Abstract: Classification of target from a mixture of multiple target information is quite challenging. In this paper, we have used supervised Machine learning algorithm namely Linear Regression to classify the received data which is a mixture of target-return with the noise and clutter. Target state is estimated from the classified data using Kalman filter. Linear Kalman filter with constant velocity model is used in this paper. Minimum Mean Square Error (MMSE) analysis is used to measure the performance of the estimated track at various Signal to Noise Ratio (SNR) levels. The results state that the error is high for Low SNR, for High SNR the error is Low.

Keywords: Kalman Filter, Linear Regression, Target Classification.

I. INTRODUCTION

In radar and sonar systems, target tracking plays an important role to determine the number of targets and target trajectory information such as positions and heading angle. Many algorithms are exists for target detection from noise-corrupted received signal. These detections are used further for target tracking. Received signal may include mixture of target-return with the noise and clutter [9]. To obtain the better estimate, tracking may be divided into different stages such as gating, data association, and state estimation.

The simplest form of gating is the mean-square distance of rectangular gating. A second, more accurate, form of gating is ellipsoidal gating, which uses the normalized distance. For association, Nearest Neighbor (NN) selects the one measurement that is nearest the predicted measurement and uses that measurement to update the track. In most cases, the algorithm should associate each measurement to only one track. This creates problems, when for instance, two tracks are in close proximity [1,5]. Instead of using only one measurement among the received ones and discarding the others, an alternative approach is to use all of the validated measurements with different weights (probabilities), known as Probabilistic Data Association (PDA) [11]. Here it is assumed that here is only one target of interest and among the possibly several validated measurements, at most one of them can be target-originated. When several targets as well as clutter or false alarms are present in the same neighborhood, PDA fails to address this scenario. Joint Probabilistic Data Association (JPDA) algorithm is the extension of PDA, to track several targets. This is handled by the observation to track association and track maintenance functions. Assumption of the JPDA is that the number of targets to be tracked should be known. In JPDA track coalescence can occur if the tracks are close to each other for an extended time [10].

Multiple hypothesis Tracking (MHT) [2,10] algorithm considers the association of sequences of measurements and evaluates the probabilities (or likelihoods) of all association hypotheses. This approach is measurement oriented in the sense that the probability that an established target or a new target gave rise to a certain measurement sequence is obtained. This feature allows inclusion of track initiation for new targets within the frame work of the algorithm. Once the assigning a measurement to each existing track, the next is to use the measurement to update the track. This is achieved by using Kalman filtering which is the most established method of track updating [7, 8]. Because of its tremendous processing requirements in all these three stages, this paper focused to reduce the computational complexity in the target classification.

Zheng, J. et. al.[12] proposed Random Finite Set (RFS) which exploits the Gaussian Mixture Probability Hypothesis Density (GMFHD) filter and linear regression method to extract the road map in the ground target tracking. Spingarn, K. and Weidemann, H.L. [13], used Cartesian coordinate linear regression filter for tracking maneuvering aircraft targets. In the same way, Ganggang Dong and Gangyao Kuang [3], in their work used local linear regression for classification in SAR Imagery. S. Vashui and V. Vaidehi [6] proposed support vector machine (SVM) for target tracking and show the better performance. Q. Zhao, and J. C. [5] Principle also used SVM for SAR automatic target recognition. But SVM is computationally complex than the linearregressing. Instead of using the linear regression as a combination with other algorithms, here we proposed machine learning methodology based linear regression to classify the different track data. We observed the comparatively better classification.

In this paper Section-II explains the proposed technique, Linear Regression model and the Kalman Filter.

II. PROPOSED METHOD

The target tracks are synthesized in a way that the data contains target-return, cluttered data and noise data at all $n$ time intervals with $m$ data in each interval, so that for each track has data $D_t \in \mathbb{R}^{m \times n}$. All the measurements are assumed to be range measurements.
Machine learning based track classification and estimation using Kalman Filter

The synthesized data of both tracks $D_1 \in R^{m \times n}$ and $D_2 \in R^{m \times n}$ were combined randomly to generate a new data $D \in R^{2m \times n}$ which contains the range data measured up to $n$ times on both tracks. Block diagram representation of target state estimation is shown in Figure 1.

Linear regression algorithm is used to classify the both tracks. Then nearest neighbor (NN) approach is used to select one measurement that is nearest to the predicted measurement and is used as a target measurement. Kalman filter is used to estimate the optimized target state in each measurement. Following section explains the mathematics involved in the linear regression method.

