DOA Estimation of Wideband Cyclic MUSIC Algorithm under Rayleigh Fading Environment in MIMO Systems

N. V. S. V. Vijay Kumar, K. Raja Rajeswari, P. Rajesh Kumar

Abstract- Direction of arrival (DOA) estimation has, for quite some time been a challenging situation in most of the wireless communication applications, radar and sonar. The resolution of the direction of arrival estimation can be increased using the help of array signal processing. The performance of the direction of arrival estimation for multiple input multiple output(MIMO) radar systems has been reviewed for cyclic Multiple Signal Classification(MUSIC), extended cyclic MUSIC and Wideband cyclic MUSIC under Rayleigh fading environment. MUSIC and its variants have been taken into consideration for the analysis as these have been a very good parameter estimation technique due its low cost, high resolution and stability. Direction of Arrival estimation clubbed with cyclostationarity has been included into the new algorithm because of its immunity to noise and interference. The new algorithm along with cyclic correlations when applied to these signals, improves the performance of the entire system substantially. The performance of this wideband cyclic MUSIC high resolution direction of arrival estimation algorithm over the Rayleigh fading is analyzed in this paper. The simulation results citing the three methods show the performance of these methods in presence of the fading environments.

Index Terms - Direction of arrival, Multiple Input Multiple Output(MIMO) radar, cyclostationarity, cyclic MUSIC, extended cyclic MUSIC, Wideband Cyclic MUSIC, Rayleigh fading.

I. INTRODUCTION

Radar signal processing has been an area where the direction of arrival estimation has been of prime importance that estimates the angle of the signal to be imposed on the antenna array[1]. Direction of arrival refers to the direction in which the beampattern focuses on a particular point called the beamforming[2-3]. With the advancement of technology many high resolution beamforming techniques are available like the Cyclic MUSIC, Extended mode of cyclic MUSIC and the Wideband cyclic MUSIC [4-8] are being investigated for the exact evaluation of the direction of arrival which is useful for the target detection. There is always a mismatch between the direction of the signal and the inbound steering vector. So, with proper selection of antenna array algorithm the beampattern can be analyzed. The challenge faced by these direction of arrival estimation algorithms is that many signals are received simultaneously with different amplitudes and orientations. Noise also plays a vital role at the receiving end. So, it is very important to have an efficient estimation algorithm with high resolution and accuracy, like the various MUSIC algorithms.

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MUSIC is a very high resolution direction of arrival estimation approach that is optimized for antenna array signal processing [9-14]. The Eigen Vector Decomposition(EVD) is done for the covariance matrix which extracts the original signal and the noise subspaces that are orthogonal to each other. Various high resolution variants of cyclic MUSIC have been proposed which are very effective in direction of arrival estimation that use cyclostationarity in signals due to their immunity to noise and interference. Using this method that uses cyclostationarity, the signals' evaluation of the direction of arrival[15-16] can be improved accordingly, under low SNR and also subsequently reduces the number of computations.

Most of the wireless communications channels have a predominantly significant multipath propagation which cause the signal to fade at the receiver. A fading channel parameter is applied to the cyclic MUSIC methods. Rayleigh fading model[17-18] is introduced into the model here to analyze and enhance the performance of the direction of arrival estimation.

II. CYCLIC MUSIC

The algorithms used here for the analysis are the different variants of Cyclic MUSIC direction of arrival algorithms.

For the Cyclic MUSIC method, consider n_α sources are emitting cyclostationary signals[12] bearing a cyclic frequency of α, where $n_\alpha \leq n$. For preliminary assumption consider f(t) consists of the n_α signals which indicate the cyclic frequency α, excluding the $n - n_\alpha$ signals that do not have the cycle frequency α. Any sort of noise signals taken into consideration are summed up into a vector n(t). The Cyclic autocorrelation and the conjugate autocorrelation function at the cycle frequency α with a delay τ are non zero and can be determined by

$$\mathrm{R}_{\mathrm{xx}}^{\alpha}(\tau) = \frac{1}{\mathrm{M}}\sum_{m=1}^{\mathrm{M}}[\mathrm{x}(t_m + {}^{\tau}/_{2})\mathrm{x}^{H}(t_m - {}^{\tau}/_{2})]e^{-i2\pi\alpha\, t_m} \tag{1}$$

$$\mathrm{R}_{\mathrm{xx}}^{\alpha\,*}(\tau) = \frac{1}{\mathrm{M}}\sum_{m=1}^{\mathrm{M}}[\mathrm{x}(t_m + {}^{\tau}/_{2})\mathrm{x}^{T}(t_m - {}^{\tau}/_{2})]e^{-i2\pi\alpha\, t_m} \tag{2}$$

where M is the total number of samples.

