

# Direct Adaptive Controllers for a Multi Input Multi Output Process

L. Thillai Rani

**Abstract:** Control of nonlinear dynamic systems is of great practical importance, since virtually all real world systems belong to this class. The prevalent approach is to use a model of the process linearized about the steady state operating point to design linear controllers. The Multi Input Multi Output (MIMO) process considered in this paper has varying system dynamics. This paper focuses on the performance evaluation of multi-loop Adaptive MIT (AMIT) controller and Adaptive PI (API) controller with vanishing adaptation technique for a laboratory interacting coupled tank MIMO process.

**Keywords :** Coupled Tank, MIMO, MRAC, AMIT, API.

## I. INTRODUCTION

The MIMO system chosen in this work is interacting coupled tank system. Model Reference Adaptive Control (MRAC) comes under Direct Adaptive Controller (DAC) [1]. Asan Mohideen *et al.* designed a MRAC for a coupled tank system [2]. Priyank Jain and Nigam dealt with the design of a Model Reference Adaptive Scheme (MRAS) for a second order system [3]. In this paper, process models are identified using open loop responses for different operating regions in real time. Using these process models, the proposed AMIT controller and API controller are designed and their performances are compared.

In this paper, the description of coupled tank process is given in section 2. Modelling and identification of the chosen process are outlined in section 3. In Section 4, design of Adaptive MIT controller is discussed. Design of Adaptive PI controller is presented in section 5. The real time responses of the designed controllers are given in Section 6 and 7. Finally, conclusion is given in Section 8.

## II. PROCESS DESCRIPTION

Fig. 1 shows the setup of the chosen process in this work. This process contains two cylindrical tanks namely Tank1 and Tank2. These two tanks are connected by an interconnecting valve. Water comes from reservoir tank to Tank1 and Tank2. To measure the inflow rates, rotameters (R1 and R2) are used. Motorized Control Valves (MCV1 and MCV2) are used to adjust the flow rates. To sense the level in the two tanks Level transmitters (LT1 and LT2) are used. Data Acquisition Card (DAQ) VDAS-01 receives input from these transmitters. The control signal from VDAS-01 actuates the motorized control valves to control the levels in the tank.



Fig. 1. Laboratory Coupled tank process

## III. IDENTIFICATION OF PROCESS

Fig. 2 shows the interfacing diagram of the process. The inflow rate into the tank1 is  $q_{in1}$ . The inflow rate into the tank2 is  $q_{in2}$ . The outflow rate from tank1 is  $q_{o1}$ . The outflow rate from tank2 is  $q_{o2}$ . The height of the liquid in tank1 is  $h_1$  and the height of the liquid in tank2 is  $h_2$ .

VDAS-01 card is used to acquire open loop input/output data from the process for specific operating ranges. These input/output data are used to estimate the parameters of the model.

Step changes in flow rates in terms of Litres Per Hour (LPH) are entered in Personal Computer (PC). The digital control signal from PC is converted into voltage using DAQ card and the inbuilt signal converter is used to convert this voltage to current signal to actuate the MCVs. These Motorized Control valves manipulate the inflow rates to the tanks. The height of the tanks are obtained from LTs. The level transmitter outputs are send to the PC via DAQ card.

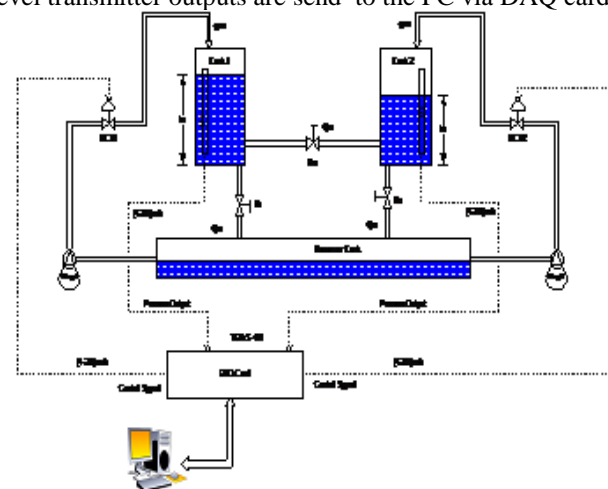


Fig. 2. Real time Interfacing diagram of Interacting coupled tank process

Revised Manuscript Received on April 30, 2020.