**Fig.1 Block diagram representation of the proposed methodology.**

A. Linear Regression in Machine Learning

The supervised learning algorithm used in this paper is linear regression model. This method gives a linear regression line which can separate the data in to two parts as track 1 and track 2 data. To find the regression line from the training set $(x^{(i)}, y^{(i)})$, assume the hypothesis given as

$$h_{\theta} = \theta_0 + \theta_1 x$$  \hspace{1cm} (1)

Where $x$ is the input (measured data) feature, $y$ is the output variable (expected separation point in each observation). $(x^{(i)}, y^{(i)})$ is the $i^{th}$ training data. $\theta_0$ and $\theta_1$ are unknown parameters. The values of $\theta_0$ and $\theta_1$ are found recursively using the cost function $J(x)$, which is designed based on minimum mean square error (MMSE) and is given as

$$J(x) = \frac{1}{2m} \sum_{i=1}^{m} (h_{\theta}^{(i)} - y^{(i)})^2$$  \hspace{1cm} (2)

In order to find the optimized values of $\theta_0$ and $\theta_1$, the cost function has to be minimized. Gradient descent algorithm is one of the methods used to minimize the cost function and is explained in the next section.

B. Gradient descent algorithm

It is an iterative algorithm. The cost function given in (2) is calculated for different values of the $\theta_0$ and $\theta_1$. The values at which the cost function is minimum is considered to be the optimized parameters to get the hypothesis as given in (1) to separate the two track data. Gradient decent algorithm is MMSE based iterative algorithm follows the steps given below to obtain the optimized parameters [14].

Algorithm:
1. Assign Random Values for $\theta_0$ and $\theta_1$.
2. $\theta_j \leftarrow \theta_j - \alpha \times \frac{\partial}{\partial \theta_j} (J(x))$; $j=0,1$
3. $J(\theta) = \frac{1}{2m} \sum_{i=1}^{m} (\theta_0 + \theta_1 x^{(i)} - y^{(i)})^2$

Where $\alpha$ is learning rate, if the value of $\alpha$ is very large gradient descent can overshoot the minimum and it fails to converge. The algorithm works very slowly if $\alpha$ is very small. Choose value of $\alpha$ in the range $[0.003$ to $0.3]$; 

C. KALMAN FILTER

To estimate the target state from the classified data, Kalman filter is used. As the Kalman filter uses the $x$, $y$ positions and corresponding velocities for predicting the state equation and measurement, the $x$-position, $y$-position was found using basic trigonometry. Here both the targets are assumed to be moved with constant velocity.

Kalman filter estimates the target state using the state updates and measurement updates. State model equation is given as

$$X(K) = FX(K-1) + W(K) \hspace{1cm} (3)$$

The measurement equation is given as

$$Z_m(K) = HX(K) + V(K) \hspace{1cm} (4)$$

Where:
- $X(K)$- state vector at $K^{th}$ sample.
- $F$ - state transition matrix.
- $W$ - process noise (white noise with zero mean and a known initial co-variance $Q$).
- $Z_m$ - measurement vector.
- $H$ - observation vector.
- $V$ - measurement noise (white noise with zero mean and a known co-variance $R$).

The process noise and the measurement noises are independent to each other. The covariance of process and measurement noise are given as

$$Q = E(W(k)W(k)^T) \hspace{1cm} and \hspace{1cm} R = E(V(k)V(K)^T) \hspace{1cm} (5)$$

The transition matrix and observation matrix are given as:

$$F = \begin{bmatrix} 1 & 0 & \Delta t & 0 \\ 0 & 1 & 0 & \Delta t \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \hspace{1cm} \Delta t = t_{k+1} - t_k$$

$$H = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \hspace{1cm} (6)$$

It is required to estimate the target state vector from the measurements in an optimum way. Two estimates of the state are distinguished. Optimized estimate $\hat{X}(k|k)$ is a conditional estimate, conditioned on the measurement history up to time kalogram with the predicted state $\hat{X}(k|k-1)$. It is an estimate, conditioned on the measurement sample history up the $k-1$ time.

