

# Sensor FDD based FTC Design for Time-Delay PEMFC Systems



Vikash Sinha, Sharifuddin Mondal

**Abstract:** An FTC design based on sensor FDD is presented for time-delay PEMFC systems with disturbances and uncertainties. A robust fault detection observer with a fault diagnosis technique is used. The Lyapunov principle derives the adaptive law and estimates sensor fault. A scheme based on FDD adaptive algorithm is derived to obtain fault in the sensor. The PEMFC systems simulations show the utility of the algorithm. Time-delays are considered in deriving both adaptive observer and FTC algorithm which is not incorporated in previous works on PEMFC systems. The simulated results can estimate the fault and validate the FTC design.

**Keywords:** FDD, FTC, PEMFC, time-delays.

## I. INTRODUCTION

PEMFC systems are renewable sources to get reliable power under steady state. These are used as distributed application based generation sources. The increase desire for less emission vehicles has made these systems alluring for automobile applications. These are very quick to start and possess high power. Some works already exist on steady state and dynamic modelling of fuel cells [1-6]. The fault detection and diagnosis (FDD) has been paid much attention in dynamic systems in the last few decades. This is due to an increased demand to get high reliability and safety in industrial processes. Reliability and safety are very important measures of performance of the PEMFC systems. Fault tolerant control (FTC) has emerged as an effective method to enhance system safety and reliability. This has caught the attention of the research community in recent years. It is useful in advanced control and research methodology in last few decades. It is classified into passive and active FTC [5-17]. The passive one applies control algorithms for making the system robust in response to probable faults. These probable faults are modelled as uncertainties. Active strategy works on real-time fault scenario supplied by fault estimator. Based on the faulty scenario, an additional control algorithm is designed for the system stability to preserve an acceptable performance of the faulty system [5-17].

Observers applying FDD techniques are the efficient approaches as these generate residuals for FDI.

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Adaptive approaches give a solution to FDD and hence adaptive observers have been used in many cases. Adaptive tuning is used to estimate the faults. In [7], an adaptive observer is described for a linear system having actuator and sensor faults by using adaptive laws to directly estimate the faults. Also, adaptive observers estimate sensor faults with their thresholds in non-linear systems. In [8], an FTC algorithm is designed for the exogenous disturbances with actuator faults. In [9], by using the radial basis functions online learning capability, an adaptive observer is designed. However, time-delay is not considered in these previous works.

Time-delay appears in various mechanical systems, chemical reactions, electric circuits and many control systems [10]. Several studies inspect the problems of time-varying delays because it degrades system performance and stability. Research works on adaptive tuning for such systems have been done recently [10-12]. In [10], for a constant time-delay system, a non-linear observer is proposed with a dynamic gain. This relaxes solutions on systems without time-delays. In [11], for SISO systems, the adaptive parameter is proposed. The parametric system is formed to develop adaptive tuning. [12] used adaptive logic for time-delay non-linear systems with uncertainties and disturbances.

Many results exist for the faulty systems with time-delays by various adaptive laws [13-17]. In [13], robust FDD problems are studied for faulty sensor and actuator systems with parametric uncertainties. The fault is detected by an adaptive threshold depending upon the inputs. In [14], an adaptive diagnosis method based on Takagi-Sugeno is proposed for systems with sensor faults. PEMFC systems estimation designs with time-delays have not been explored [15, 16]. Schultze and Horn [16] developed an UKF strategy combined with prediction, which compensates the effect of time-varying delay on the estimated PEMFC actual variables. A measure of oxygen volume in the cathode channel adds a significant amount of time-varying delay. Another work by [16] introduced a state estimator with time-varying delays with feedback control method for cathode pressure model of PEMFC system. Its weak point is the utility of linear state feedback control. But in literature, a robust framework of FTC based adaptive design observer for sensor fault diagnosis of these systems with time-varying delays is still lacking [5, 6]. Also, research in which the application of the adaptive laws for faulty sensor time-delay systems has not been taken into consideration till now. Here, an adaptive algorithm estimates the sensor fault.



This diagnosis is based on FTC for these systems with uncertainties and disturbances [5, 6].

In this work, an adaptive algorithm estimates the fault in the sensor for a PEMFC system with external disturbances and time-delays. The Lyapunov principle gives the solution. This generalizes the past solutions without time-delays [7] and actuator fault algorithm [5, 6]. This work proposes a sensor fault diagnosis approach [5, 6] to design an FTC algorithm. Finally, cathode pressure model of PEMFC system is used to show the efficacy of the algorithm.

