

# Fuzzy $\alpha - \psi^*$ - Irreducible Spaces

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**Abstract:** In this paper, the concept of  $\psi^*$  operator on a family of fuzzy  $\alpha$ -open sets in a fuzzy topological space is introduced. Also, the concepts of fuzzy  $\alpha - \psi^*$ -irreducible spaces, fuzzy  $\alpha - \psi^*$ -generic sets and fuzzy  $\alpha - \psi^*$ -quasi-Sober spaces are initiated and some properties are discussed.

**Keywords :** operator on  $F\alpha O(X, \tau)$ , fuzzy  $\alpha - \psi^*$ -generic sets, fuzzy  $\alpha - \psi^*$ -irreducible spaces, fuzzy  $\alpha - \psi^*$ -quasi-Sober spaces  
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## I. INTRODUCTION

L.A. Zadeh[8] initiated fuzzy set in 1965. In 1968, Chang [2] characterized fuzzy topological space. Njastad [5] introduced  $\alpha$ -open sets. In the same sprit Bin Shahna [1] defined fuzzy  $\alpha$ -open sets and fuzzy  $\alpha$ -closed sets. The idea of an irreducible or hyperconnected topological space has been studied by T. Thompson. In this paper, the concept of  $\psi^*$  operator on a family of fuzzy  $\alpha$ -open sets in a fts is introduced. Also, the concepts of fuzzy  $\alpha - \psi^*$ -irreducible spaces, fuzzy  $\alpha - \psi^*$ -generic sets and fuzzy  $\alpha - \psi^*$ -quasi-Sober spaces are initiated and some properties are discussed.

## II. PRELIMINARIES

This section contains basic definitions and preliminary results needed for this paper.

**Definition 2.1 [4]** Let  $X$  be a topological space.

(i)  $X$  is irreducible, if  $X \neq \Phi$ , and whenever  $X = Z_1 \cup Z_2$  with  $Z_i$  closed,  $X = Z_1$  or  $X = Z_2$ .

(ii)  $Z \subset X$  is an irreducible component of  $X$  if  $Z$  is a maximal irreducible subset of  $X$ .

**Definition 2.2 [3]** A subset  $F \neq \Phi$  of a topological space is irreducible if,  $F \subseteq A \cup B$  where  $A$  and  $B$  then  $F \subseteq A$  or  $F \subseteq B$ .

**Definition 2.3 [4]** A topological space  $X$  is said to be quasi-sober if for every irreducible closed subset has a generic point.

**Definition 2.4 [6]** A fuzzy set  $\mu_A$  is quasi-coincident with the fuzzy set  $\mu_B$  iff  $\exists x \in X$  such that  $\mu_A(x) + \mu_B(x) > 1$ .

**Definition 2.5 [7]** A fuzzy set  $\lambda$  in a fuzzy topological space  $(X, T)$  is called fuzzy dense if there exists no fuzzy closed set

$\mu$  in  $(X, T)$  such that  $\lambda < \mu < 1$ . That is.,  $cl(\lambda) = 1$  in  $(X, T)$ .

**Definition 2.6 [7]** A fuzzy set  $\lambda$  in a fuzzy topological space  $(X, T)$  is called fuzzy nowhere dense if there exists no non-zero fuzzy open set  $\mu$  in  $(X, T)$  such that  $\mu < cl(\lambda)$ . That is.,  $int(cl(\lambda)) = 0$  in  $(X, T)$ .

## III. FUZZY $\alpha - \psi^*$ - IRREDUCIBLE SPACES

Throughout this paper, fuzzy topological space is shortly denoted by fts. Then  $F\alpha O(X, \tau)$ ,  $F\alpha C(X, \tau)$  and  $\mathcal{FP}(X)$  denote set of all fuzzy  $\alpha$ -open sets, fuzzy  $\alpha$ -closed sets in  $(X, \tau)$  and fuzzy points over  $X$  respectively.

