

Optimal Synthesis of six bar Mechanism using Particle Swarm Optimization

Abdul Qaiyum, Aas Mohammad



Abstract: This paper presents the optimal synthesis of a six-bar Watt-II mechanism with rotational joints in which a tracing point on the coupler traces a desired path. The objective of the work is to synthesize a six-bar mechanism whose coupler point generates the path as close to the given path as possible. The tracing path is the combination of two circular arcs with eighteen precision points with prescribed timing. A mathematical model is formed by two vector loop closure equations, and design constrained are imposed to improve the result quality. 'Particle Swarm Optimization' (PSO), an evolutionary technique, is used to obtain the optimal solution of the given six bar mechanism at first time, which has been used by various authors in case of four bar mechanism. Final design parameters and convergence rate of the optimal solutions are presented.

Keywords: Particle swarm optimization, Mechanism synthesis, Evolutionary technique, Six bar mechanism, Watt-II mechanism.

I. INTRODUCTION

In various engineering fields, the design of mechanical system required the synthesis of various mechanism to perform a particular operation. On the basis of the operation for which the mechanism is to be designed, the synthesis problem can be categorized into three groups; function generation, if a specific input-output relationship is needed; motion generation, if the objective to guide a rigid body through a series of configuration; and path generation, if the mechanism is required to trace a prescribed trajectory through a point on the coupler. Four bar mechanism is the most commonly used mechanism. However, many time a four mechanism is failed to perform a particular task efficiently, a six-bar mechanism would be a choice the for designers. All single degree of freedom six bar linkages (with revolute and prismatic joints) are inversions of the Watt and Stephenson six link chain [1,2]. Several methods have been proposed by researchers for synthesizing the mechanism: analytical methods [3], numerical methods [4], graphical methods [5]

and computer -aided methods [6,7].

The proposed method is the optimal synthesis method in which the synthesis process is based on a global optimum search [8]. A proper cost function, based on design error between desired and tracing trajectory, is usually defined and minimized the error by means of penalty method which is employed in order to deal with equality and inequality constraints, while a particle swarm algorithm is used as an optimum searching technique. In this work the six bar Watt-II mechanism is selected, which is the combination two four bar mechanism and formed by adding one dyad to the coupler point of the four mechanism. The output link (rocker) of first four bar mechanism acts as the input link of the second four bar mechanism.

II. PARTICLE SWARM OPTIMIZATION

PSO is a population based stochastic optimization technique, given by Kennedy and Eberhart [9] in 1995. This algorithm works on the inspiration of searching action of bird's population (swarm) in N dimensional space. Due to its simplicity and good convergence, PSO has remained one of the widely used optimization technique. Serdar Kucuk [10] applied PSO to reduce actuator power consumption in fully planar parallel manipulator. Acharyya and Mandal [11] used PSO during comparative study of four-bar mechanism for path generation. R.V. Ram et al. [12] applied PSO technique to find the optimum inverse kinematic solution for the moving manipulator. R.M. Dougall and S. Nokleby [13] used PSO technique to synthesize a general four-bar mechanism.

A. Implementation of PSO

PSO is implemented through intelligent collective behavior of flocking of birds. A population or swarm of particle is defined where each particle's position in N dimensional space problem represents the solution of the optimization problem. The movement of these particles shows how well it solves the problem. Each particle knows its own best value, called ' P_{best} ' and the best value in the group (among ' P_{best} ') ' G_{best} '. By considering current values, like current positions (S_x, S_y), current velocities (v_x, v_y), and the individual intelligence (P_{best}), and the group intelligence (G_{best}) each particle tries to modify its position. The following equations are utilized, in computing the position and velocities, in the X-Y plane:

$$v_j^{n+1} = w \times v_j^n + C_1 \times r_1 \times (P_{bestj} - S_j^n) + C_2 \times r_2 \times (G_{best} - S_j^n) \quad (1)$$

$$S_j^{n+1} = S_j^n + v_j^{n+1} \quad (2)$$

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where, v_j^{n+1} is the velocity of (k + 1)th iteration of jth individual, v_j^n the velocity of nth iteration of jth individual, w the inertial weight, C_1, C_2 are positive constants, having values [0,2], r_1, r_2 are the random numbers selected between 0 and 1, $Pbest_j$ is the best position of the jth individual, $Gbest$

the best position among the individual (group best) and S_j^n is the position of jth individual at nth iteration. The weighting factor 'w' is modified using equation (3) to enable quick convergence.