The remaining paper is followed in this manner. In Section II, the PEMFC system is described with an observer for FDI. An FDD technique is used and an adaptive law by the Lyapunov principle is obtained. These designs are in Section III. A sensor fault diagnosis based FTC algorithm is designed in Section IV. Simulation results of PEMFC system followed by some conclusions are presented in Sections V and VI respectively.

## II. PEMFC SYSTEM

Consider a PEMFC system [2-6] with state delay, sensor fault, disturbances and uncertainties as

$$\dot{x}(t) = Ax(t) + A_d x(t-h(t)) + Bu(t) + f(x, u, t) \quad (1)$$

$$y(t) = Cx(t) + Dg(\theta, u, t) \quad (2)$$

In (1)-(2),  $x(t)$ ,  $u(t)$ , and  $y(t)$  being vectors of state, input, and output.  $A, A_d, B, C$ , and  $D$  are assumed to be suitable dimensions matrices [2-6].  $h(t)$  is the time-delay,  $f(x, u, t)$  and  $g(\theta, u, t)$  denotes states uncertainties and sensor properties respectively.  $\theta(t)$  changes when sensor fault occurs. For fault-free condition,  $\theta(t) = \theta_H$  holds in which  $\theta_H$  is the healthy sensor vector. In faulty condition,  $\theta(t) \neq \theta_H$  holds. Thus, the FDD alarms when system fault occurs so as to estimate  $\theta(t)$ , which characterizes the faulty sensor. Before defining FDD based observer design problem for (1)-(2), these assumptions are as stated.

*Assumption 1*

$(A, B)$  is controllable and  $(A, C)$  is observable.

*Assumption 2*

The time-varying delay  $h(t)$  satisfies the following condition as

$$\dot{h}(t) \leq 1 - \delta, \quad 0 < \delta \leq 1, \quad h \text{ and } \delta \text{ are known.}$$

*Assumption 3*

$$\|f(x, u, t)\| \leq \bar{f}.$$

*Assumption 4*

$$\|\theta(t)\| \leq k, \text{ where } k \text{ is known.}$$

Assumptions 1 and 2 are used for adaptive observer [2-6]. Assumptions 3 and 4 are commonly considered for time-delay systems [2-6].

Now the FDD observer for (1)-(2) is defined as follows:

$$\dot{\hat{x}}(t) = A\hat{x}(t) + A_d \hat{x}(t-h(t)) + Bu(t) + K(y(t) - \hat{y}(t)) \quad (3)$$

$$\hat{y}(t) = C\hat{x}(t) + Dg(\theta_H, u, t) \quad (4)$$

$\hat{x}(t)$ ,  $\hat{y}(t)$  being estimated state, output

vectors.  $(A, C)$  follows Assumption 1 and  $K$  makes  $(A - KC)$  to be stable asymptotically.

$$e_x(t) = x(t) - \hat{x}(t) \quad (5)$$

$$r(t) = e_x(t) + D[g(\theta, u, t) - g(\theta_H, u, t)] \quad (6)$$

In fault-free condition, the second term of equation (6) disappears. The FDI interprets

(a) if  $\|r(t_f)\| < \lambda$ , fault-free scenario.

(b) if  $\|r(t_f)\| \geq \lambda$ , faulty scenario.

Here,  $\lambda$  is the threshold value and  $t_f$  is the fault occurrence time. The FDD device is designed as

$$\dot{\hat{x}}'(t) = A\hat{x}'(t) + A_d \hat{x}'(t-h(t)) + Bu(t) + K(y(t) - \hat{y}'(t)) \quad (7)$$

$$\hat{y}'(t) = C\hat{x}'(t) + Dg(\hat{\theta}, u, t) \quad (8)$$

Here  $\hat{\theta}(t)$  is estimated such that in fault-free condition

$$\hat{\theta}(t) = \theta_H.$$

The diagnosis state error is

$$e'_x(t) = x'(t) - \hat{x}'(t) \quad (9)$$

Then,

$$\dot{e}'_x(t) = (A - KC)e'_x(t) + A_d e'_x(t-h(t)) +$$

$$KD[g(\theta, u, t) - g(\hat{\theta}, u, t)] + f(x, u, t) \quad (10)$$

$$\text{with } e'_g(t) = [g(\theta, u, t) - g(\hat{\theta}, u, t)].$$

Using Assumption 4, we get

$$g(\theta, u, t) - g(\hat{\theta}, u, t) = [\partial g(\hat{\theta}, u, t) / \partial \hat{\theta}(t)](\theta(t) - \hat{\theta}(t)) + \varphi \quad (11)$$

where  $\|\varphi\| \leq k_1$ ,  $k_1$  is a constant.