**Definition 3.1** Let  $(X, \tau)$  be a fts. A fuzzy operator

$$\psi^*: F\alpha O(X, \tau) \rightarrow I^X$$

is defined as, if for each  $\mu \in F\alpha O(X, \tau)$  with  $\mu \neq 0_X$ ,  $F int(\mu) \leq \psi^*(\mu)$  and  $\psi^*(0_X) = 0_X$ .

**Remark 3.1** It is easy to check that some examples of fuzzy operators on  $F\alpha O(X, \tau)$  are the well known fuzzy operators viz.  $F int$ ,  $F int(Fcl)$ ,  $Fcl(F int)$ ,  $Fint(Fcl(Fint))$  and  $Fcl(F int(Fcl))$ .

**Definition 3.2** Let  $(X, \tau)$  be a fts and  $\psi^*$  be a fuzzy operator on  $F\alpha O(X, \tau)$ . Then any  $\mu \in F\alpha O(X, \tau)$  is called fuzzy  $\alpha - \psi^*$ -open if  $\mu \leq \psi^*(\mu)$ . Then  $1_X - \mu$  is called fuzzy  $\alpha - \psi^*$ -closed set.

**Notation 3.1** The family of all fuzzy  $\alpha - \psi^*$ -open (resp. fuzzy  $\alpha - \psi^*$ -closed) sets in  $(X, \tau)$  is notated by  $F\alpha - \psi^* O(X, \tau)$  (resp.  $F\alpha - \psi^* C(X, \tau)$ ).

**Definition 3.3** For any  $\mu \in I^X$  in a fts  $(X, \tau)$  and  $\psi^*$  be a fuzzy operator on  $F\alpha O(X, \tau)$ , the fuzzy  $\alpha - \psi^*$ -interior of  $\mu$  (briefly,  $F\alpha - \psi^* int(\mu)$ ) is defined by

$$F\alpha - \psi^* int(\mu) = \vee \{ \sigma : \sigma \leq \mu \text{ and } \sigma \in F\alpha - \psi^* O(X, \tau) \}.$$

**Definition 3.4** For any  $\mu \in I^X$  in a fts  $(X, \tau)$  and  $\psi^*$  be a fuzzy operator on  $F\alpha O(X, \tau)$ , the fuzzy  $\alpha - \psi^*$ -closure of  $\mu$  (briefly,  $F\alpha - \psi^* cl(\mu)$ ) is defined by

$$F\alpha - \psi^* cl(\mu) = \wedge \{ \sigma : \sigma \geq \mu \text{ and } \sigma \in F\alpha - \psi^* C(X, \tau) \}.$$

**Definition 3.5** Any fts  $(X, \tau)$  is said to be a fuzzy  $\alpha - \psi^*$ -irreducible space, where  $\psi^*$  is a fuzzy operator on  $F\alpha O(X, \tau)$  if, for any  $\mu_1, \mu_2 \in F\alpha - \psi^* O(X, \tau)$  where  $\mu_1 \neq 0_X, \mu_2 \neq 0_X$  and  $\mu_1 q \mu_2$ .

**Definition 3.6** Any  $\lambda \in I^X$  in a fts  $(X, \tau)$  is said to be fuzzy  $\alpha - \psi^*$ -irreducible,

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where  $\psi^*$  is a fuzzy operator on  $F\alpha O(X, \tau)$  if,  $\lambda \neq 0_X$  and  $\lambda \leq (\mu_1 \vee \mu_2)$  where  $\mu_1, \mu_2 \in F\alpha - \psi^* C(X, \tau)$ , then either  $\lambda \leq \mu_1$  or  $\lambda \leq \mu_2$ . Then the set of all fuzzy  $\alpha - \psi^*$ -irreducible sets is noted by  $F\alpha - \psi^* I(X, \tau)$

**Definition 3.7** Any  $\lambda \in I^X$  is called a fuzzy  $\alpha - \psi^*$ -maximal irreducible set of fts  $(X, \tau)$ , where  $\psi^*$  is a fuzzy operator on  $F\alpha O(X, \tau)$ , if there is no  $\mu \in F\alpha - \psi^* I(X, \tau)$  such that  $\mu > \lambda$ . Then collection of all fuzzy  $\alpha - \psi^*$ -maximal irreducible sets is denoted by  $F\alpha - \psi^* MI(X, \tau)$

