

Interval Arithmetic based Adaptive Filtering Technique for Removal of Noise in Audio Signal



Akshay V. Nagashetti, Soumya S. Patil, Rajashekar B. Shettar

Abstract: Active noise cancellation is one of the fundamental problems in acoustic signal processing. The proposed work focuses on the enhancement of audio signal quality by cancelling the noise using interval analysis (arithmetic). An adaptive filters basically works on the concept of optimal weight calculations which is an optimization problem. This optimization problem can be more effectively solved using interval analysis. Interval analysis gives the boundary of the weight coefficients. Using interval Newton method, the weight coefficients are found. This algorithm is tested for noise cancellation of speech signal. The three adaptive filters algorithm used for comparison with the obtained results are Least Mean Square (LMS), Recursive Mean Square (RMS) filters and Kernel based filters. It is observed that the parameters mean square error is very less. The speed of convergence and signal to noise ratio is improved as compared to kernel methods. But processing time is very high and computational cost is doubled, as interval data includes infimum and supremum values. This algorithm can be used in noise cancelling headphones.

Index Terms: Adaptive noise cancellation (ANC), LMS algorithm, Kernel adaptive filter, RLS algorithm, Interval analysis

I. INTRODUCTION

ADAPTIVE filters are one of the best useful in cases where conditions of signal or parameters of system are slower changes and for filters for the adjustment to compromise for the change. The easily understood but very great power filter can be called as combiner linear adaptive, for which will be nothing more compared to adjustable FIR filter. The criterion of LMS can be a search algorithm which will be using for providing the strategy for adjustment for the coefficients of filters. In the IIR and FIR conventional digital filters, which will be assumed for the parameters process for determination of the filter characteristics is known.

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They will be changed according to time, except the variation of the nature is known to be assumed. The filter coefficient can be practically compensated to adjust according to the changes in the output [10], [15]. An adaptive system can learn the characteristics of the signal to keep track of the slow changes.

Kernel adaptive filter will be very helpful when there is uncertainty about the signal characteristics or when these change in the characteristics. Adaptive kernel filters are being used within the ALE and

ANC structures and are basically used in the linear algorithms such as the Recursive Least Squares (RLS) and the Least Mean Square (LMS) algorithms [4], [18]. Practically these models provide satisfactory performances for Gaussian noise, but the performances deteriorate dramatically for non-Gaussian noises. Most of the existing system produces non linear signals. This adds more complexity and degrades the signals quality to address the signal recovery and Extraction of signals from these nonlinear signals, implementation of Volterra utilizing filters, neurofuzzy methods and neural networks would help [2], [25], [27], [28]. Though the adaptive filters show a very good performance in a wide range of applications, they are not suitable for implementation in nonlinear models as the time taken for convergence is more and there is possibility of getting assured to the local minima. There exists alternative method of implementing the adaptive kernel filter, where the data can be transformed by the help of Reproductive Kernel Hilbert spaces (RKHS) filtering techniques which is related to the input original spaces in a nonlinear manner [20]. The significance in the evolution of the adaptive kernel filtering algorithms can be known as the Kernel LMS (KLMS) algorithm [6], [23], [28], [28]. In these adaptive filter methods the improvement can be done on mean square error by using interval analysis based optimization algorithms. The audio signal samples can be represented using minimum and maximum bounds as in interval arithmetic. Interval analysis is one of the powerful tools to handle the rounding errors and floating point errors [29]. The optimization of the filter is done by using Interval Newton method. The calculated weights of the filter are bound to certain values. This helps in enhancing the efficiency of the system [30], [31].

The rest of the paper is organised as follows, section 1 includes, introduction to adaptive algorithm, section 2 discusses the role of adaptive filter algorithm in cancelling the noise.

A review of LMS, RLS and Kernel based adaptive algorithms is briefed in Section 3. Section 4, focuses on the advantages of using interval adaptive filters in the design. Discussion and analysis of the results obtained is carried out in Section 5. Section 6 concludes the work and highlights on the work that can be carried out in future.

II. ADAPTIVE FILTER

Active Noise Cancellation (ANC) output can be used as feedback to minimise noise distribution by generating the anti noise signal.

The production of an opposite noise (180° phase shifted version) helps in cancelling out with the original signal using adaptive filters. Reduction of noise component from original signal is a critical issue which is achieved by the implementation of ANC. A demonstration of noise cancellation using a single channel feedback is shown in the Figure 1. The resultant waveform remains the same when compared with target noise, except that it will have 180 degree phase shift. If such a wave form is combined together, the resultant is a much weak residual waveform.