## III. FAULT DIAGNOSIS

**Theorem 1.** If  $P$  and  $Q$  exist such that the ARE is given by

$$(A - KC)^T P + P(A - KC) + \left(\frac{1}{ab}\right) P A_d A_d^T P + 2aI_n + Q = 0 \quad (12)$$

holds where  $a$  and  $b$  are known with adaptive FDD tuning

$$\dot{\hat{\theta}}(t) = -\sigma \hat{\theta}(t) - [\partial g(\hat{\theta}, u, t) / \partial \hat{\theta}(t)]^T D^T K^T P e'_x(t) \quad (13)$$

where  $\sigma$  is a constant making (10) stable.

**Proof:** The function is

$$V = \left(\frac{1}{2}\right) [e'^T_x(t) P e'_x(t)] + \left(\frac{1}{2}\right) [\tilde{\theta}^T(t) \tilde{\theta}(t)] + a \int_{t-h(t)}^t e'^T_x(s) P e'_x(s) ds \quad (14)$$

Then,

$$\dot{V} = \left(\frac{1}{2}\right) \dot{e}'^T_x(t) P e'_x(t) + \left(\frac{1}{2}\right) e'^T_x(t) P \dot{e}'_x(t) + \tilde{\theta}^T(t) \dot{\tilde{\theta}}(t) + a e'^T_x(t) P e'_x(t) - a e'^T_x(t-h(t)) P e'_x(t-h(t)) (1 - \dot{h}(t)) \quad (15)$$

Substituting (10) into (15), we get

$$\dot{V} = \left(\frac{1}{2}\right) e_x^T(t) \left[ (A-KC)^T + P(A-KC) \right] e_x'(t) + e_x^T(t) PA_d e_x'(t-h(t)) + \tilde{\theta}^T(t) \dot{\hat{\theta}}(t) + ae_x^T(t) e_x'(t) - ae_x^T(t-h(t)) e_x'(t-h(t)) (1-\dot{h}(t)) + e_x^T(t) PKD \left[ g(\theta, u, t) - g(\hat{\theta}, u, t) \right] - e_x^T(t) Pf(x, u, t) \quad (16)$$

The following inequality can be easily shown like

$$2e_x^T(t) PA_d e_x'(t-h(t)) \leq \left(\frac{1}{ab}\right) e_x^T(t) PA_d A_d^T e_x'(t) + abe_x^T(t-h(t)) e_x'(t-h(t)) \quad (17)$$

Then, it interprets

$$\dot{V} \leq \left(\frac{1}{2}\right) e_x^T(t) \left[ (A-KC)^T P + P(A-KC) \right] e_x'(t) + \left(\frac{1}{2ab}\right) \left[ e_x^T(t) PA_d A_d^T P e_x'(t) \right] + \left(\frac{1}{2}\right) abe_x^T(t-h(t)) e_x'(t-h(t)) + \tilde{\theta}^T(t) \dot{\hat{\theta}}(t) + ae_x^T(t) e_x'(t) - ae_x^T(t-h(t)) e_x'(t-h(t)) (1-\dot{h}(t)) + e_x^T(t) PKD \left[ g(\theta, u, t) - g(\hat{\theta}, u, t) \right] - e_x^T(t) Pf(x, u, t) \quad (18)$$

Adding  $\left(\frac{1}{2}\right) abe_x^T(t-h(t)) e_x'(t-h(t))$  to RHS of (18), we get

$$\dot{V} \leq \left(\frac{1}{2}\right) e_x^T(t) \left[ (A-KC)^T P + P(A-KC) \right] e_x'(t) + \left(\frac{1}{2ab}\right) \left[ e_x^T(t) PA_d A_d^T P e_x'(t) \right] + abe_x^T(t-h(t)) e_x'(t-h(t)) + \tilde{\theta}^T(t) \dot{\hat{\theta}}(t) + ae_x^T(t) e_x'(t) - abe_x^T(t-h(t)) e_x'(t-h(t)) + e_x^T(t) PKD \left[ g(\theta, u, t) - g(\hat{\theta}, u, t) \right] - e_x^T(t) Pf(x, u, t) = \left(\frac{1}{2}\right) e_x^T(t) \left[ (A-KC)^T P + P(A-KC) \right] e_x'(t) + \left(\frac{1}{2ab}\right) \left[ e_x^T(t) PA_d A_d^T P e_x'(t) \right] + \tilde{\theta}^T(t) \dot{\hat{\theta}}(t) + ae_x^T(t) e_x'(t) + e_x^T(t) PKD \left[ g(\theta, u, t) - g(\hat{\theta}, u, t) \right] - e_x^T(t) Pf(x, u, t) \quad (19)$$