Dr. L. Thillai Rani, Assistant Professor, Department of EIE, Annamalai University, Tamilnadu, India. Email: thillairani72@gmail.com .

Here the manipulated input variables are inflow rates. The controlled output variables are levels. The open loop responses are shown in Fig. 3 and Fig. 4.

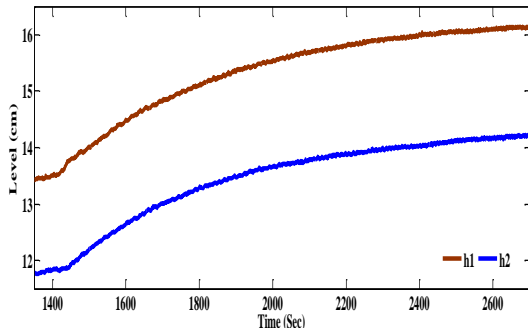


Fig. 3. Response of the process in open loop for change in  $q_{in1}$ .

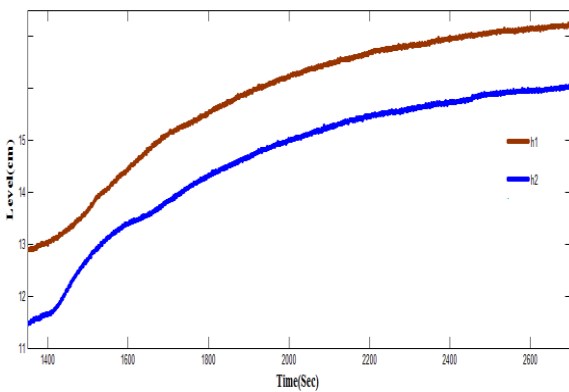


Fig. 4. Response of the process in open loop for change in  $q_{in2}$ .

The process transfer functions are calculated using Fig.3 and Fig. 4. They are given in Table-I[4].

Table-I : Process Transfer Functions

Operating region(OR)	Tank1 (Keeping $q_{in2}$ constant)			Tank2 (Keeping $q_{in1}$ constant)			Process Transfer Function $G_p(s)$
	$\Delta q_{in1}$ (LPH)	$\Delta h_1$ (cm)	$\Delta h_2$ (cm)	$\Delta q_{in2}$ (LPH)	$\Delta h_1$ (cm)	$\Delta h_2$ (cm)	
OR1	50-65	13.5- 16.1	11.6- 14.2	50-65	13.5- 17.2	11.6- 16.0	$\left[ \begin{array}{c} \frac{0.683}{399s^2 + 64s + 1} \quad \frac{0.48}{207s^2 + 72s + 1} \\ \frac{0.428}{983s^2 + 97s + 1} \quad \frac{0.6}{300s^2 + 65s + 1} \end{array} \right]$
OR2	65-80	16.1- 18.1	14.2- 15.9	65-80	17.2- 18.5	16.0- 17.4	$\left[ \begin{array}{c} \frac{0.8}{375s^2 + 80s + 1} \quad \frac{0.6}{600s^2 + 70s + 1} \\ \frac{0.44}{113s^2 + 100s + 1} \quad \frac{0.68}{618s^2 + 90s + 1} \end{array} \right]$
OR3	80-95	18.1- 18.8	15.9- 16.6	80-95	18.5- 19.0	17.4- 17.9	$\left[ \begin{array}{c} \frac{0.9}{171s^2 + 60s + 1} \quad \frac{0.56}{900s^2 + 100s + 1} \\ \frac{0.46}{120s^2 + 22s + 1} \quad \frac{0.8}{112s^2 + 58s + 1} \end{array} \right]$

The partial derivatives of error are given in (8) and (9)

$$\frac{\partial e_i}{\partial \theta_{1i}} = \frac{b_i}{s^2 + 2sa_{m1i} + a_{m2i}} h_{spi} \quad (8)$$

$$\frac{\partial e_i}{\partial \theta_{2i}} = \frac{-b_i}{s^2 + 2sa_{m1i} + a_{m2i}} h_i \quad (9)$$

#### IV. MULTI-LOOP ADAPTIVE MIT CONTROLLER

The set point of height of two tanks are  $h_{spi}$ . The controlled outputs are  $h_i$  (where  $i=1,2$  represents the index for two tanks) Here the reference model is  $(h_m)$  [5]-[7]. In the reference models, the damping ratio is chosen as 0.7. The undamped natural frequency is chosen as 1.