Given the estimated state $\hat{X}(k-1|k-1)$ and the estimated state covariance $\hat{P}(k-1|k-1)$ at $k-1^{th}$ time the predicted state vector $\hat{X}(k|k-1)$ and the prediction state covariance $\hat{P}(k|k-1)$ at $k^{th}$ time are given as:

$$\hat{X}(k|k-1) = \hat{X}(k-1|k-1)$$

$$\hat{P}(k|k-1) = \hat{P}(k-1|k-1) + \hat{F} \hat{W} \hat{F}^T$$

$$K(k) = \hat{P}(k|k-1)H^T (HH^T + R)^{-1}$$

$$\hat{X}(k|k) = \hat{X}(k|k-1) + K(k) (Z(k) - H \hat{X}(k|k-1))$$

$$\hat{P}(k|k) = \hat{P}(k|k-1) - K(k) \hat{P}(k|k-1) K(k)^T$$
\[
\hat{x}(k|k-1) = F\hat{x}(k-1|k-1)(7)
\]

\[
\hat{p}(k|k-1) = F\hat{p}(k-1|k-1)F^T + Q (8)
\]

Then using the predicted state \(\hat{x}(k|k-1)\) given in (7) and the current measurement \(Z_m(k)\) given (4), the estimated state vector is a linear combination of \(\hat{x}(k|k-1)\) and \(Z_m(k)\) as:

\[
\hat{x}(k|k) = \hat{x}(k|k-1) + K\theta(k)(9)
\]

Where, \(\theta(k)\) known as innovation is defined as.

\[
\theta(k) = Z_m(k) - H\hat{x}(k|k-1)(10)
\]

The covariance matrix for the estimation error is given as

\[
\hat{p}(k|k) = E(\hat{\theta}(k)\hat{\theta}(k)^T)
\]

\[
\hat{\theta}(k) = (I - KH)\hat{p}(k|k-1)(11)
\]

The Kalman gain \(K\) used in (9) and (11) is chosen in such a way that the estimation error has a minimum variance. It is given as

\[
K = \hat{p}(k|k-1)H^TS^{-1}(12)
\]

The Linear Kalman Filter Algorithm is given as

Table-I: Linear Kalman Filter Algorithm

<table>
<thead>
<tr>
<th>Step</th>
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### III. RESULTS AND SIMULATION

To synthesize the track data following assumptions are made. X-band (8-12GHz) short range mono pulse radar is considered for the target reception. Two tracks are synthesized as shown in fig. 2, where track 1 has the initial range of 1.075 Km with the moving velocity of 1.5 m/sec. Similarly track 2 has the initial range of 1.125 Km with the moving velocity of 2.5 m/sec. Observations are repeated 20 times and are consolidated for every 3 sec. Total data collected is collected for 1 minute. (i.e.) The number of available data in one minute duration is 20×20=400 data. 20 times repetition data is synthesized in a way that the data contains the target return data, cluttered data and noise data. The target return data is created using the ideal track added with additive white Gaussian noise. The cluttered data is created using a random track which follows the Poisson distribution with fixed mean and variance and the noise of the received data is created by adding additive white Gaussian noise (awgn) with SNR of 20dB. The collective data in one minute interval is shown in fig. 3. To show the data collections pertaining to each track, fig. 4 plotted with different symbols to isolate the track 1 and 2 data.
Linear Regression is applied on to the received data, using the algorithm defined in section-II by repeatedly updating the hypothesis given in (1) till the cost function shown in (2) got minimized. When the final regression line is plotted in the observed data, it classifies the two tracks accurately as shown in Fig.5.

Mean Square Error (MSE) is used to analyse the performance of the estimated track with the ideal tracks. It is always non-negative and close to zero is better. Consider $\hat{y}$ is estimated track and $y$ is the ideal track then,

$$MSE = E((\hat{y} - y)^2)$$

MSE is used to compare the performance at various SNR Levels; Fig.8 shows the MMSE graph for tracks at SNR value from -10 dB to 80 dB. From -10dB to 5 dB, estimation error is more in track 2 than track 1 and from 10 dB to 80 dB the performance is better. This shows that the amount of noise present plays a major role in target estimation.

**IV. CONCLUSION**

This paper mainly emphasizes on the application of linear regression to classify two tracks. Simulated results show the better classification for the received data with SNR of 20dB. MSE analysis shows that up to 0 dB SNR it works well. If the received data has more than two-track information, then supervised Machine Learning algorithm like Support Vector Machine (SVM), Unsupervised learning algorithms like K-mean cluster model, KNN model can be used. In future to synthesize the clutter points, one can use Rayleigh, Weibull distribution instead of Poisson distribution. Further we can improve this by using clutter separation or suppression algorithms like Low Rank Matrix Optimization (LRMO) and Independent Component Analysis (ICA) and filtering techniques to remove noises.

**REFERENCES**


AUTHORS PROFILE

B. Sai Tejeswar Reddy currently a final year B.Tech student of Electronics and Communication Engineering at Vellore Institute of Technology affiliated to VIT University. He is highly focused on research work and a complete academic career. He has done notable interdisciplinary projects relating to Signal processing, Image Processing and Embedded Systems.

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