Thus the following expression is obtained,

$$\dot{V} \leq \left(\frac{1}{2}\right) e_x^T(t) \left[ (A-KC)^T P + P(A-KC) + \left(\frac{1}{ab}\right) PA_d A_d^T P + 2aI_n \right] e_x'(t) + \tilde{\theta}^T(t) \dot{\hat{\theta}}(t) + e_x^T(t) PKD \left[ g(\theta, u, t) - g(\hat{\theta}, u, t) \right] - e_x^T(t) Pf(x, u, t) \quad (20)$$

From (12), we have

$$(A-KC)^T P + P(A-KC) + \left(\frac{1}{ab}\right) PA_d A_d^T P + 2aI_n = -Q \quad (21)$$

An adaptive law is

$$\dot{\hat{\theta}}(t) = -\sigma \hat{\theta}(t) - \left[ \partial g(\hat{\theta}, u, t) / \partial \hat{\theta}(t) \right]^T D^T K^T P e_x'(t) \quad (22)$$

Substituting (21) and (22) into (20) gives

$$\dot{V} \leq \left(-\frac{1}{2}\right) e_x^T(t) Q e_x'(t) - e_x^T(t) Pf(x, u, t) + e_x^T(t) PKD \left\{ \left[ \partial g(\hat{\theta}, u, t) / \partial \hat{\theta}(t) \right] \left[ \theta(t) - \hat{\theta}(t) \right] + \phi \right\} + \tilde{\theta}^T(t) \left\{ -\sigma \hat{\theta}(t) - \left[ \partial g(\hat{\theta}, u, t) / \partial \hat{\theta}(t) \right]^T D^T K^T P e_x'(t) \right\} = \left(-\frac{1}{2}\right) e_x^T(t) Q e_x'(t) - \sigma \tilde{\theta}^T(t) \hat{\theta}(t) - e_x^T(t) Pf(x, u, t) + e_x^T(t) PKD \phi \quad (23)$$

Due to Assumptions 3 and 4,

$$\dot{V} \leq \left(-\frac{1}{2}\right) \lambda_{\min}(Q) \|e_x'(t)\|^2 + 2\sigma k^2 + \|e_x'(t)\| \|P\| (k_i \|K\| \|D\| - \bar{f}) \quad (24)$$

where  $\tilde{\theta}^T(t) \hat{\theta}(t) = \sigma \left[ \theta(t) - \hat{\theta}(t) \right]^T \hat{\theta}(t) \leq 2\sigma k^2$ .

Suppose,  $\left(-\frac{1}{2}\right) \lambda_{\min}(Q) = \gamma_1$ ,

$\|P\| (k_i \|L\| \|D\| - \bar{f}) = \gamma_2$  and  $2\sigma k^2 = \gamma_3$ . Then,

$$\dot{V} \leq \gamma_1 \|e_x'(t)\|^2 + \gamma_2 \|e_x'(t)\| + \gamma_3 = \gamma_1 \left[ \|e_x'(t)\| + (\gamma_2 / 2\gamma_1) \right]^2 - (\gamma_2^2 / 4\gamma_1) + \gamma_3 \quad (25)$$

By (22),

$$\|e_x'(t)\| \leq \left[ (\gamma_2^2 - 4\gamma_1 \gamma_3)^{\frac{1}{2}} - \gamma_2 \right] / 2\gamma_1, \dot{V} \leq 0 \quad (26)$$

#### IV. FAULT TOLERANT CONTROLLER DESIGN

The FTC algorithm presented here is designed based on adaptive fault diagnosis tuning rate (13). Since state variables are generally unmeasurable, a robust adaptive observer based FTC is derived for stabilizing the closed loops system when faults are present. Based on the feedback law for the FTC is

$$u(t) = -K_1 \hat{x}(t) - D_1 Dg(\hat{\theta}, u, t) \quad (27)$$

where  $K_1 \in R^{m \times n}$  is the matrix gain and  $D_1 \in R^{m \times n}$  is a matrix which is stated later. The FTC gain matrix  $K_1$  is calculated in such a manner that  $(A - BK_1)$  becomes stable using Assumption 1. The condition which is sufficient for the existence of  $K_1$  is satisfied. For stable  $(A - BK_1)$  and  $W = W^T > 0$  with matrix  $H, G = G^T > 0$  of the Lyapunov equation is given as

$$(A - BK_1)^T G + G(A - BK_1) + H = -W \quad (28)$$

Using [17] and references therein,  $span(D) \subseteq span(B)$  gives

$$(I_n - BD_1)D = 0 \quad (29)$$

$span(D) \subseteq span(B)$  states that  $\bar{D} \in R^{m \times r}$  exists such that

$$D = B\bar{D}. \text{ Since } B \text{ is of full rank, } B^+ = (B^T B)^{-1} B^T.$$