Variants of the Cyclic MUSIC method use the Singular Value Decomposition(SVD) for deriving the correlation matrix rather than the Eigen Value Decomposition(EVD) used by MUSIC algorithm for deriving the covariance matrix.

Cyclic MUSIC algorithm can be implemented for lesser number of samples by the following procedure:

a) Estimate the matrix $R_{xx}^{\alpha}(\tau)$ by using (1) or the matrix $R_{xx}^{\alpha\,*}(\tau)$ by using (2)
b) Calculate SVD using

$$[V_s \quad V_m]\begin{bmatrix}\sum_s & 0\\ 0 & \sum_m\end{bmatrix}[W_s \quad W_m]^H \qquad (3)$$

where $[V_s \quad V_m]$ and $[W_s \quad W_m]^H$ are the singular and the cross sectional elementary values of the respective matrices $\sum_s$ and $\sum_m$ which are organized in the descending order. $\sum_m$ reduces to zero as the count on the total samples increase.

c) Calculate the minimum and maximum values of $\|V_m^H c(\phi)\|^2$ and $\|V_s^H c(\phi)\|^2$.

III. EXTENDED CYCLIC MUSIC

The conventional model is extended in a manner such that the cyclostationary of the inbound signals is exploited to form an extended value data vector.

$$M_{cc}(t) = [m(t) \quad m^*(t)] \qquad (4)$$

Cyclic correlation matrix estimate for the extended data vector is given by

$$R_{cc} = \frac{1}{M}\sum_{m=1}^{M} I_{2N}^{\alpha}(t_m) x_{cc}(t_m + \tau/2) x_{cc}^H(t_m - \tau/2) \qquad (5)$$

where $I_{2N}^{\alpha}(t)$ is the time dependent matrix is defined by

$$I_{2N}^{\alpha}(t) = \begin{bmatrix} I_N e^{-i2\pi\alpha t} & 0\\ 0 & I_N e^{+i2\pi\alpha t}\end{bmatrix} \qquad (6)$$

Finally, calculating the Singular Value Decomposition of R_{cc} that is much similar to Cyclic MUSIC method, the power spectrum of the Extended mode of the cyclic MUSIC method is determined spatially by

$$S(\phi) = \frac{1}{c^H(\phi)V_m c(\phi) - \|c^T(\phi)V_m c(\phi)\|} \qquad (7)$$

This method ispurely used only for cyclostationary signals and it is a signal selective method for the signals of interest(SOI) and has no proper limitation.

IV. WIDEBAND CYCLIC MUSIC

The Wideband Cyclic MUSIC algorithms' main functionality is to extract the data information from the spectral cross correlation density of the sensor arrays. The spectral cross correlation density matrix is given by

$$M_k^{\beta}(f) \triangleq \mathcal{E}\{\vec{Y}\left(f+\tfrac{\beta}{2}\right)\vec{Y}^{H}\left(f-\tfrac{\beta}{2}\right)\} \qquad (8)$$

where $M_k^{\beta}(f)$ is the spectral cross correlation density of the signals of interest for cyclostationary frequency β. Let the singular value decomposition of $M_f^{\beta}(f)$ be

$$M_k^{\beta}(f) = X(f)\Sigma(f)W^{H}(f) \qquad (9)$$

The projection variable that is directed on the signal arrays at frequency value of f are obtained from the spectral cross correlation density matrix for the frequencies pertaining to $f+\frac{\beta}{2}$.