The error equation is given in (1).

$$e_i = h_i - h_m \quad (1)$$

The controller parameters ( $\theta_{ni}$ ) are given by (2).

$$J(\theta_{ni}) = \frac{1}{2} e_i^2 \quad (2)$$

where  $n=1,2$  represents controller parameter. The Adaptive MIT (AMIT) algorithm is given in (3)[8].

$$\frac{d\theta_{ni}}{dt} = -\gamma e_i \frac{\partial J}{\partial \theta_{ni}} = -\gamma e_i \frac{\partial e_i}{\partial \theta_{ni}} \quad (3)$$

The parameter  $\frac{\partial e_i}{\partial \theta_{ni}}$  is the derivative of the error with respect to controller parameters ( $\theta_{ni}$ ). The parameter,  $\gamma$  is the adaptation rate. The linearized model is given in (4).

$$G_{pi} = \frac{h_i}{u_i} = \frac{b_i}{s^2 + a_{1i}s + a_{2i}} \quad (4)$$

where  $b_i, a_{1i}, a_{2i}$  are the parameters of the process. The control signal  $u_i$ , is given in (5).

$$u_i = \theta_{1i} h_{spi} - \theta_{2i} h_i \quad (5)$$

By substituting (5) in (4), the closed loop transfer function of Adaptive MIT controller is given in (6).

$$\frac{h_i}{h_{spi}} = \frac{b_i \theta_{1i}}{s^2 + 2sa_{m1i} + a_{m2i}} \quad (6)$$

The transfer function of the reference model is given by

$$G_m = \frac{h_m}{h_{spi}} = \frac{b_{m1i}s + b_{m2i}}{s^2 + 2sa_{m1i} + a_{m2i}} \quad (7)$$

where  $b_{m1}, b_{m2}, a_{m1}$  and  $a_{m2}$  are the reference model parameters.

Substituting (8) and (9) in the MIT algorithm in (3), the parameters of the controller are obtained and are given in (10) and (11).

$$\theta_{1i} = -\frac{\gamma'}{s} e_i \frac{1}{s^2 + 2sa_{m1i} + a_{m2i}} h_{spi} \quad (10)$$

$$\theta_{2i} = \frac{\gamma'}{s} e_i \frac{1}{s^2 + 2sa_{m1i} + a_{m2i}} h_i \quad (11)$$

where  $\gamma' = \gamma b_i$ .

### V. MULTI-LOOP ADAPTIVE PI CONTROLLER

In Adaptive PI (API) control, which is a type of DAC, the performance index is given in terms of a reference model [9]. An error signal between the desired and actual output is employed to update controller parameters in online. The PI control law used in this controller is given in (12).

$$u_i = K_{ci}(h_{spi} - h_i) + \frac{K_{ii}}{s}(h_{spi} - h_i) \quad (12)$$

The Adaptive PI (API) controller transfer function is given in (13).

$$\frac{h_i}{h_{spi}} = \frac{bK_{ci}s + bK_{ii}}{s^2 + s[bK_{ci} + a] + K_{ii}b} \quad (13)$$

The second order reference model is presented in (14).

$$\frac{h_m}{h_{spi}} = \frac{b_{m1i}s + b_{m2i}}{s^2 + 2sa_{m1i} + a_{m2i}}$$

The condition for model to be matched is given in (15)

$$s^2 + s[bK_{ci} + a] + K_{ii}b = s^2 + 2sa_{m1i} + a_{m2i} \quad (15)$$

By substituting (13) and (14) in (16), the model matching condition is obtained as given in (17).

$$e_i = h_i - h_m \quad (16)$$

$$e_i = \frac{bsK_{ci} + bK_{ii}}{s^2 + s[bK_{ci} + a] + bK_{ii}} h_{spi} - \frac{b_{m1i}s + b_{m2i}}{s^2 + 2sa_{m1i} + a_{m2i}} h_{spi} \quad (17)$$

Then,

$$\frac{\partial e_i}{\partial K_{ci}} = \frac{sb[h_{spi} - h_i]}{s^2 + 2sa_{m1i} + a_{m2i}} \quad (18)$$

$$\frac{\partial e_i}{\partial K_{ii}} = \frac{b[h_{spi} - h_i]}{s^2 + 2sa_{m1i} + a_{m2i}} \quad (19)$$