Hence,

$$(I_n - BB^+)B = 0 \quad (30)$$

Then,

$$(I_n - BB^+)D = 0 \quad (31)$$

From above, all the poles of the system are assigned by control feedback law due to Assumption 1. Hence, the proposed FTC design assigns all poles to the LHS for gaining stability. Now, (1) and (2) is stabilized by FTC (27).

Applying FTC algorithm (27) to equation (1) gives the closed loop dynamics given below

$$\dot{x}(t) = Ax(t) + A_d x(t-h(t)) - BD_1 Dg(\hat{\theta}, u, t) + Dg(\theta, u, t) + f(x, u, t) = (A - BK_1)x(t) + A_d x(t-h(t)) + BK_1 e_x(t) + (I_n - BD_1) Dg(\hat{\theta}, u, t) + De'_g(t) + f(x, u, t) \quad (32)$$

such that  $(A - BK_1)$  becomes stable.

Using (29) into (32) gives

$$\dot{x}(t) = (A - BK_1)x(t) + A_d x(t-h(t)) + BK_1 e_x(t) + De'_g(t) + f(x, u, t) \quad (33)$$

Then,

$$V(t) = x^T(t)Gx(t) + \int_{t-h(t)}^t x^T(s)Hx(s)ds + e_x^{T'}(t)Pe'_x(t) + a \int_{t-h(t)}^t e_x^{T'}(s)e'_x(s)ds + \sigma e_g^{T'}(t)e'_g(t) \quad (34)$$

$$\dot{V}(t) = 2x^T(t)G\dot{x}(t) + x^T(t)Hx(t) - (1-\dot{h}(t))x^T(t-h(t))Hx(t-h(t)) + 2e_x^{T'}(t)Pe'_x(t) + ae_x^{T'}(t)e'_x(t) - a(1-\dot{h}(t))e_x^{T'}(t-h(t))e'_x(t-h(t)) + 2\sigma e_g^{T'}(t)e'_g(t) \quad (35)$$

From (33), it gives

$$\begin{aligned} \dot{V}(t) = & x^T(t)((A - BK_1)^T G + G(A - BK_1) + H)x(t) + 2x^T(t)GBK_1 e'_x(t) + 2x^T(t)GDe'_g(t) + 2x^T(t)Gf(x, u, t) + \\ & 2x^T(t)GA_d x(t-h(t)) - (1-\dot{h}(t))x^T(t-h(t))Hx(t-h(t)) + e_x^{T'}(t)(P(A - KC) + (A - KC)^T P)e'_x(t) + 2e_x^{T'}(t)PA_d e'_x(t-h(t)) - \\ & a(1-\dot{h}(t))e_x^{T'}(t-h(t))e'_x(t-h(t)) + 2e_x^{T'}(t)Pf(x, u, t) + 2\sigma e_g^{T'}(t)e'_g(t) - 2\sigma e_g^{T'}(t)D^T P(A - KC)e'_x(t) - 2\sigma e_g^{T'}(t)D^T PDe'_g(t) - 2\sigma e_g^{T'}(t)D^T Pf(x, u, t) - \\ & 2\sigma e_g^{T'}(t)g(\theta, u, t) + 2\sigma e_g^{T'}(t)\dot{g}(\theta, u, t) \end{aligned} \quad (36)$$