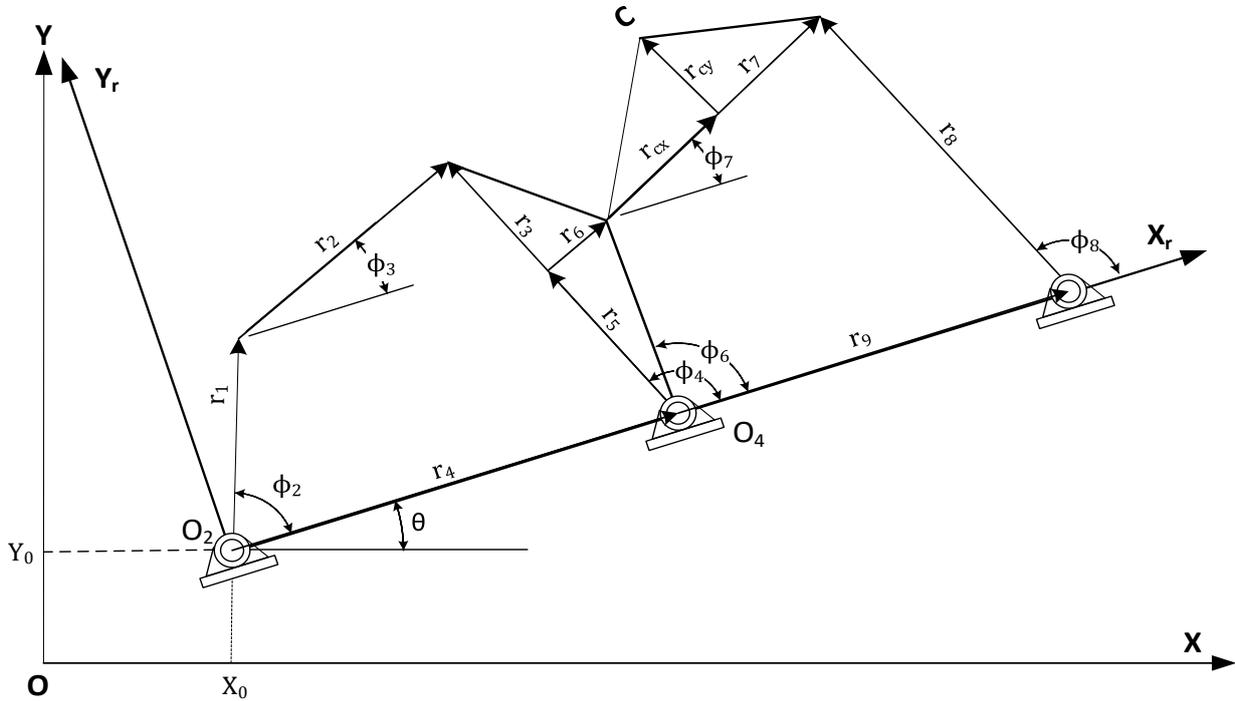


Fig. 1. Design parameters of six bar Watt-II mechanism

$$w = w_{max} - \frac{w_{max} - w_{min}}{iter_{max}} \times iter \quad (3)$$

where w_{max} represents the initial weight, w_{min} the final weight, $iter$ the current iteration number and $iter_{max}$ is the maximum iteration number.

B. Advantages of PSO algorithm

PSO has numerous advantages when compared with other mathematical algorithm and heuristic optimization techniques. PSO has very few parameters to run the algorithm. It has ability to run parallel and high probability to reach the optimized value. It takes little time to converge and compute the result. PSO can be very efficient for solving problems presenting difficulty to find accurate mathematical model.

III. PROPOSED SCHEME

Our aim is to evaluate the dimensional synthesis of six bar Watt-II mechanism to trace a given curve as close to desired path as possible. Fig 1 shows the selected Watt-II mechanism inclined θ angle with global coordinate OXY. This mechanism can be divided into two identical four bar mechanisms in which the output of first four bar mechanism used as the input of second four bar mechanism.