**Definition 3.8** Let  $\psi^*$  be a fuzzy operator on  $F\alpha O(X_1, \tau_1)$  and  $F\alpha O(X_2, \tau_2)$  in a ftss  $(X_1, \tau_1)$  and  $(X_2, \tau_2)$  respectively. Any function  $f : (X_1, \tau_1) \rightarrow (X_2, \tau_2)$  is said to be a fuzzy  $\alpha - \psi^*$ -continuous function if for every  $\mu \in F\alpha - \psi^* O(X_2, \tau_2)$ ,  $f^{-1}(\mu) \in F\alpha - \psi^* O(X_1, \tau_1)$ .

**Proposition 3.1** Let  $\psi^*$  be a fuzzy operator on  $F\alpha O(X_1, \tau_1)$  and  $F\alpha O(X_2, \tau_2)$  in a ftss  $(X_1, \tau_1)$  and  $(X_2, \tau_2)$  respectively. Let  $f : (X_1, \tau_1) \rightarrow (X_2, \tau_2)$  be a bijective and fuzzy  $\alpha - \psi^*$ -continuous function. If  $\lambda \in F\alpha - \psi^* I(X_1, \tau_1)$  then  $f(\lambda) \in F\alpha - \psi^* I(X_2, \tau_2)$

**Proof.** Let  $\mu_1, \mu_2 \in F\alpha - \psi^* C(X_2, \tau_2)$  such that  $f(\lambda) \leq (\mu_1 \vee \mu_2)$ . Then

$f^{-1}(f(\lambda)) \leq f^{-1}(\mu_1 \vee \mu_2)$ , which implies that  $\lambda \leq f^{-1}(\mu_1 \vee \mu_2)$ . Then  $f^{-1}(f(\lambda)) = \lambda$ , since  $f$  is one-one. Thus  $\lambda \leq f^{-1}(\mu_1) \vee f^{-1}(\mu_2)$ . Since  $\lambda \in F\alpha - \psi^* I(X_1, \tau_1)$ ,  $\lambda \leq f^{-1}(\mu_1)$  or  $\lambda \leq f^{-1}(\mu_2)$ . Thus  $f(\lambda) \leq f(f^{-1}(\mu_1))$  and hence  $f(\lambda) \leq \mu_1$  as  $f$  is onto,  $f(f^{-1}(\mu_1)) = \mu_1$ .

or  $f(\lambda) \leq f(f^{-1}(\mu_2))$  and so  $f(\lambda) \leq \mu_2$ . Therefore  $f(\lambda) \in F\alpha - \psi^* I(X_2, \tau_2)$ .

**Remark 3.2** For any  $\lambda \in I^X$  in a fts  $(X, \tau)$  and  $\psi^*$  be a fuzzy operator on  $F\alpha O(X, \tau)$ ,  $\lambda \leq F\alpha - \psi^* cl(\lambda)$ .

**Proof.** Proof is obvious from the definition of fuzzy  $\alpha - \psi^*$ -closure of a fuzzy set  $\lambda \in I^X$ .

**Proposition 3.2** If  $\lambda \in F\alpha - \psi^* I(X, \tau)$  in fts  $(X, \tau)$  and  $\psi^*$  be a fuzzy operator on  $F\alpha O(X, \tau)$ , then  $F\alpha - \psi^* cl(\lambda) \in F\alpha - \psi^* I(X, \tau)$ .