Using [17], (36) can be converted to

$$\begin{aligned} \dot{V}(t) \leq & -e_x^T(t)Me_x(t) + \frac{1}{\beta_1} e_x^{T'}(t)PPe'_x(t) + \beta_1 f^T(x, u, t)f(x, u, t) + 2\sigma e_g^{T'}(t)e'_g(t) + \frac{\sigma}{\beta_2} e_g^{T'}(t)D^T PDe'_g(t) + \sigma \beta_2 e_x^{T'}(t)(A - KC)^T (A - KC)e'_x(t) - 2\sigma e_f^T(t)D^T PDe'_g(t) + \frac{1}{\beta_3} e_x^T(t)PPe_x(t) + \beta_3 f^T(x, u, t)f(x, u, t) + \frac{\sigma}{\beta_4} e_g^{T'}(t)e'_g(t) + \beta_4 \sigma g^T(\theta, u, t)g(\theta, u, t) + \frac{\sigma}{\beta_5} e_g^{T'}(t)e'_g(t) + \beta_5 \sigma \dot{g}^T(\theta(t), u(t))\dot{g}(\theta(t), u(t)) - x^T(t)Wx(t) + \frac{1}{\beta_6} x^T(t)GBK_1 K_1^T B^T Hx(t) + \beta_6 e_x^{T'}(t)e'_x(t) + \frac{1}{\beta_7} x^T(t)GDD^T Gx(t) + \beta_7 e_g^{T'}(t)e'_g(t) + \frac{1}{\beta_8} x^T(t)GGx(t) + \beta_8 f^T(x, u, t)f(x, u, t) + \frac{1}{\beta_9} e_x^{T'}(t)PA_d A_d^T Pe_x(t) + \beta_9 e_x^{T'}(t-h(t))e'_x(t-h(t)) - a(1-\dot{h}(t))e_x^{T'}(t-h(t))e'_x(t-h(t)) + \frac{1}{\beta_{10}} x^T(t)GA_d A_d^T Gx(t) + \beta_{10} x^T(t-h(t))x(t-h(t)) - (1-\dot{h}(t))x^T(t-h(t))Hx(t-h(t)) \end{aligned} \quad (37)$$

where  $\beta_1 - \beta_{10}$  are scalars with  $M, L, S_1$  as same in [17].

Then,

$$\dot{V}(t) \leq -c_1 \|x(t)\|_2^2 - c_2 \|e'_x(t)\|_2^2 - c_3 \|e'_g(t)\|_2^2 + \delta, \quad (38)$$

where  $c_1 - c_3$  and  $\delta$  are calculated in same manner as in [17].

From (38),  $\dot{V}(t) < 0$  for

$$c_1 \|x(t)\|_2^2 + c_2 \|e'_x(t)\|_2^2 + c_3 \|e'_g(t)\|_2^2 > \delta \quad (39)$$

Therefore, since  $\beta_1 - \beta_{10}$  can be any scalars according to [17], it can be shown that by choosing these constants in (38) always hold. And,  $\dot{V}(t) < 0$  can be shown in a similar

manner as in [17] so that the FTC (27) ensures that the system is stable in faulty scenario.

## V. SIMULATION RESULTS

The cathode pressure model of the PEMFC system [2-6] is used as follows:

$$\dot{x}(t) = Ax(t) + A_d x(t-h(t)) + Bu(t) + f(x, u, t) \quad (40)$$

$$y(t) = Cx(t) + Dg(\theta, u, t) \quad (41)$$

The system matrices using [2-6] are

$$A = \begin{bmatrix} -22.9610 & -22.9610 & -22.9610 \\ -46.4930 & -46.4930 & -46.4930 \\ -0.3295 & -0.3295 & -0.3295 \end{bmatrix}$$

$$A_d = 10^{-3} \times \begin{bmatrix} -22.9610 & -22.9610 & -22.9610 \\ -46.4930 & -46.4930 & -46.4930 \\ -0.3295 & -0.3295 & -0.3295 \end{bmatrix}$$

$$B = \begin{bmatrix} 367.5 & 7.7390 & -0.2296 \\ 367.5 & 29.104 & -0.4649 \\ 367.5 & 0.1180 & -942.225 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Using Assumption 1 here, sensor faults are considered in such a way that faults occur in the output assuming  $D = B$ .

The delay is  $h(t) = 0.2\sin(t)$ . The function  $f(x, u, t)$  represents the energy bounded system disturbances. The simulation is simplified by replacing it with white noise and  $g(\theta, u, t) = \tan^{-1}(\theta, u, t)$ .  $0.5 \leq |\theta(t)| \leq 2$  so that  $k = 2$ .

$$g(\theta_1, u, t) - g(\theta_2, u, t) = [\partial g(\theta, u, t) / \partial \theta]_{\theta=\bar{\theta}} (\theta_1 - \theta_2) \quad (42)$$

where  $\theta_1$  and  $\theta_2 \in [0.5 \ 2]$  and  $\bar{\theta} \in [0.5 \ 2]$  such that

$$\begin{aligned} & |\tan^{-1}(\theta_1, u, t) - \tan^{-1}(\theta_2, u, t) - [\partial \tan^{-1}(\theta, u, t) / \partial \theta]_{\theta=\bar{\theta}} (\theta_1 - \theta_2)| \\ & \leq 16/|u(t)| \leq k_1 \end{aligned} \quad (43)$$