$$\begin{aligned} Q(f) &= X_M\left(f+\tfrac{\beta}{2}\right)X_M^{H}\left(f-\tfrac{\beta}{2}\right)\\ &= W_M\left(f+\tfrac{\beta}{2}\right)W_M^{H}\left(f-\tfrac{\beta}{2}\right)\end{aligned} \qquad (10)$$

where X_M and W_M bear the X and W columns of the M non zero singular values.
The direction of arrivals are derived from the estimated values of the cyclic cross spectral density $\widehat{M}_f^{\beta}(f)$, using singular value decomposition, given by

$$\widehat{M}_k^{\beta}(f) = \hat{X}(f)\hat{\Sigma}(f)\widehat{W}^{H}(f) \qquad (11)$$

The direction of arrival estimation can be derived from the estimated noise subspace or from the average of the direction of arrival estimate of the multiple evaluation frequencies. These multiple evaluation frequencies are basically used to avoid interference. Using the Wideband mode of the Cyclic MUSIC method the direction vectors values can be obtained through the spectral cross correlation matrix, given by

$$\begin{aligned} M_k^{\beta}(f) &= \mathcal{E}\left\{\vec{Y}\left(f+\tfrac{\beta}{2}\right)\vec{Y}^{H}\left(f-\tfrac{\beta}{2}\right)\right\}\\ &= M_z^{\beta}(f)\vec{c}\left(f+\tfrac{\beta}{2}\right)\vec{c}^{H}\left(f-\tfrac{\beta}{2}\right)\end{aligned} \qquad (12)$$

where $M_z^{\beta}(f)$ is the signals of interest cyclic cross spectral density, and $\vec{c}\left(f+\frac{\beta}{2}\right)$ are the required direction vectors.

V. PROPOSED METHOD

The performance analysis of MIMO radar primarily is based on direction of arrival estimation. The angle of incidence depends on many factors like the SNR, polarization and estimating the number of signals. The carrier synchronization is the most important factor that is performed by various methods like the Fast Fourier Transform(FFT) and phase differentiation. But these methods are computationally more complex. So we propose various variants of Cyclic MUSIC for the detection and estimation of direction of arrivalin the presence of fading. The Rayleigh fading [19-21]is introduced which is very useful when there is no proper Line of Sight(LOS) .
MIMO radar model is given by

$$z_p(t) = \sum_{m=1}^{M}\alpha_p(m)y_{pm}(t) + n_p(t) \qquad (13)$$

where $\alpha_p(m) = \alpha(m)b_p(\theta)$, $b_p(\theta)$ is the output response of the antenna and $y_{pm}(t)$ is signal that is transmitted from the mth user to the pth signal. $\alpha_p(m)$ is the Rayleigh fading coefficient and $n_p(t)$ is the complex Gaussian noise. Here a non coherent element by element channel fading is considered through which each of the antenna gets a replica of the signal that is transmitted with a different fading parameter. Huge objects just scatter the signals that are transmitted by the transmitter. For the given Rayleigh fading model, all the frequency components that have equal amount of fading have to be analyzed. MUSIC variants have been seen to derive the signals even in flat fading environments. Signals passing through a fading channel experience Doppler shift and lower SNR values. The problem of identifying different copies of the same signal in a fading environment, arriving at the receiver, can be distinguished using cyclic MUSIC algorithms. The algorithms are used to find the difference between the spectra of these signals.The signals that are taken into consideration here are the signals with different frequencies which are passed through a Rayleigh fading channel.

VI. SIMULATION RESULTS

The DOA estimation performance is majorly dependent on the spatial covariance matrix of a received array signal and the accurate array modeling. A signal with higher SNR is to be received from target to estimate the accurate direction of arrival estimation of received incoming signal. Because of the interference between signals and also contribution from the environmental noises, the direction of arrival estimation of the signals is limited by the low SNR values. To present the simulation values, consider a uniform linear array(ULA) of N =16 array which are equally spaced by $\lambda/2$ distance. The received signals are finally added with additive White Gaussian noise with SNR of 10dB. Higher bit rates are taken for the given signal and lower bit rates are taken for the signal which is introduced as interference. The simulations for the algorithms are given to emphasize and analyze the performance analysis of the proposed method. QPSK signals are introduced which perform better than FM signals because the bandwidth requirement is less for QPSK signals. Also the number of samples of data required by the conventional MUSIC is more than the cyclic MUSIC and Wideband MUSIC algorithms to obtain similar RMSE values.