A. First loop closure equation

Loop closure vector equation for first close lope in fig. 1 is given by,

$$\vec{r}_1 + \vec{r}_2 - \vec{r}_3 - \vec{r}_4 = 0 \quad (4)$$

Where r_1, r_2, r_3, r_4 are link lengths. The equation (4) can be written in complex number form,

$$r_1 e^{i\varphi_2} + r_2 e^{i\varphi_3} - r_3 e^{i\varphi_4} - r_4 e^{i\varphi_1} = 0 \quad (5)$$

whereas φ_2 is the input (crank) and φ_4 is the output angle. The complex equation (5) can be divided into real and imaginary parts as follow (by putting $\varphi_1 = 0$),

$$r_1 \cos\varphi_2 + r_2 \cos\varphi_3 - r_3 \cos\varphi_4 - r_4 = 0 \quad (6a)$$

$$r_1 \sin\varphi_2 + r_2 \sin\varphi_3 - r_3 \sin\varphi_4 = 0 \quad (6b)$$

Now, applying Freudenstein's equation and find the values link lengths combinations and prescribed set of input (crank) angles φ_2 , the values of angles φ_3 and φ_4 :

$$C_1 \cos\varphi_4 - C_2 \cos\varphi_2 + C_3 = \cos(\varphi_2 - \varphi_4) \quad (7a)$$

$$C_1 \cos\varphi_3 + C_4 \cos\varphi_2 + C_5 = \cos(\varphi_2 - \varphi_3) \quad (7b)$$

where C_1, C_2, C_3, C_4 and C_5 are:

$$C_1 = \frac{r_4}{r_1}, C_2 = \frac{r_4}{r_3}, C_3 = \frac{r_1^2 + r_3^2 + r_4^2 - r_2^2}{2r_1 r_3}, C_4 = \frac{r_4}{r_2}$$

$$C_5 = \frac{-r_1^2 + r_3^2 - r_4^2 - r_2^2}{2r_1 r_2}$$

On solving the above equations, the angles φ_3 and φ_4 can be expressed as:

$$\varphi_{3,2} = 2 \tan^{-1} \left(\frac{-E \pm \sqrt{E^2 - 4DF}}{2D} \right) \quad (8a)$$

$$\varphi_{3,2} = 2 \tan^{-1} \left(\frac{-B \pm \sqrt{B^2 - 4AC}}{2A} \right) \quad (8b)$$

In the above expressions,

$$A = \cos \varphi_2 - C_1 - C_2 \cos \varphi_2 + C_3$$

$$B = -2 \sin \varphi_2$$

$$C = C_1 - (C_2 + 1) \cos \varphi_2 + C_3$$

$$D = \cos \varphi_2 - C_1 + C_4 \cos \varphi_2 + C_5$$

$$E = -2 \sin \varphi_2$$

$$F = C_1 + (C_4 - 1) \cos \varphi_2 + C_5$$

B. Second loop closure equation

Second loop is identical with first loop of four bar mechanism. The input angle φ_6 depends on output angle φ_4 of first four bar mechanism. From equation 8, it is clear that there are two values of φ_4 namely: φ_{4_1} and φ_{4_2} after solving the quadratic equation 7. Since φ_6 depends on the angle φ_4 , therefore, two input angles will be available for second four bar mechanism which are as follows:

$$\varphi_{6_1} = \varphi_{4_1} - \varphi_5 \text{ and } \varphi_{6_2} = \varphi_{4_2} - \varphi_5$$

where

$$\varphi_5 = \tan^{-1} \left(\frac{r_6}{r_5} \right)$$

the values of angles φ_7 and φ_8 is determined by following equations:

$$C_6 \cos \varphi_8 - C_7 \cos \varphi_6 + C_8 = \cos(\varphi_6 - \varphi_8) \quad (9a)$$

$$C_6 \cos \varphi_7 + C_9 \cos \varphi_6 + C_{10} = \cos(\varphi_6 - \varphi_8) \quad (9b)$$

where C_6, C_7, C_8, C_9 and C_{10} are:

$$C_6 = \frac{r_4}{\sqrt{r_5^2 + r_6^2}}, C_7 = \frac{r_9}{r_8}, C_8 = \frac{r_5^2 + r_6^2 + r_8^2 + r_9^2 - r_7^2}{2r_8 \sqrt{r_5^2 + r_6^2}},$$

$$C_9 = \frac{r_9}{r_7}, C_{10} = \frac{-r_5^2 - r_6^2 + r_8^2 - r_9^2 - r_7^2}{2r_7 \sqrt{r_5^2 + r_6^2}}$$