The parameters of the controller are given in (20, 21).

$$K_{ci} = -\frac{\gamma'}{s} e_i \frac{s}{s^2 + 2sam_{1i} + am_{2i}} [h_{spi} - h_i] \quad (20)$$

$$K_{ii} = -\frac{\gamma'}{s} e_i \frac{1}{s^2 + 2sam_{1i} + am_{2i}} [h_{spi} - h_i] \quad (21)$$

### VI. RESPONSES OF API CONTROLLERS

The control algorithms thus designed are implemented in real time process using USB based MATLAB interfacing card VDAS-01. Fig. 5. presents the servo output of  $h_1$  and  $h_2$  for  $\Delta h_{sp1}$  (set point change in  $h_1$ ) and their corresponding level variations to tank1 and tank2 with API controllers. The Initial condition of  $h_1$  is 13.5cm and  $h_2$  is 11.6cm. In servo1, a set point is applied at 600<sup>th</sup> sec in  $h_1$ . As a result,  $h_1$  increases and settles in its set point. Due to interaction there is a considerable fluctuation in  $h_2$  and settles in its set point due to the controller action in loop2. At 1100<sup>th</sup> sec,  $h_{sp1}$  (servo2) is increased from 14 to 14.5cm. Due to interaction  $h_1$  and  $h_2$  both increases and tracks to their set points due to the action of controllers in loop1 and loop2.

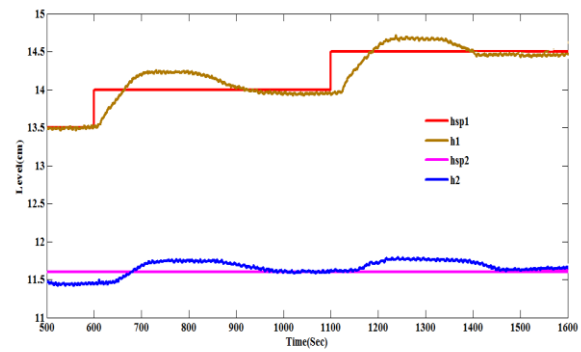


Fig. 5. Servo Responses of  $h_1$  for  $\Delta h_{sp1}$

The corresponding responses of API controllers for set point changes in  $h_1$  are shown in Fig. 6. For servo1 and servo2,  $q_{in1}$  rises and  $q_{in2}$  decreases in order to track the set point change in  $h_1$ .

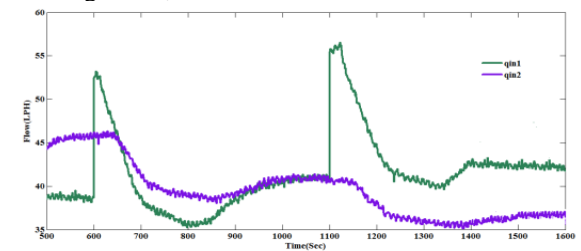


Fig. 6. Responses of API Controllers

The adaptation of controller parameter ( $K_c$ ) in API controllers is given in Fig. 7.

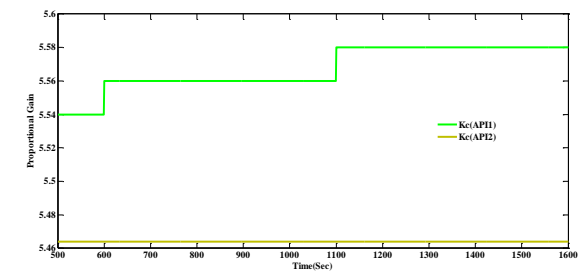


Fig. 7. Adaptation of controller parameters

The adaptation of controller parameter ( $K_i$ ) in API controllers is given in Fig. 8.

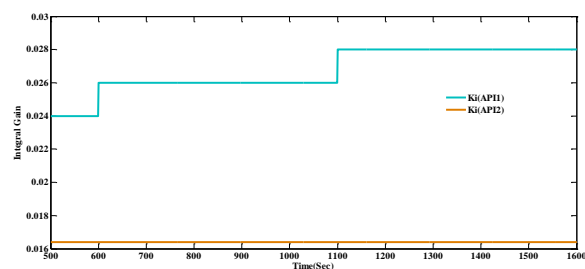


Fig. 8. Adaptation of controller parameters

Fig. 9. presents the servo responses of  $h_2$  and  $h_1$  for  $\Delta h_{sp2}$  (change in set point of  $h_2$ ) and its corresponding level variations in tank1 and tank2 with API controllers. The initial conditions of  $h_1$  is 13.5cm and  $h_2$  is 11.6cm. In (servo3), a set point change is applied at 1000<sup>th</sup> sec in  $h_2$ . As a result,  $h_2$  increases and settles in its set point. Due to interaction there is a fluctuation in  $h_1$  and settles in its set point.