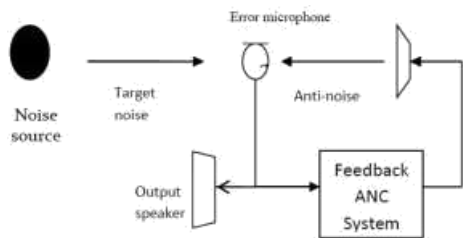


Fig. 1. Single channel feedback active noise cancellation

Enhancement of speech is one of the challenging tasks in several applications such as cellular environments, military, forensic applications, hearing aids, telecommunication signal enhancement, front-ends for speech recognition system, etc. For the enrichment of speech, concentration should not only be on the reduction of noise but also to separate signal from noise. Enhancement of speech signal is a fundamentals research area in digital signal processing applications. Enhancement of speech is a crucial problem for two reasons.

- 1) The nature of noise signals and their characteristics can change with the time.
- 2) The definitions of performance measurement differ from one variable to another variable.

Figure 2 represents the effect of ANC on the target noise. The resultant output is the residual waveform which is certain to be obtained at the receiving end or the microphone. The anti-noise signal acts upon the target noise signal by predicting the nature of the target noise.

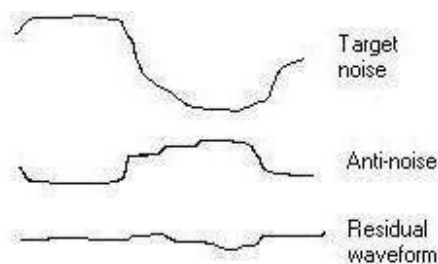


Fig. 2. Description of active noise control

As per the survey carried out, there are lot of ongoing

works for enhancement of speech signal. Adaptive filtering techniques are proven to be popular and effective ways adopted to improve the quality of the speech signal. This is because they are capable of detecting the variable parameters along with tracking the dynamic variations. The co-efficient of these digital filters change in an order such that the filter maintains an optimal state of convergence. The cost function obtained is a Mean Square of the error signal. The difference between the output signal and the derived signal is termed as an error signal. The Mean Square of this error signal gives the cost function more the adaption of the filter co-efficient, minimal is the convergence value of the Mean Square Error (MSE). This state defines that the filter has adapted by altering the coefficients to converge. At this stage the output of the filter $Y(k)$ gets closer to the desired signal $d(k)$. The input data and their characteristics can be regarded as the environment of the filter. The filter generates an advanced setof coefficients based on the change in filter environment. As the specification of the adaptive filter increases, the number of co-efficient increases. In such cases though the error converges to zero, there is an increment in the convergence delay of the filter.

III. ADAPTIVE ALGORITHMS FOR NOISE CANCELLATION

This section describes several adaptive filter algorithms such as the Least Mean Square (LMS) gradient approximation method. Recursive Least Square (RLS) adaptive algorithm and Kernel Adaptive filter algorithm, their computational models, error estimation methods and the approximation methods used.

A. Least Mean Square (LMS) Algorithm

These filter algorithm follow a stochastic gradient descent method where the adaption is based only on the error at the present time. Having been invented by Bernard Widrow in 1960, it makes use of the available data for calculating the gradient. The filter weights are updated to approach the optimum filter weights. Starting with small weights which are a zero in several cases, the weights are updated at each step by discovering the gradient of the Mean Square Error (MSE). Positive value of MSE gradient indicates the same weight is used in the iterations ahead there is a possibility of increment in the error. It is therefore necessary to reduce weight. At the same time, there is need to raise the weight in cases where the MSE-gradient values are negative. The weights are updated based on the relation (1)

$$W_{k+1} = W_n - \mu \nabla_{\epsilon}[k] \quad (1)$$

μ represents the convergence coefficient and the mean square error is represented by ϵ . Hence the weight gets updated by maintaining the stability of the system. The filter output are computed based on the relation (2) and

$$\eta'_k = \sum w(i)x_{k-i} \quad (2)$$

the corresponding weights of the filter gets updated as per the (3)

$$w_{k+1}(i) = w_k(i) + 2\mu\epsilon_k x_{k-1} \quad (3)$$

In adaptive algorithms, the LMS attains the popularity for its simplicity and the behaviour depends on the data convergence factor. The speed

of the algorithm is comparatively lower. The step size of iterations in the algorithm being fixed tends to be a disadvantage for altering the parameters. Therefore the signal input statistics have to be analyzed before implementing them to the adaptive filters [28].