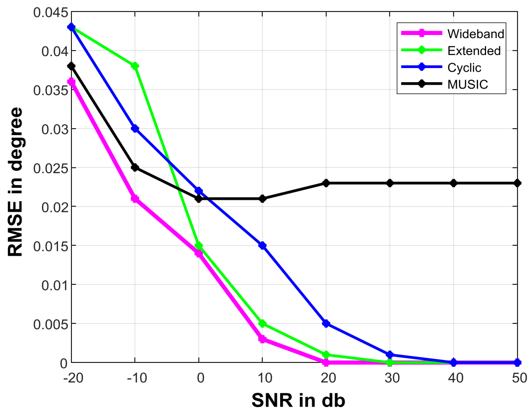

Fig.1. RMSE comparison for various MUSIC variants.

Fig.1 shows the performance comparison of Wideband cyclic MUSIC with the remaining MUSIC variants for all considered range of SNR values. The wideband cyclic MUSIC yields better direction of arrival estimation performance than other MUSIC variants as it produce better RMSE for a given SNR value also it can be noted from Fig.1 that RMSE values of wideband cyclic MUSIC tends to zero for SNR more than 20 dB which indicates a predominant performance enhancement compared to other MUSIC variants. For, an instance from Fig.1, at 0dB and 10db SNR values Wideband cyclic MUSIC are almost 0.005 lesser RMSE than the conventional cyclic MUSIC methods. As we are aware that sudden spurious peak noise addition to a signal received from target decreases the overall estimation accuracy, it is worthy to consider the simulation performance of Wideband cyclic MUSIC with addition of spurious peak noise with the received low SNR values.

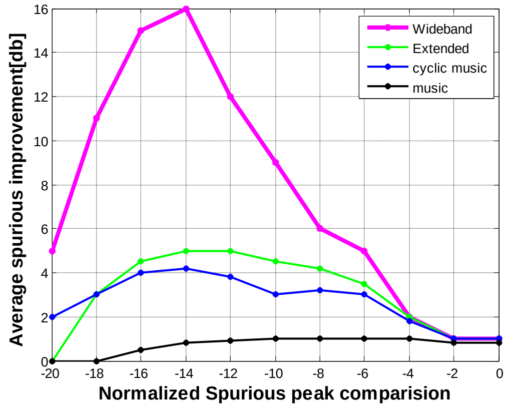

Fig. 2 Spurious peak characteristics for the Wideband and cyclic MUSIC algorithms.

Fig. 2. displays the performance comparison of the Wideband cyclic MUSIC at low SNR, with spurious noise peak in the direction of arrival spectrums that increase the ambiguity in measuring the accuracy in the direction of arrival estimation. To estimate the performance of Wideband cyclic MUSIC with other variants, a parameter called average spurious improvement is calculated at various normalized spurious peak values of noise for all the proposed and considered MUSIC variants and plotted in the Fig.2. and it can be observed that the Wideband cyclic MUSIC dominates the other considered MUSIC variants. For an instance from Fig.2. considering the spurious peak noise value of 5 added to received signal with low SNR of -10 dB the average spurious improvement for Wideband cyclic MUSIC is more than 9 dB which is almost 4dB more than extended MUSIC variant which indicates that it is possible to estimate the direction of arrival with more accuracy using proposed than extended variant.

VII. CONCLUSION

The most important problem of the direction of arrival estimation for multi target scenario in MIMO radar under fading conditions has got addressed. Wideband MUSIC is far more responsive and doesn't require large number of antenna elements than the conventional MUSIC algorithms. High resolution and high signal receptiveness is achieved using Wideband MUSIC. The algorithm uses cyclostationarity that improves the signal detection in a noisy Rayleigh fading environment in MIMO radar. Wideband cyclic MUSIC also gives the exact detection of signals of interest and eliminates the interference signals. The performance comparison of Wideband cyclic MUSIC with other MUSIC variants in terms of RMSE and Average spurious improvement is simulated under different SNR values. The simulation results show that Wideband cyclic MUSIC is far more superior than MUSIC. Also, Wideband cyclic MUSIC is more advantageous when greater narrowband interferences are present.