**Proof.** Assume that  $\lambda \in I^X$  and  $F\alpha - \psi^* cl(\lambda) \leq \mu_1 \vee \mu_2$  where  $\mu_1, \mu_2 \in F\alpha - \psi^* C(X, \tau)$ . Since  $\lambda \leq F\alpha - \psi^* cl(\lambda)$  and  $\lambda \in F\alpha - \psi^* I(X, \tau)$ ,  $\lambda \leq \mu_1$  or  $\lambda \leq \mu_2$ . Then  $F\alpha - \psi^* cl(\lambda) \leq \mu_1$  or  $F\alpha - \psi^* cl(\lambda) \leq \mu_2$ . Hence  $F\alpha - \psi^* cl(\lambda) \in F\alpha - \psi^* I(X, \tau)$

**Proposition 3.3.** If  $\lambda \in F\alpha - \psi^* MI(X, \tau)$ , then  $\lambda \in F\alpha - \psi^* C(X, \tau)$ .

**Proof.** Let  $\lambda \in F\alpha - \psi^* MI(X, \tau)$ . Then, there is no  $\mu \in F\alpha - \psi^* I(X, \tau)$  such that  $\mu > \lambda$ . By Proposition 3.2,  $F\alpha - \psi^* cl(\lambda) \in F\alpha - \psi^* I(X, \tau)$ . Since

$\lambda \leq F\alpha - \psi^* cl(\lambda)$ , the only possibility is  $\lambda = F\alpha - \psi^* cl(\lambda)$ . Hence  $\lambda \in F\alpha - \psi^* C(X, \tau)$ .

**Proposition 3.4** Let  $\lambda \in F\alpha - \psi^* MI(X, \tau)$  in fts  $(X, \tau)$ , where  $\psi^*$  be a fuzzy operator on  $F\alpha O(X, \tau)$ . If  $\mu \in F\alpha - \psi^* I(X, \tau)$ , then  $\mu \leq \lambda$ .

**Proof.** Let  $\lambda \in F\alpha - \psi^* MI(X, \tau)$ . Let  $\mathcal{A}$  be the set of all fuzzy  $\alpha - \psi^*$ -irreducible sets  $\lambda_j \in I^X$  such that  $\lambda < \lambda_j, j \in J$  where  $J$  is an indexed set. Let  $\mathcal{A}' \subset \mathcal{A}$  be the set of fuzzy  $\alpha - \psi^*$ -irreducible sets  $\lambda_k, k = 1, 2, \dots, l$  such that  $\lambda_1 < \lambda_2 < \dots < \lambda_l$ .

Let  $\lambda' = \bigvee_k \lambda_k, k = 1, 2, \dots, l$ . It is enough to show that  $\lambda' \in F\alpha - \psi^* I(X, \tau)$ . Let  $\lambda' \leq \mu_1 \vee \mu_2$ , where  $\mu_1, \mu_2 \in F\alpha - \psi^* C(X, \tau)$ . Since each  $\lambda_k \in F\alpha - \psi^* I(X, \tau), i = 1, 2, \dots, l$ , we get  $\lambda_i \leq \mu_1$  or  $\lambda_i \leq \mu_2$ . Suppose that for some  $i_0 \in J, \lambda_{i_0} \in \mathcal{A}'$  and  $\lambda_{i_0} \leq \mu_1$ , then  $\lambda_i \leq \mu_2$  for all  $i \in J$ . Thus  $\lambda' \leq \mu_2$ . Therefore  $\lambda' \in F\alpha - \psi^* I(X, \tau)$  and  $\lambda' \in F\alpha - \psi^* MI(X, \tau)$ .

**Proposition 3.5** If  $(X, \tau)$  is fuzzy  $\alpha - \psi^*$ -irreducible, then for any  $\lambda \in F\alpha - \psi^* O(X, \tau)$ ,  $F\alpha - \psi^* cl(\lambda) = 1_X$ .