To extend the application of adaptive linear filters, modelling of nonlinear systems can be considered. Implementation of these nonlinear parameters in the Adaptive Line Enhancement (ALE) design method would help in updating the coefficient of FIR filter. This helps in addressing variable step sizes and also provides the exact gradient values. Hence it can help in enhancing the speech signal quality. The Volterra series is a commonly used model to address the non-linear characteristics of the system. This can further be accomplished by implementing the kernel adaptive algorithms. The kernel methods used in these methods help in mapping the signal to a higher dimension of linear feature space. The nonlinear behaviour can be comparatively taken as the sum over this kernel, the domain of which is the feature space. This method can be regarded as a universal approximation for addressing nonlinear characteristics if it can be executed in a Reproducing Kernel Hilbert Spaces (RKHS). The Kernel methods possess the recognition of bearing the convex loss functions without any local minima. The implementation is also moderately complex. Hence in the proposed design the performance of nonlinear algorithms are exploited using kernel filters with the help of ALE for enhancement of speech signal even in the presence of noise. KLMS algorithm which is the kernel version of least mean square is therefore attracting the researches as it helps in stabilizing the system.

The functionality of the proposed adaptive algorithm where the interval values are considered can be briefed as follows:

- 1) The loop variables and weight parameters (W) are set to zero.
- 2) The parameters get updated based on the input signal values obtained at the microphone.
- 3) The step size 'μ' (in terms of time intervals) at which the outputs are expected from the filters are computed and are used further for estimating the errors in the signal.
- 4) The weights are updated based on the fixed step sizes. The above steps are repeated so that the loop parameters get equal to the buffer sizes. The functionality of the proposed interval adaptive algorithm which works on the basis of adaptive algorithms using kernel filters is as shown in Figure 3.

The consideration of LMS values helps in enhancing the stability of the system, the condition for which is $0 \leq \mu \leq 2$. The stepsizes can be varied to larger values with enhancement in the adaptive rates.

B. Recursive Least Square (RLS) Adaptive filter

The Recursive Least Square (RLS) is an algorithm used to estimate the coefficient in order to get a reduced weighted linear least square cost function in relation with the input signals. The algorithm attempts to update the cross-correlation and auto matrices directly by the techniques of Wiener Hopf equation. Though being complex as compared with the Normalized Least Mean

Square (NLMS) and LMS algorithm, the RLS algorithm proves to perform better in terms of error generation and convergence rates.

RLS uses an adaptive filter which determines the coefficient in the recursive way and helps in reducing the weighted linear least square in relation with input signals. This provides the cost function and helps in proving the RLS performs better with varying environmental conditions and varying time. The algorithm is computationally complex and has problems associated with stability. The following steps explain the adaptivity of the output parameters $d(n)$ with adaption in the input signal $x(n)$.

- 1) The weight parameters are initially set to zero.
- 2) $0 \leq \lambda \leq 1$ is regarded as the forgetting factor. This helps in reducing the weights exponentially as compared to the earlier error samples.
- 3) For any given input $x(n)$, the weighted covariance matrix is given by the relation (4)

$$R_x(n)w_n = r_dx(n) \quad (4)$$

where $R_x(n)$ represents the weighted co-variance matrix $r_dx(n)$ gives a nearly matching estimate for the cross covariance between $d(n)$ and $x(n)$.

- 4) The error is estimated by the relation

$$e(n) = x(n) - w^T(n-1)y(n) \quad (5)$$

- 5) The gain vector is given by

$$k(n) = \lambda^{-1}P(n-1)x(n)[1+x^T(n)(\lambda^{-1})P(n-1)x(n)]^{-1} \quad (6)$$

$$k(n) = P(n-1)x(n)[\lambda + x^T(n)P(n-1)x(n)]^{-1} \quad (7)$$

- 6) The co-efficient vector that reduces the cost function is evaluated based on the relation

$$w(n) = w(n-1) + \Delta w(n-1) \quad (8)$$

where $\Delta w(n-1)$ represents the correlation factor at (n-1) time interval

- 7) Considering the gain vector, the recursive function that can be used for updation of the correlation matrix is given by

$$P(n) = \lambda^{-1}P(n-1) - k(n)x^T(n)\lambda^{-1}P(n-1) \quad (9)$$

C. Kernel adaptive algorithm

The adaptive kernel filtering is a way of carrying the particular tasks that uses filtering by adaptive techniques for nonlinear problems systems in general. This is an existing generalised method of adaptive linear filtering for the reproduction of Hilbert kernel spaces. Adaptive kernel filters are online methods of kernel, which is closer in relation with few neural artificial networks such as regularization networks and radial basis function networks. The algorithm for KLMS can be a gradient stochastic methodology for solving the RKHS least squares problems in least squares method. Since the updated equation could be written with the inner product, input space are efficiently computed in KLMS. The high standard of approximated abilities for the stems of KLMS from the data transformed, include



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possible features of infinite original data. This is non parametric model and a good technique which can change the data inputs into a feature of high dimensional space through a kernel reproduction for which the production of inner operation in the feature space could be calculated in an effective way through the kernels.