On solving the second loop closure equation (9), four values of coupler angle φ_7 and output angle φ_8 are obtained in case of second four bar mechanism:

$$\varphi_{8,2} = 2 \tan^{-1} \left(\frac{-H \pm \sqrt{H^2 - 4GI}}{2G} \right) \quad (10a)$$

$$\varphi_{8,2,4} = 2 \tan^{-1} \left(\frac{-O \pm \sqrt{O^2 - 4NP}}{2N} \right) \quad (10b)$$

$$\varphi_{7,2} = 2 \tan^{-1} \left(\frac{-L \pm \sqrt{L^2 - 4JM}}{2J} \right) \quad (10c)$$

$$\varphi_{7,3,4} = 2 \tan^{-1} \left(\frac{-R \pm \sqrt{R^2 - 4QS}}{2Q} \right) \quad (10d)$$

In the above expressions,

$$G = \cos \varphi_{6_1} - C_6 - C_7 \cos \varphi_{6_1} + C_8$$

$$H = -2 \sin \varphi_{6_1}$$

$$I = C_6 - (C_7 + 1) \cos \varphi_{6_1} + C_8$$

$$J = \cos \varphi_{6_1} - C_6 + C_9 \cos \varphi_{6_1} + C_{10}$$

$$L = -2 \sin \varphi_{6_1}$$

$$M = C_6 + (C_9 + 1) \cos \varphi_{6_1} + C_{10}$$

$$N = \cos \varphi_{6_2} - C_6 - C_7 \cos \varphi_{6_2} + C_8$$

$$O = -2 \sin \varphi_{6_2}$$

$$P = C_6 - (C_7 + 1) \cos \varphi_{6_2} + C_8$$

$$Q = \cos \varphi_{6_2} - C_6 + C_9 \cos \varphi_{6_2} + C_{10}$$

$$R = -2 \sin \varphi_{6_2}$$

$$S = C_6 + (C_9 + 1) \cos \varphi_{6_2} + C_{10}$$

C. Coupler equation

The coupler equation can be written in two parts for easy calculations. There are four different values for a single point which have come from the combinations of different coupler and output angles.

$$\begin{bmatrix} P_{x_1} \\ P_{x_2} \\ P_{x_3} \\ P_{x_4} \end{bmatrix} = \begin{bmatrix} r_1 \\ r_2 \\ -r_3 + r_5 \\ r_6 \end{bmatrix} \begin{bmatrix} \cos \varphi_2 & \cos \varphi_{3_1} & \cos \varphi_{4_1} & \sin \varphi_{4_1} \\ \cos \varphi_2 & \cos \varphi_{3_1} & \cos \varphi_{4_2} & \sin \varphi_{4_2} \\ \cos \varphi_2 & \cos \varphi_{3_2} & \cos \varphi_{4_1} & \sin \varphi_{4_1} \\ \cos \varphi_2 & \cos \varphi_{3_2} & \cos \varphi_{4_2} & \sin \varphi_{4_2} \end{bmatrix} \quad (11a)$$

$$\begin{bmatrix} P_{y_1} \\ P_{y_2} \\ P_{y_3} \\ P_{y_4} \end{bmatrix} = \begin{bmatrix} r_1 \\ r_2 \\ -r_3 + r_5 \\ -r_6 \end{bmatrix} \begin{bmatrix} \sin \varphi_2 & \sin \varphi_{3_1} & \sin \varphi_{4_1} & \cos \varphi_{4_1} \\ \sin \varphi_2 & \sin \varphi_{3_1} & \sin \varphi_{4_2} & \cos \varphi_{4_2} \\ \sin \varphi_2 & \sin \varphi_{3_2} & \sin \varphi_{4_1} & \cos \varphi_{4_1} \\ \sin \varphi_2 & \sin \varphi_{3_2} & \sin \varphi_{4_2} & \cos \varphi_{4_2} \end{bmatrix} \quad (11b)$$

And

$$\begin{bmatrix} S_{x_1} \\ S_{x_2} \\ S_{x_3} \\ S_{x_4} \end{bmatrix} = \begin{bmatrix} r_{10} \\ -r_{11} \end{bmatrix} \begin{bmatrix} \cos \varphi_{7_1} & \sin \varphi_{7_1} \\ \cos \varphi_{7_2} & \sin \varphi_{7_2} \\ \cos \varphi_{7_3} & \sin \varphi_{7_3} \\ \cos \varphi_{7_4} & \sin \varphi_{7_4} \end{bmatrix} \quad (12a)$$

$$\begin{bmatrix} S_{y_1} \\ S_{y_2} \\ S_{y_3} \\ S_{y_4} \end{bmatrix} = \begin{bmatrix} r_{10} \\ r_{11} \end{bmatrix} \begin{bmatrix} \sin \varphi_{7_1} & \cos \varphi_{7_1} \\ \sin \varphi_{7_2} & \cos \varphi_{7_2} \\ \sin \varphi_{7_3} & \cos \varphi_{7_3} \\ \sin \varphi_{7_4} & \cos \varphi_{7_4} \end{bmatrix} \quad (12b)$$