**Proof.** Since  $(X, \tau)$  is fuzzy  $\alpha - \psi^*$ -irreducible, for any two  $\lambda, \mu \in F\alpha - \psi^* O(X, \tau)$  where  $\lambda \neq 0_X$  and  $\mu \neq 0_X$ ,  $\lambda q \mu$ . Let  $\lambda_1 \in F\alpha - \psi^* O(X, \tau)$  such that  $\lambda_1 \neq 0_X$ . If  $\lambda_1 = 1_X$ , then  $F\alpha - \psi^* cl(\lambda_1) = 1_X$ . Assume that  $\lambda_1 \neq 1_X$  and  $x_{t_1} \in \mathcal{FP}(X)$  be such that  $x_{t_1} \neq \lambda_1$ . Let  $\mu_1 \in F\alpha - \psi^* O(X, \tau)$  and  $\mu_1 \neq 0_X$  such that  $x_{t_1} \leq \mu_1$ . By hypothesis,  $\lambda_1 q \mu_1$ . That is, there exist some  $x_{t_i} \leq \lambda_1$  where  $i \neq 1$  and  $i \in J, J$  is an indexed set such that  $x_{t_i} \leq \mu_1$  with  $x_{t_i} \tilde{q} x_{t_1}$ . Since  $\mu_1$  is an arbitrary fuzzy  $\alpha - \psi^*$ -open set in  $(X, \tau)$ , it follows that every fuzzy  $\alpha - \psi^*$ -open  $\mu_1$  in  $(X, \tau)$  is such that  $x_{t_i} \leq \lambda_1, i \neq 1$  and  $i \in J$  such that  $x_{t_i} \leq \mu_1$  with  $\lambda_1 q \mu_1$ . Therefore  $x_{t_1}$  is a fuzzy limit point of  $\lambda_1$ . So every fuzzy point over  $X$  is a fuzzy limit point of  $\lambda_1$ . Thus, every  $x_{t_1} \in \mathcal{FP}(X)$  is such that  $x_{t_1} \leq F\alpha - \psi^* cl(\lambda), \lambda \in F\alpha - \psi^* O(X, \tau)$ . Hence, fuzzy  $\alpha - \psi^*$ -closure of every fuzzy  $\alpha - \psi^*$ -open set is  $1_X$ .

#### IV. FUZZY $\alpha - \psi^*$ -QUASISOBER SPACE

**Definition 4.1** Let  $\psi^*$  be a fuzzy operator on  $F\alpha O(X, \tau)$  in a fts  $(X, \tau)$ . Let  $\lambda \in I^X$  be a fuzzy  $\alpha - \psi^*$ -irreducible closed set. Any fuzzy set  $\mu \in I^X$  with  $\mu \leq \lambda$ , is said to be a fuzzy  $\alpha - \psi^*$ -generic set of  $\lambda$  if  $F\alpha - \psi^* cl(\mu) = \lambda$ .

**Definition 4.2** Let  $\psi^*$  be a fuzzy operator on  $F\alpha O(X, \tau)$  in a fts  $(X, \tau)$ . Then  $(X, \tau)$  is said to be a fuzzy  $\alpha - \psi^*$ -quasi-Sober space, if for every fuzzy  $\alpha - \psi^*$ -irreducible closed set there exists a fuzzy  $\alpha - \psi^*$ -generic set.

**Remark 4.1** Let  $\psi^*$  be a fuzzy operator on  $F\alpha O(X, \tau)$  in a fts  $(X, \tau)$ . Let  $Y \subset X$  and  $(Y, \tau_Y)$  be a fuzzy subspace of  $(X, \tau)$ . Then  $\psi^*$  be a fuzzy operator on  $F\alpha O(Y, \tau_Y)$ .

**Definition 4.3** Let  $\psi^*$  be a fuzzy operator on  $F\alpha O(X, \tau)$  in a fts  $(X, \tau)$ . If for  $\lambda, \mu \in I^X$  with  $\lambda q \mu$  there exists a  $\delta \in F\alpha - \psi^* C(X, \tau)$  such that either  $\mu \leq \delta$ ,  $\lambda q \delta$  or  $\lambda \leq \delta$ ,  $\mu q \delta$ , then  $(X, \tau)$  is called fuzzy  $\alpha - \psi^*$ -Kolmogorov.

**Proposition 4.1** Let  $\psi^*$  be a fuzzy operator on  $F\alpha O(X, \tau)$  in a fts  $(X, \tau)$ . Let  $Y \subset X$  and  $(Y, \tau_Y)$  be a fuzzy subspace of  $(X, \tau)$ . If  $(X, \tau)$  is fuzzy  $\alpha - \psi^*$ -Kolmogorov, then  $(Y, \tau_Y)$  is fuzzy  $\alpha - \psi^*$ -Kolmogorov.