In the recent times the linear methods of appropriation are being applied on data transformation. This is advantages as compared with the traditional methods. This approach helps in achieving the desired signal by adopting kernel principle for analysis of the components using Support Vector Machines (SVM). SVM's are proven to perform well in terms of accuracy and in the enhancement of speech activity [26].

IV. INTERVAL ADAPTIVE FILTER

Using interval analysis the adaptive filter is designed in the proposed paper. The use of interval analysis is as follows.

- 1) In computers, the signal amplitude can be disturbed by rounding off during quantization or the processor floating point round off errors
- 2) In the real world the data errors due to measurement may occur and hence may give imprecise data

Floating point arithmetic suffers from round off errors, because of the limitations of the computer data representation. Also the other reasons are representing the real numbers using IEEE 754 format (Ex. 1/3), truncation errors etc. Such errors can be neglected if the numbers of calculations are less, but for huge number of calculations the error accumulates and may lead to disasters.

However, numerical mathematics research shows that, with a more optimal vector arithmetic and comprehensive, reliable results can be more easily obtained when dealing with extensive and huge problems. Computers with this kind of arithmetic have proved the significance of this development in many successful applications. Until now, it has been assumed that an optimal vector arithmetic could not be implemented on supercomputers. The users, therefore, had to choose between either lengthy computation times and accurate results on general purpose computers or comparatively short computation times and possibly wrong results obtained on supercomputers [15].

The basic operations of interval arithmetic are, for two intervals $[a, b]$ and $[c, d]$ that are subsets of the real line $(-\infty, \infty)$.

- 1) $[a, b] + [c, d] = [a + c, b + d]$
- 2) $[a, b] - [c, d] = [a - d, b - c]$
- 3) $[a, b] * [c, d] = [\min(a * c, a * d, b * c, b * d), \max(a * c, a * d, b * c, b * d)]$
- 4) $[a, b] / [c, d] = (a \div c, a \div d, b \div c, b \div d)$ when $0 \notin [c, d]$ is not in $[c, d]$

Interval division containing zero is not found in the basic arithmetic interval. The multiplication and addition operations are associative, commutative and sub-distributive: the set $X(Y + Z)$ is a subset of $XY + XZ$.

Apart from working with an uncertainty real work with the both ends of the interval $[a, b]$ can contains $x:x$ which lies between a and b , or could be among one of them.

Likewise a function which is applied to it is also be uncertain. Instead, of working in interval arithmetic produces an interval $[c,d]$ which may be the possible values $f(x)$ for all $x [a,b]$.

A. Proposed Interval adaptive algorithm:

Stage 1: Find the optimum weight using Interval Newton Method during learning stage
 Stage 2: Identify the box width of a signal and the weight matrix from
 Stage 3: Estimate the signal using set inversion via interval analysis (SIVIA). The algorithm using SIVIA is as shown in figure 3