Now the coupler equation for given six bar Watt-II mechanism will be the combination of equation 11 and 12, which is given by:

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$$\begin{bmatrix} C_{x_1} \\ C_{x_2} \\ C_{x_3} \\ C_{x_4} \\ C_{x_5} \\ C_{x_6} \\ C_{x_7} \\ C_{x_8} \end{bmatrix} = \begin{bmatrix} P_{x_1} \\ P_{x_1} \\ P_{x_2} \\ P_{x_2} \\ P_{x_3} \\ P_{x_3} \\ P_{x_4} \\ P_{x_4} \end{bmatrix} + \begin{bmatrix} S_{x_1} \\ S_{x_2} \\ S_{x_1} \\ S_{x_2} \\ S_{x_3} \\ S_{x_4} \\ S_{x_3} \\ S_{x_4} \end{bmatrix} \quad \& \quad \begin{bmatrix} C_{y_1} \\ C_{y_2} \\ C_{y_3} \\ C_{y_4} \\ C_{y_5} \\ C_{y_6} \\ C_{y_7} \\ C_{y_8} \end{bmatrix} = \begin{bmatrix} P_{y_1} \\ P_{y_1} \\ P_{y_2} \\ P_{y_2} \\ P_{y_3} \\ P_{y_3} \\ P_{y_4} \\ P_{y_4} \end{bmatrix} + \begin{bmatrix} S_{y_1} \\ S_{y_2} \\ S_{y_1} \\ S_{y_2} \\ S_{y_3} \\ S_{y_4} \\ S_{y_3} \\ S_{y_4} \end{bmatrix} \quad (13)$$

$$\begin{bmatrix} C_x \\ C_y \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} C_{x_r} \\ C_{y_r} \end{bmatrix} + \begin{bmatrix} X_0 \\ Y_0 \end{bmatrix} \quad (14)$$

where $r=1, 2, \dots, 8$.

From the above equation (14), there are eight different coordinates for a single coupler point obtained for tracing curve whereas only one point will be considered on the basis of minimum difference between desired and these obtained coordinates.

IV. MATHEMATICAL MODELING

The objective function is the sum of two parts, one is related to primary objective function and next one is penalty terms to take care of constraints. The first part computes the design error which is the sum of the squares of the Euclidean distances between each desired points C_d^i and actual obtained points C_a^i where:

$$C_d^i = [C_{xd}^i; C_{yd}^i]^T \text{ and}$$

$$C_a^i = [C_x^i; C_y^i]^T = [C_x(\varphi_2^i); C_y(\varphi_2^i)]^T$$

where φ_2^i is the input crank angle

Now, the first part of the objective function is given by:

$$f_{obj} = \left\{ \sum_{i=1}^N [(C_{Xd}^i(X) - C_X^i)^2 + (C_{Yd}^i(X) - C_Y^i)^2] \right\}$$

where N is the number of precision points

The geometric magnitudes of six-bar mechanism are described in Fig. 1: the design variables, $r_1, r_2, r_3, r_4, r_5, r_6, r_7, r_8, r_9, r_{cx}, r_{cy}, \theta, x_0, y_0$, and the input angle, φ_2 . The second part of the objective function is derived from the constraints imposed on the mechanism. Thus, the next step is to set these constraints. Among them one could point out:

- (a) The Grashof condition, to allow for the entire turn of at least one link,
- (b) The range for the design variables,
- (c) The range of variation for the input angle

Now, the problem is completely defined by:

Minimize:

$$f_{obj} = \left\{ \sum_{i=1}^N [(C_{Xd}^i(X) - C_X^i)^2 + (C_{Yd}^i(X) - C_Y^i)^2 + Mh(X)] \right\}$$

Subjected to:

- (a) Grashof criterion for first four bar mechanism to ensure full rotation of at least one link that can act as input (or crank),

$$[\max(r_1, r_2, r_3, r_4)] + [\min(r_1, r_2, r_3, r_4)] < \{\text{sum}(r_1, r_2, r_3, r_4) - [\max(r_1, r_2, r_3, r_4)] + [\min(r_1, r_2, r_3, r_4)]\};$$