REFERENCES

1. Krem, M. Vibergh, "Two decades of array signal procesing research: the parametric approach," *IEEE Signal Processing Magazine.*, vol. 13, no. 4, pp. 67–94, 1996.
2. John. Li, P. Stoicaa, "MIMO radar with co - located antennas," *IEEE Signal Processing Magazine.*, vol. 24, pp. 106–114, 2007.
3. John. Li, P. Stoicaa, "*MIMO Radar Signal Processing"*. Wiley 2009.
4. Gardener. W.A, "Simplification of MUSIC and ESPRIT by exploitation of cyclostatonarity," *Processing. IEEE*, vol. 76, pp. 845–847, 1988.
5. P. Stoicaa, A. Nehoray, "MUSIC, Maximum Likelihood and Cramer-Rao Bound'' *IEEE Transaction ASSP*, ASSP-37 720-741, 1989
6. Schel, S.V."Performance analysis of the Cyclic MUSIC method of DirectionEstimation for Cyclostationary Signals. Trans. 1994
7. P. Charge, Yidei Wang and J. Sailard, "An extcnded cyclic MUSIC algorithm," IEEE Trans. Sigrid Prucessing, Vol. 5 I, 2003
8. Kevin D. Mauk, "Wideband Cyclic Music", IEEE 1993.
9. R. Schmidth, "Multiple Emitter Location and Signal Parameter Estimation", IEEE Transactions on Antennas and Propagation, Vol. AP-34, No. 3, pp. 276-280, 1986
10. Gucluoglu, T. M. Duman. "Performance Analysis of Transmit and Receive Antenna Selection over Flat Fading Channels", *IEEE Transactions on Wireless Communications*, volume 7, pp. 3056–3065, 2008
11. H. Dai, Q. Zhou, "Asymptotic analysis in MIMO diversity, systems," Processing International Symposium. Intelligent Signal Processing Communication Systems, 2005.
12. Xu, G., T. Kailath, "DirectionofArrival Estimation via Exploitation of Cyclostationarity – A Combination of Temporal and Spatial Processing," *IEEE. SP* 40 1992, 1775-1786.
13. A. Hasanien and S. A. Vorobyov, "Transmit energy focusing for DOA estimation in MIMO radar with colocated antennas," *IEEE Trans. Signal Processing*, vol. 59, no. 6, pp. 2669–2682, 2011.
14. A. Khabazibasmenj, A. Hasanien, S. Vorobyov, and M. Morrency, "Efficient transmit beamspace design for search-free based DOA estimation in MIMO radar," *IEEE Trans. Signal Processing*, vol. 62, no. 6, pp. 1490–1500, 2014.
15. L. Xu, John. Li, and P. Stoicaa, "Target detection and parameter estimation for MIMO radar systems," *IEEE Trans. Aerosp. Electron. Syst.*, vol. 44, no. 3, pp. 927–939, 2008
16. G. Hua and S. S. Abeyskera, "MIMO radar transmit beampattern design with ripple and transition band control," *IEEE Trans. Signal Processing*, vol. 61, no. 11, pp. 2963–2974, 2013
17. P. A. Dighi, R. K. Mallick, S. S. Jamuar, "Analysis of transmit-receive diversity in Rayleigh fading," IEEE Trans. Commun, vol. 51, no. 4 2003, pp. 694–703.
18. Y. X. Li and X. J. Haung, "The simulation of independent Rayleigh faders," IEEE Trans. Commun., vol. 50, no. 9, pp. 1503–1514, 2002.
19. Julio Araiuz, "Discrete Rayleigh Fading Channel Modeling", Department of Information Sciences and Telecom, University of Pittsburgh,135 N. Belle field Ave., 2002.
20. Yahong Rosa Zhing and Chenshan Xiao, Senior Member IEEE, "Simulation Models With Correct Statistical Properties for Rayleigh Fading Channels", IEEE Transactions on Communications, Vol. 51, No. 6, 2003.
21. C. S. Patil, G. L. Styuber, and T. G. Pratt, "Comparative analysis of statistical models for the simulation of Rayleigh faded cellular channels", IEEE Transacton Communication., vol. 53, pp. 1017-1026, 2005.