**Proof.** Let  $\lambda, \mu \in I^X$  with  $\lambda q \mu$ . Then there exists a  $\delta \in F\alpha - \psi^* C(X, \tau)$  such that  $\mu \leq \delta$ ,  $\lambda q \delta$  or  $\lambda \leq \delta$ ,  $\mu q \delta$ . Let  $\lambda|_Y, \mu|_Y \in I^Y$  with  $\lambda|_Y q \mu|_Y$ . Since  $\delta \in F\alpha - \psi^* C(X, \tau)$ ,  $\delta|_Y \in F\alpha - \psi^* C(Y, \tau_Y)$ . Also, since  $\lambda|_Y \leq \lambda$ ,  $\lambda \leq \delta$  and  $\mu|_Y \leq \mu$ ,  $\mu \leq \delta$ ,  $\lambda|_Y \leq \delta|_Y$  with  $\mu|_Y q \delta|_Y$  or  $\mu|_Y \leq \delta|_Y$  with  $\lambda|_Y q \delta|_Y$ . Hence  $(Y, \tau_Y)$  is fuzzy  $\alpha - \psi^*$ -Kolmogorov.

**Notation 4.1** Let  $\psi^*$  be a fuzzy operator on  $F\alpha O(X, \tau)$  and  $F\alpha O(Y, \sigma)$  where  $(X, \tau)$  and  $(Y, \sigma)$  are any two fts. For any  $\lambda \in I^X$ ,  $\mu \in I^Y$ ,  $F\alpha - \psi^* cl(\lambda)$  with respect to  $(X, \tau)$  and  $(Y, \sigma)$  are denoted by  $F\alpha - \psi^* cl(\lambda)$  and  $F\alpha - \psi^* cl(\mu)$  respectively. Then the collection of all fuzzy  $\alpha - \psi^*$ -irreducible closed in  $(X, \tau)$  id denoted by  $F\alpha - \psi^* IC(X, \tau)$

**Proposition 4.2** Let  $\psi^*$  be a fuzzy operator on  $F\alpha O(X, \tau)$  in a fts  $(X, \tau)$ . Let  $Y \subset X$  and  $(Y, \tau_Y)$  be a fuzzy  $\alpha - \psi^*$ -closed subspace of  $(X, \tau)$ . If  $(X, \tau)$  is fuzzy  $\alpha - \psi^*$ -quasi-Sober, then  $(Y, \tau_Y)$  is fuzzy  $\alpha - \psi^*$ -quasi-Sober.

**Proof.** Let  $\lambda \in F\alpha - \psi^* IC(Y, \tau_Y)$ . Then  $\lambda \in F\alpha - \psi^* IC(X, \tau)$ . Since  $(X, \tau)$  is fuzzy  $\alpha - \psi^*$ -quasi-Sober, there exists a fuzzy  $\alpha - \psi^*$ -generic set  $\mu \in I^X$  such that  $F\alpha - \psi^* cl(\mu) = \lambda$ . Thus  $F\alpha - \psi^* cl(\mu) \wedge 1_Y = \lambda \wedge 1_Y$ . This implies that  $F\alpha - \psi^* cl(\mu) \wedge 1_Y = \lambda$ . Therefore  $F\alpha - \psi^* cl(\mu) = \lambda$ . Hence  $(Y, \tau_Y)$  is fuzzy  $\alpha - \psi^*$ -quasi-Sober.

**Proposition 4.3** Let  $\psi^*$  be a fuzzy operator on  $F\alpha O(X_1, \tau_1)$  and  $F\alpha O(X_2, \tau_2)$  where  $(X_1, \tau_1)$  and  $(X_2, \tau_2)$  are any two fts. Let  $f: (X_1, \tau_1) \rightarrow (X_2, \tau_2)$  be a injective and fuzzy  $\alpha - \psi^*$ -continuous function. If  $\lambda \in F\alpha - \psi^* IC(X_1, \tau_1)$ , then  $F\alpha - \psi^* cl(f(\lambda)) \in F\alpha - \psi^* IC(X_2, \tau_2)$ .