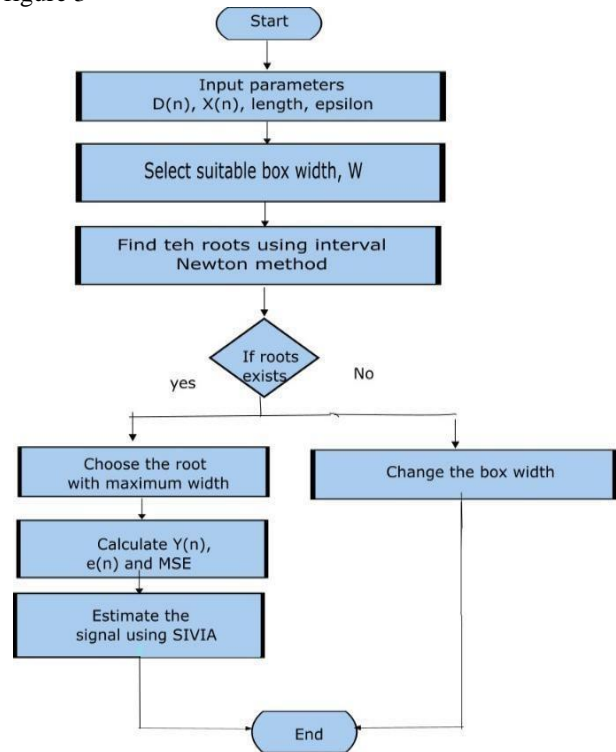


Fig. 3. Block diagram of proposed interval adaptive filter

B. Implementation Details

The core of our solver is written mainly in MATLAB with the use of MEX (Matlab Executable) files for communication with the parts written in C++. Interval computations are handled by INTLAB [15] and the Boost interval arithmetic library [7]. The visualization tool is written entirely in C++ and contains both the main function of C++ application and the gateway function for MEX, so it can also be used as a self- standing application outside of the MATLAB environment. The tool is accepting input generated by other solvers as well, if their solutions are pavings. The tool allows visualizing several pavings at once. The CImg Library [16] is used by the graphical user interface for image processing and interaction with the user.

```

Algorithm SIVIA (in: f, Y, [x], ε, inout: [ Xl, Xu ])
1 if [f]([x]) ∩ Y = ∅ return;
2 if [f]([x]) ∈ Y then
3 { Xl := Xl ∪ [x]; Xu := Xu ∪ [x]; return };
4 if w([x]) < ε then { Xu ∪ [x]; return };
5 SIVIA (in: f, Y, [x], ε, inout: [ Xl, Xu ]);
  
```

Fig. 4. Algorithm of set

V. EXPERIMENTAL RESULTS

The plot of input signal, noisy signal, recovered signal and the error are as shown in Figure 5. It is observed that the input signal with noise and the recovered signals are almost

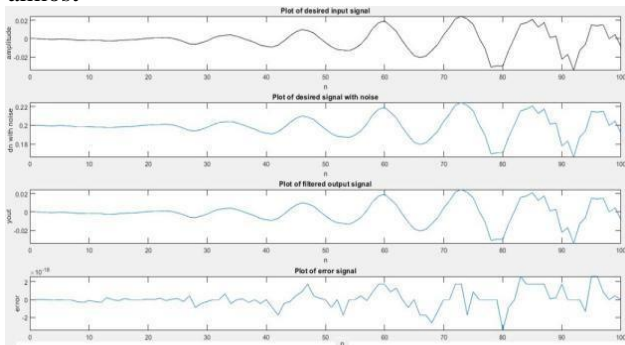


Fig. 5. Plot of input signal, input signal added with noise, recovered signal and the error signal

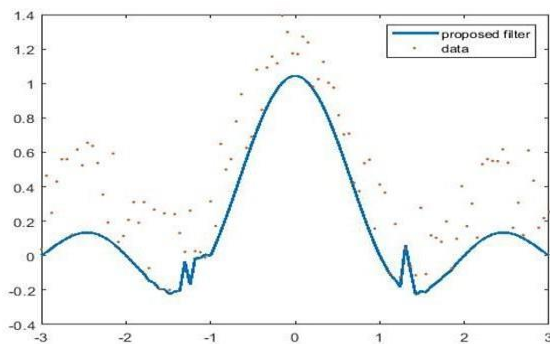


Fig. 6. Plot of estimated data for sinc function using SIVIA algorithm

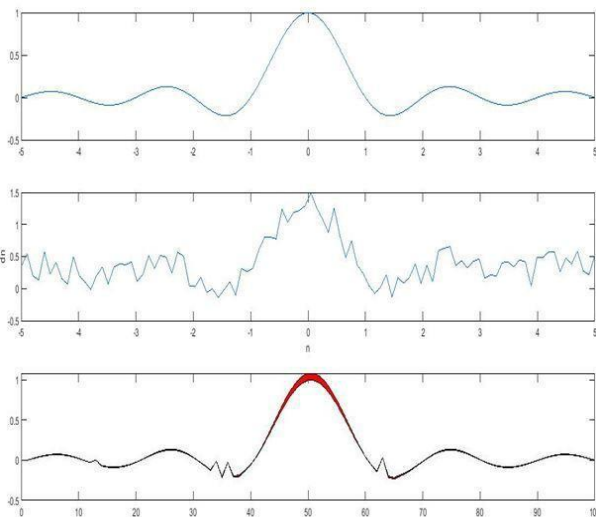


Fig. 7. Plot of sinc function input and the recovered signal

same. The same algorithm was tried for sinc function. Sinc function is one of the highly non linear function. The interval adaptive filter shows the better recovery. The plot of original sinc function, noisy sinc function and the recovered sinc mean square error verses number of iteration is shown in Figure 8. It is observed that the interval adaptive filter is function are as shown in Figure 7. Similarly the data estimated for sinc function is as shown in Figure 6. The comparison of showing better performance as compared to

filters. The comparison of MSE in decibels versus number of iterations is as shown in Figure 9. The interval adaptive algorithm is converging faster as compared to KLMS algorithm.