Using (43),  $g(\theta, u, t)$  satisfies Assumption 4. Assuming

$a = 0.5$ ,  $b = 0.5$ , and  $\sigma = 0.01$  in (12), we get

$$L = \begin{bmatrix} 0.3295 & -1.1177 & -0.1068 \\ -46.4930 & 1.7099 & -1.3542 \\ -0.3295 & -0.5426 & 1.1642 \end{bmatrix}$$

$$P = 10^9 \times \begin{bmatrix} 0.4429 & 0.1460 & 0.5995 \\ 0.1460 & 0.2350 & 0.2827 \\ 0.5995 & 0.2827 & 1.1863 \end{bmatrix}$$

The FTC gain matrices are calculated using (27) and (28) by choosing  $W = 0.5I_3$ .

$$K = \begin{bmatrix} 2.1328 & 2.7917 & 0.3792 \\ -0.9533 & -0.9422 & 1.1100 \\ -0.3671 & 0.9066 & -3.3887 \end{bmatrix}$$

$$D_1 = \begin{bmatrix} 0.0064 & -0.0017 & -0.0020 \\ -0.0803 & 0.0556 & 0.0247 \\ 0.0025 & -0.0006 & -0.0018 \end{bmatrix}$$

$$D_1 D = \begin{bmatrix} -1.0000 & 0.0000 & 0.0000 \\ -0.0000 & -1.0000 & -0.0000 \\ 0.0000 & 0.0000 & -1.0000 \end{bmatrix}$$

$\theta_H = 1$  and the fault is introduced in the system at  $t = 40s$ . The initial conditions of the observer states and the fault diagnosis algorithm are taken as zero. With these matrices, an adaptive observer is designed. Figs. 1-2, respectively, illustrate the system residuals in fault-free and faulty scenario. The simulation result for fault estimation is depicted in Fig. 3. The comparison of fault-free with FTC scenarios is shown in Fig. 4. The simulated results show that the asymptotic convergence of faults is achieved. Using the fault diagnosis algorithm in comparison to the system outputs without faults, the designed FTC ensures the system stability and performs well in faulty scenario. Simulation results can also be validated from [5, 6].