At 1550<sup>th</sup> sec,  $h_{sp2}$  (servo4) is increased from 12.1 to 12.6cm. Due to interaction, both  $h_2$  and  $h_1$  increases and tracks to their set points due to the action of controllers in loop2 and loop1.

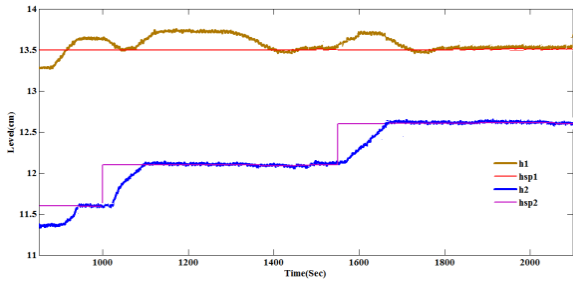


Fig. 9. Servo Responses of  $h_2$  for  $\Delta h_{sp2}$

The corresponding responses of API controllers for set point changes in  $h_2$  are shown in Fig. 10. The controllers take corrective actions depending upon the variations in  $h_1$  and  $h_2$ .

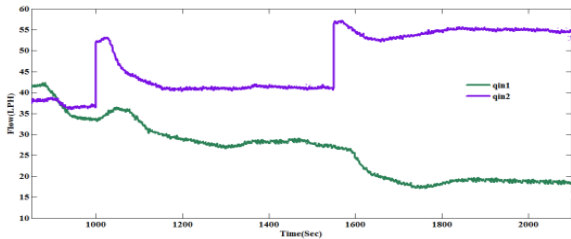


Fig. 10. Responses of API Controllers

The adaptation of controller parameter ( $K_c$ ) in API controllers is given in Fig. 11.

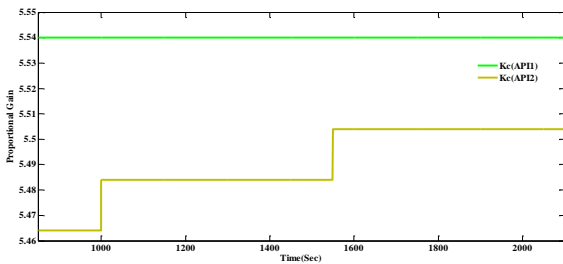


Fig. 11. Adaptation of controller parameters

The adaptation of controller parameter ( $K_i$ ) in API controllers is given in Fig. 12.

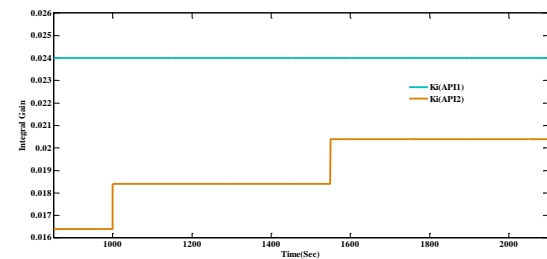


Fig. 12. Adaptation of Integral Gains

VII. RESPONSES OF AMIT CONTROLLERS

Fig. 13 presents the servo responses of  $h_1$  and  $h_2$  for  $\Delta h_{sp1}$  and its corresponding level variations in tank1 and tank2 with AMIT controllers.

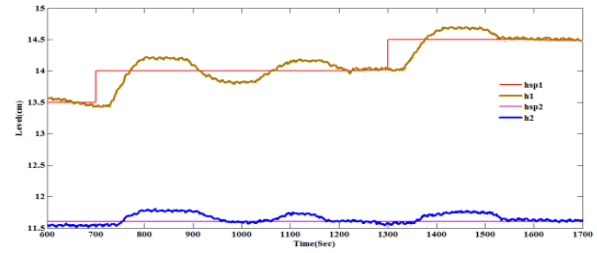


Fig. 13. Servo Responses of  $h_1$  for  $\Delta h_{sp1}$

The corresponding responses of AMIT controllers for set point changes in  $h_1$  are shown in Fig. 14.