Fig. 8. Comparison of MSE verses number of iterations of LMS, RLS and the proposed filter

The comparison of the LMS, RLS and the proposed algorithms is shown in Table I. It is observed that the proposed algorithm is showing better results and this is most suitable adaptive filter for speech processing. But the computational complexity is very high in proposed interval adaptive filter. Proposed filter is also having more stability as compared to all other algorithms. The RLS algorithm can be used for all of the facts, present and past, apart for that may be a problem if the data of past is ambiguous for the parameters of present data. Then the researcher are being looking for a rule of quantitative that at the time for the uses of one or the other, it cannot be having one. The algorithm of RLS that can be additionally for the computation to the intensive than algorithm of LMS, to the same extent for the LMS is good enough, at the time for to make secure for one to go with. The algorithms of RLS tend to meet faster, other than it can be more time varying and has the computationally intensive disadvantage.

VI. CONCLUSION

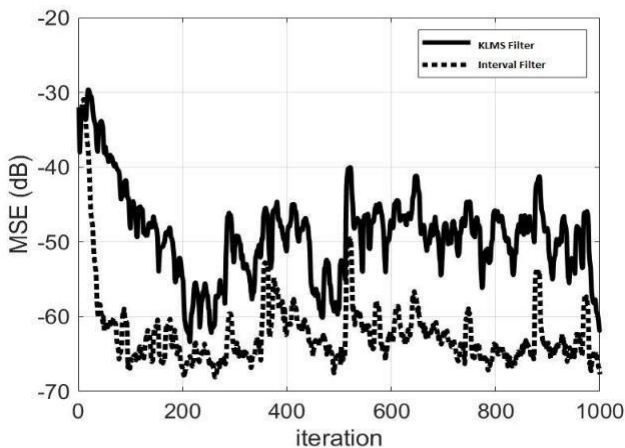


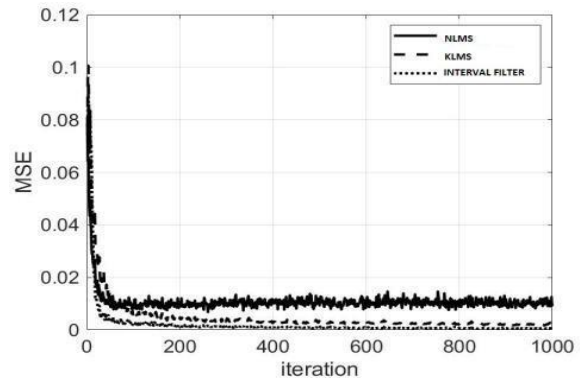
Fig. 9. Comparison of MSE in db and number of iterations

for the algorithm of adaptive filter. With this the mean square error of the system is reduced considerably. Least mean square (LMS) is one of the easiest and simplest to implement but its convergence rate is very slow. The recursive least square (RLS) is more computationally expensive than least mean square (LMS). Kernel adaptive filter is a non linear filter and the weights are adopted to its transfer function. Interval adaptive filter shows the best performance among all the floating point filters. The computational cost is doubled as interval data has infimum and supremum values. In future work, the algorithm can be implemented using GPU's to reduce computational time and to learn the new applications with less restriction on complexity.

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The interval adaptive digital filter employs active noise cancellation (ANC) scheme for the generation pure audio signals. The interval adaptive filter updates the coefficients repetitively using interval Newton method for tracking the



most desirable type of optimal solutions that are possibly used

Table-I comparison Of Various Adaptive Filter Algorithms

Algorithm	Mean square error	Complexity	Stability
LMS	$1.5 \cdot 10^{-2}$	$2N+1$	Less Stable
RLS	$6.2 \cdot 10^{-3}$	$4N^2$	Moderately stable
KLMS	$4.2 \cdot 10^{-2}$	N	Highly stable
Proposed interval adaptive filter	$3.2 \cdot 10^{-1}$	$4(2N + 1)^2$	Highly stable

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