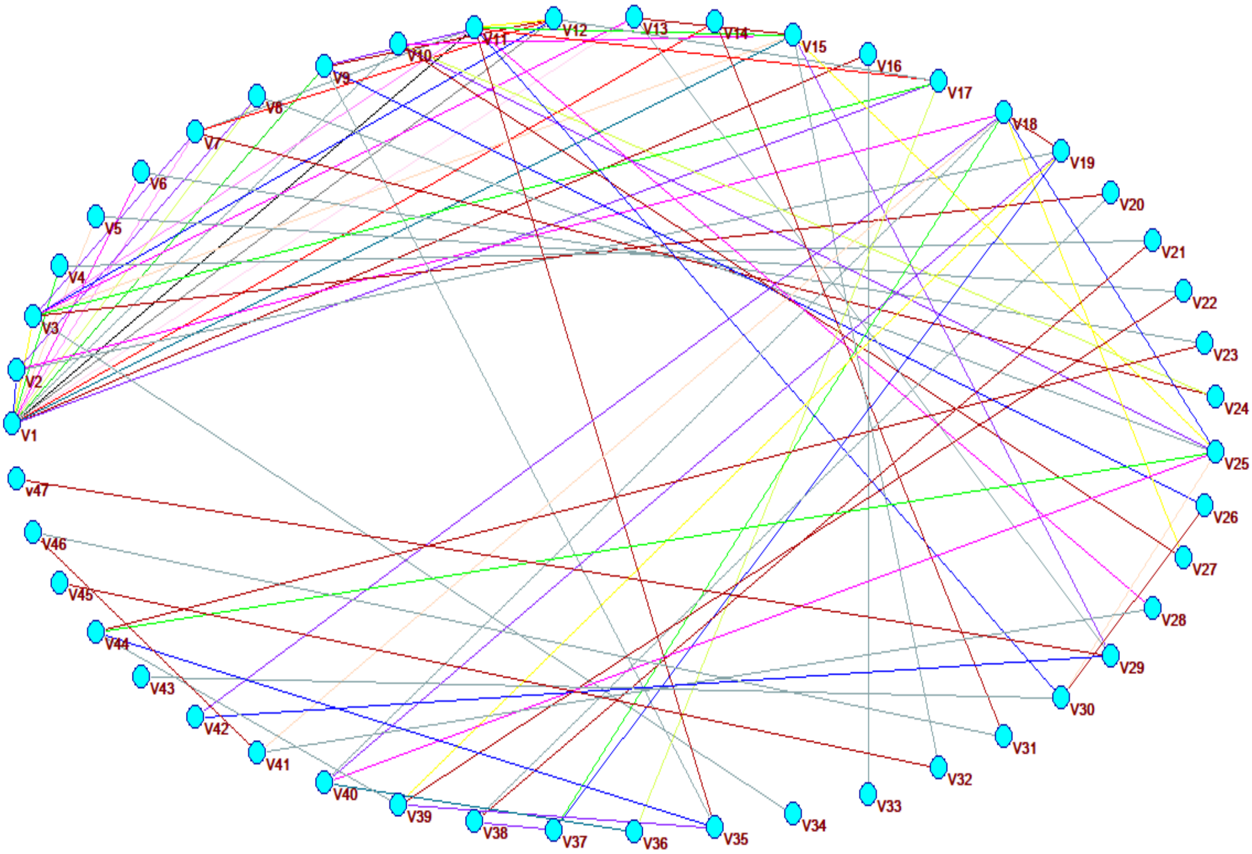

Figure. 3

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