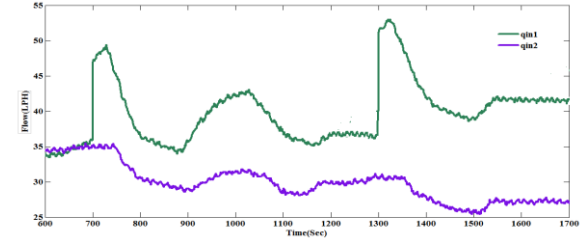


Fig. 14. Responses of AMIT Controllers

The adaptation of controller parameters ( $\theta_1$  and  $\theta_2$ ) of AMIT controllers is shown in Fig. 15.

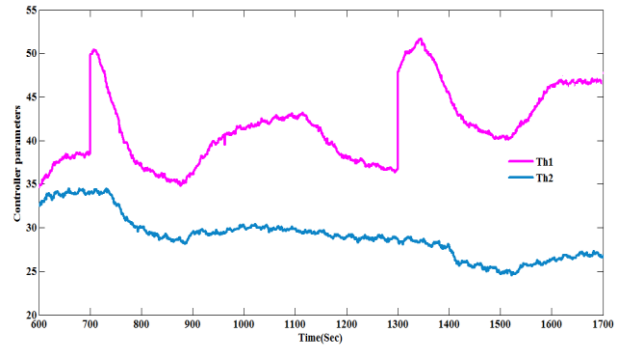


Fig. 15. Adaptation of Controller Parameters

Fig. 16 presents the servo responses of  $h_2$  for  $\Delta h_{sp2}$  (change in set point of  $h_2$ ) and its corresponding level variations in tank1 and tank2 with AMIT controllers.

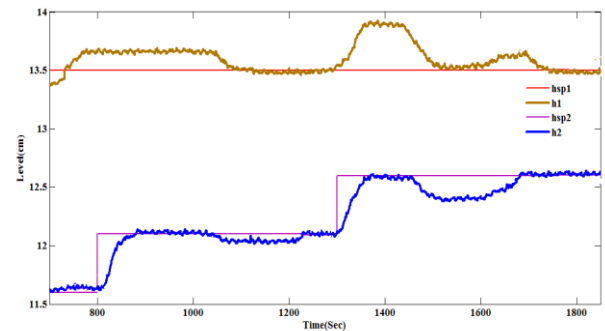


Fig. 16. Servo Responses of  $h_2$  for  $\Delta h_{sp2}$

The corresponding responses of AMIT controllers for set point changes in  $h_2$  are shown in Fig. 17. The multi-loop AMIT controllers take corrective actions depending upon the variations in  $h_1$  and  $h_2$ .

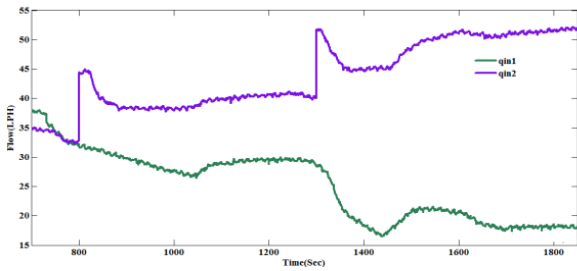


Fig. 17. Responses of AMIT Controllers

The adaptation of controller parameters ( $\theta_1$  and  $\theta_2$ ) of AMIT controllers is shown in Fig. 18.

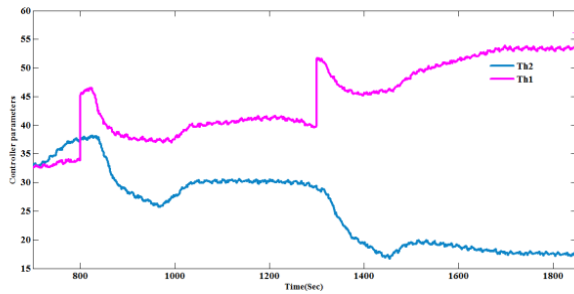


Fig. 18. Adaptation of Controller Parameters

### VIII. RESULTS OF AMIT AND API CONTROLLERS

From the servo responses of AMIT and API Controllers when implemented in coupled tank process, the time integral criteria such as ISE (Integral Square Error), IAE (Integral Absolute Error) and ITAE (Integral Time Absolute Error) are compared and the results are tabulated.