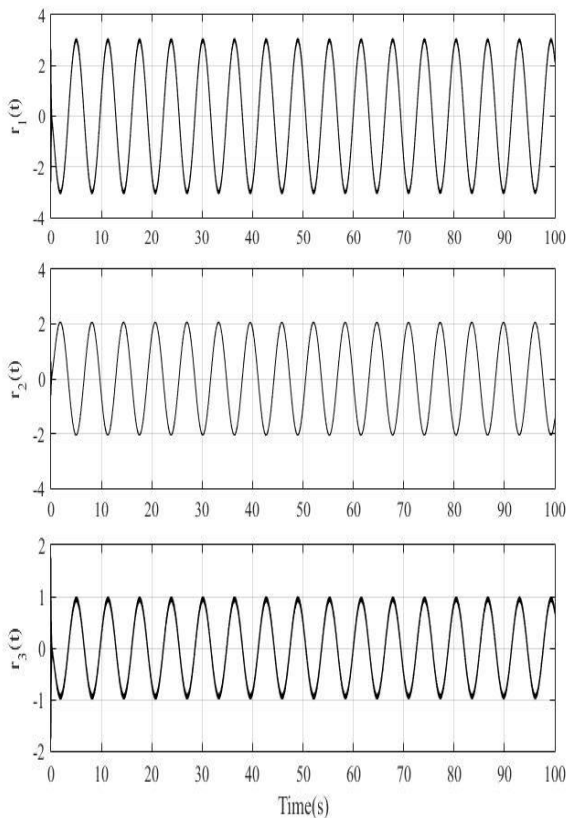


Fig. 1. System residual  $r(t)$  in fault-free scenario

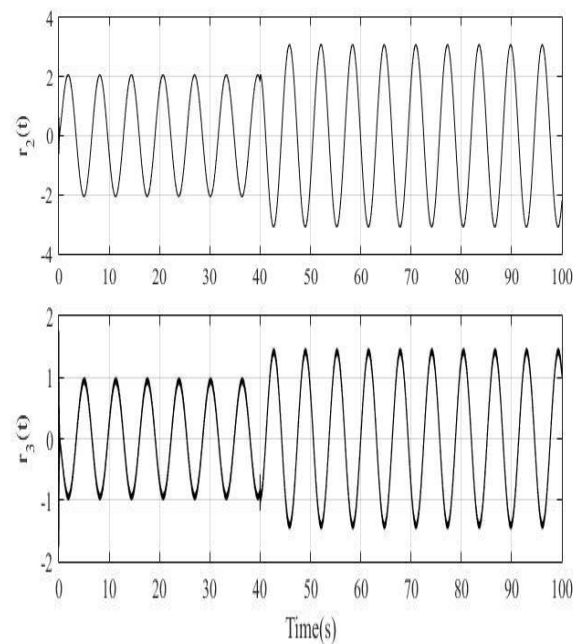
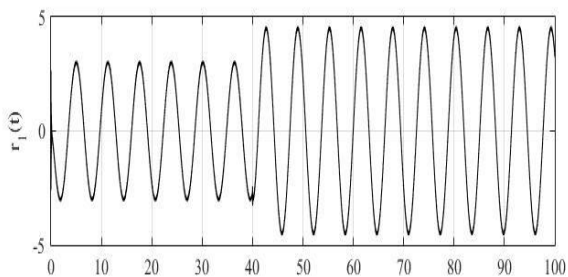


Fig. 2. System residual  $r(t)$  in faulty scenario

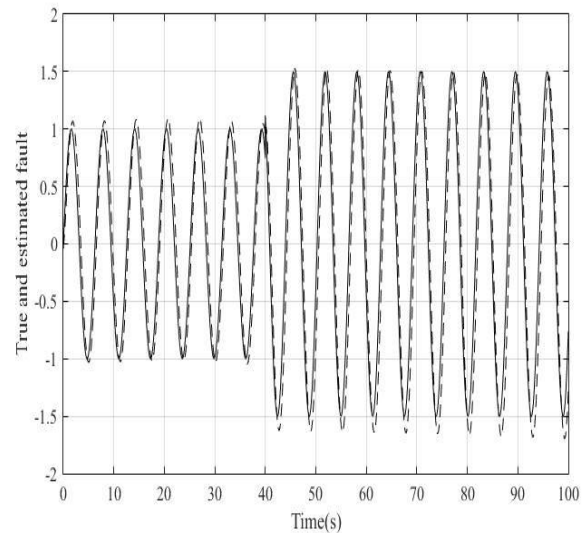
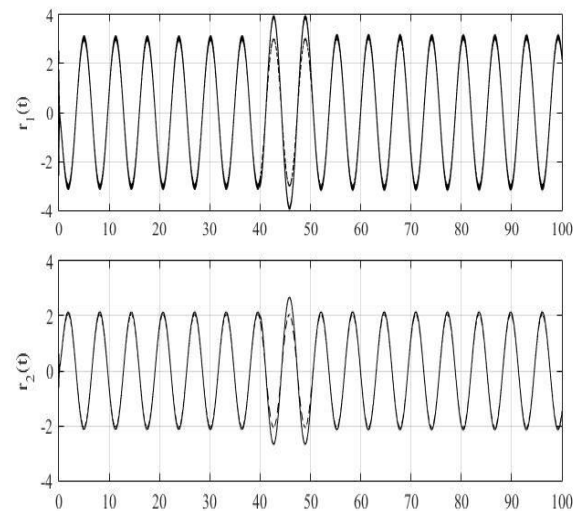


Fig. 3. True (solid line) and estimated (dotted line) fault



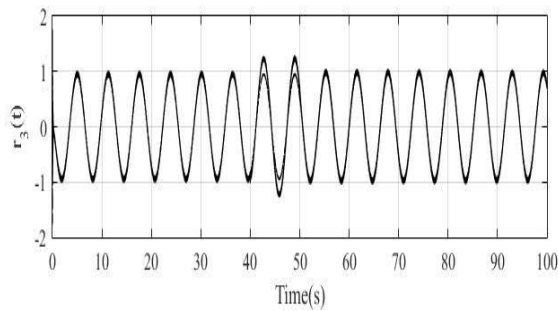


Fig. 4.  $r(t)$  in fault-free and FTC scenarios (dotted and solid lines)

## VI. CONCLUSION

This work proposed a sensor FDD algorithm based FTC design for PEMFC systems with disturbances, uncertainties and time-delays. The time-delays occur in the states. An FDI and FDD observer is constructed individually. The FDD based adaptive observer can estimate the sensor fault. The Lyapunov principle gives the required adaptive law for fault estimation. The simulations of the cathode pressure model of the PEMFC system depict the efficacy of FTC design proposed.

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