**Proof.** Let  $\lambda \in F\alpha - \psi^* IC(X_1, \tau_1)$ . Let  $\mu_1, \mu_2 \in F\alpha - \psi^* C(X_2, \tau_2)$  such that  $F\alpha - \psi^* cl(f(\lambda)) \leq \mu_1 \vee \mu_2$ . Since  $f(\lambda) \leq F\alpha - \psi^* cl(f(\lambda))$ ,  $f(\lambda) \leq \mu_1 \vee \mu_2$ . As  $f$  is injective,  $\lambda \leq (f^{-1}(\mu_1)) \vee (f^{-1}(\mu_2))$ . Then  $\lambda \leq (\lambda \wedge f^{-1}(\mu_1)) \vee (\lambda \wedge f^{-1}(\mu_2))$ . Since  $f$  is a fuzzy  $\alpha - \psi^*$ -continuous function,  $f^{-1}(\mu_1), f^{-1}(\mu_2) \in F\alpha - \psi^* C(X_1, \tau_1)$ . This implies that  $\lambda \wedge f^{-1}(\mu_1), \lambda \wedge f^{-1}(\mu_2) \in F\alpha - \psi^* C(X_1, \tau_1)$ . Since  $\lambda \in F\alpha - \psi^* IC(X_1, \tau_1)$ ,  $\lambda \leq (\lambda \wedge f^{-1}(\mu_1))$  or  $\lambda \leq (\lambda \wedge f^{-1}(\mu_2))$ . Thus  $\lambda \leq f^{-1}(\mu_1)$  or  $\lambda \leq f^{-1}(\mu_2)$ . Therefore,  $f(\lambda) \leq \mu_1$  or  $f(\lambda) \leq \mu_2$ . This implies that  $F\alpha - \psi^* cl(f(\lambda)) \leq \mu_1$  or  $F\alpha - \psi^* cl(f(\lambda)) \leq \mu_2$ . Hence  $F\alpha - \psi^* cl(f(\lambda)) \in F\alpha - \psi^* IC(X_2, \tau_2)$ .

**Proposition 4.4** Let  $\psi^*$  be a fuzzy operator on  $F\alpha O(X, \tau)$  in a fts  $(X, \tau)$ . Let  $Y \subset X$  and  $(Y, \tau_Y)$  be a fuzzy  $\alpha - \psi^*$ -open subspace of  $(X, \tau)$ . If  $(X, \tau)$  is fuzzy  $\alpha - \psi^*$ -quasi-Sober, then  $(Y, \tau_Y)$  is fuzzy  $\alpha - \psi^*$ -quasi-Sober.

**Proof.** Let  $\mu \in F\alpha - \psi^* IC(Y, \tau_Y)$ . Then  $\mu \in F\alpha - \psi^* IC(X, \tau)$ . Since  $(X, \tau)$  is fuzzy  $\alpha - \psi^*$ -quasi-Sober, there exists a fuzzy  $\alpha - \psi^*$ -generic set  $\lambda \in I^X$  such that  $F\alpha - \psi^* cl(\lambda) = \mu$ . Then  $F\alpha - \psi^* cl_X(\lambda) \wedge \chi_Y = \mu \wedge \chi_Y$ . Since  $\mu \in F\alpha - \psi^* IC(Y, \tau_Y)$ ,  $F\alpha - \psi^* cl(\lambda) = \mu$  and  $\lambda \leq \mu$ . Hence  $(Y, \tau_Y)$  is fuzzy  $\alpha - \psi^*$ -quasi-Sober.

## V. CONCLUSION

In this paper, we explored  $\psi^*$  operator on a family of fuzzy  $\alpha$ -open sets in a fuzzy topological spaces. We can extend  $\psi^*$  operator on fuzzy homotopy and fuzzy spectral spaces.

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