Table-II presents the comparison of performance criteria of laboratory interacting coupled tank system with AMIT and API controllers for  $\Delta h_{sp1}$ .

Table-II Comparison of Performance Criteria with AMIT and API Controllers for  $\Delta h_{sp1}$

Parameters	AMIT ( $\Delta h_{sp1}$ )		API ( $\Delta h_{sp1}$ )	
	Servo1	Servo2	Servo1	Servo2
Settling time (sec)	528	544	314	300
(for 1000 sec duration)				
ISE	1.965e+4		0.854e+4	
IAE	2317		878	
ITAE	6.879e+6		1.123e+6	

Table-III presents the comparison of performance criteria of AMIT and API controllers for  $\Delta h_{sp2}$

Table-III Comparison of Performance Criteria with AMIT and API Controllers for  $\Delta h_{sp2}$

Parameters	AMIT ( $\Delta h_{sp2}$ )		API ( $\Delta h_{sp2}$ )	
	Servo3	Servo4	Servo3	Servo4
Settling time (sec)	424	361	144	80
(for 1050 sec duration)				
ISE	2.582e+4		2.146e+4	
IAE	2487		2358	
ITAE	1.624e+7		1.287e+7	

### IX. CONCLUSION

The design and experimental results of multi-loop direct adaptive controllers such as AMIT and API are analysed. The

experimental results show that the proposed API controller has ability to obtain the desired dynamics. The controller parameter variations of API controller varies to a minimum extent when compared to that of AMIT controller.

### REFERENCES

1. Astrom, K.J. and Wittenmark, B., "Self-tuning controllers based on pole-zero placement", *IEEE Proceedings*, vol. 127, no.3, 1980, pp.120-130.
2. Asan Mohideen, K., Saravanakumar, G., Valarmathi, K., Devaraj, D. and Radhakrishnan, T.K., "Real-coded Genetic algorithm for system identification and tuning of a modified model reference adaptive controller for a hybrid tank system", *Applied Mathematical Modeling*, vol. 37, 2013, pp. 3829-3847.
3. Priyank Jain and Nigam, M.J., "Design of a Model Reference adaptive controller using modified MIT rule for a second order system", *Advance in Electronic and Electrical Engineering*, ISSN 2231-1297, vol.3, no.4, 2013, pp.477-484.
4. Suparook Kangwamrat, Vittaya Tipsuwannaporn and Arjin Numsomarn, "Design of PI controller using MRAC techniques for coupled tanks process", *Proceedings of International Conference on Control, Automation and Systems*, Korea, 2010, pp. 485-489.
5. Gawthrop, P.J., "Self-tuning PID controller: Algorithms and implementation", *IEEE Transaction on Automatic Control*, vol.31, no.3,1986, pp.201-209.
6. Marino, R., Peresada, S. and Tomei, P., "Adaptive output feedback control of current-fed induction motors with uncertain rotor resistance and load torque", *Automatica*, vol.34, no.5, May 1998, pp.617-624.
7. George Stephanopoulos, *Chemical Process Control- An Introduction to Theory and Practice*, PHI Learning Private Limited, New Delhi, 2005, ch 5.
8. Landau, I.D., Lozano, R. and M'Saad, M., *Adaptive Control*, Springer Verlag, London, U.K, 1997, ch 4.
9. Li, G., Ma, J. and Zhao, G., "Simulation comparisons among PID control, adaptive control and MFC on ship anti-rolling tank test platform", *Proceedings of IEEE International Conference on Mechatronics and Automation*, China, 2007, pp.3948-3953.
10. Landau, I.D., "The R-S-T digital controller design and application", *Control Engineering Practice*, vol. 6, 1998, pp.155-165.

### AUTHOR PROFILE



Dr. L. Thillai Rani was born in Chidambaram, India on 22<sup>nd</sup> June 1982. She graduated her Bachelor of Engineering degree in Electronics and Instrumentation in the year 2004 in Annamalai University, Chidambaram. She further qualified herself with Master degree in Process Control and Engineering in the year 2006 in the same institution. He received her Doctorate in Electronics and Instrumentation Engineering at Annamalai University in the year 2018. She is working as Assistant Professor in the Department of Electronics and Instrumentation Engineering at Annamalai University since 2008. Her areas of interest is in Process Control, Digital signal processing and system theory.