- (b) Feasible range for design variables,

$$x_i \in [\min(x_i), \max(x_i)] \quad \forall x_i \in X$$

Where

$$X = [r_1, r_2, r_3, r_4, r_5, r_6, r_7, r_8, r_9, r_{cx}, r_{cy}, \varphi_2, \theta, x_0, y_0]$$

$$h(X) = 0 \rightarrow \text{if Grashof condition is true}$$

$$h(X) = 1 \rightarrow \text{if Grashof condition is false}$$

where M is the constant that represent the weight of penalty function and the values of this penalty weight is taken as 1000.

V. RESULT AND DISCUSSION

This section comprises the result of the case study conducted for dimensional synthesis of six bar Watt-II mechanism using Particle Swarm Optimization. This problem was solved using an PSO algorithm programmed in MATLAB R2106a on intel Core i5 6th Gen 2.20 GHz processor. The set of parameters required for initializing the PSO algorithm is taken from [11] and are provided in Table 1.

Table-I: Initialization parameters for PSO algorithm

Parameter	PSO
Number of initial population	100
Constriction factor	0.689
Generation number	1000

A. Case Study: Path generation with prescribed timing for eighteen precision point

This problem deals with dimensional synthesis of six bar Watt-II mechanism for generation of a path which is the combination of two circular arcs containing eighteen precision points with prescribed timing and passing through eighteen tracing points. The precision points on the circular arcs are to be traced in correlation with the input crank angles. The input data of the coupler curve is shown in Table 2.

Table-II: Input data for case study

Precision point	C_{Xd}^i	C_{Yd}^i	δ_2^i
1	-0.5424	2.3708	0
2	0.2202	2.9871	15
3	0.9761	3.4633	40
4	1.0618	3.6380	60
5	0.8835	3.7226	80
6	0.5629	3.7156	100
7	0.1744	3.6128	120
8	-0.2338	3.4206	140
9	-0.6315	3.1536	160

10	-1.0000	2.8284	180
11	-1.3251	2.4600	200
12	-1.5922	2.0622	220
13	-1.7844	1.6539	240
14	-1.8872	1.2654	260
15	-1.8942	0.9448	280
16	-1.8096	0.7665	300
17	-1.6349	0.8522	320
18	-1.5187	1.6081	345

Where, Crank angle $\varphi_2^i = \varphi_2 + \delta_2^i$. In table 2, C_{xd}^i and C_{yd}^i represent the x and y-coordinates of precision points of the given coupler curve.

The range of design variables are taken as given following:

$$r_1, r_2, r_3, r_4, r_5, r_6, r_7, r_8, r_9, \in [0, 6];$$

$$r_{cx}, r_{cy}, X_0, Y_0 \in [-6, 6]; \varphi_2, \theta \in [0, 2\pi];$$

For optimized results, 20 iterations have been run in which three best results are shown in Table 3 and convergence rate are plotted in Fig 2. Iteration-III gives the closest result to the desired points with minimum error of 0.2348. The computer took 20-30 seconds for plotting the results.

Fig 3 shows the convergence plots for these three results with design errors. Result-I has started

Table-III: optimized results in three iterations

Parameter	Iteration-I	Iteration-II	Iteration-III
r_1	0.5214	0.4485	0.4555
r_2	5.9631	5.7503	4.6368
r_3	3.1137	3.4449	3.0049
r_4	3.6447	3.1965	2.3909
r_5	2.937	3.2166	1.4958
r_6	2.9128	5.4564	3.1631
r_7	5.9716	5.6176	5.554
r_8	5.0995	4.9687	4.8823
r_9	3.0785	2.818	0.7009
r_{cx}	-2.1653	-3.01	-2.8715
r_{cy}	3.7756	4.3241	2.0033
φ_2	0.0546	0.2698	3.9685
x_0	2.08	-0.4534	-1.0019
y_0	3.4311	2.9787	2.7219
θ	2.6509	2.9058	0.0377
Error	2.9604	0.5405	0.2348

convergence at very low error of 90 converged after 2200 iterations. Result-II has started beyond the 450 and converged after 4200 iterations. The most efficient result-III has started convergence at 340 error and converged at 2000 iterations. It gives the minimum error of 0.2348. The design errors are clearly shown in Fig 4.

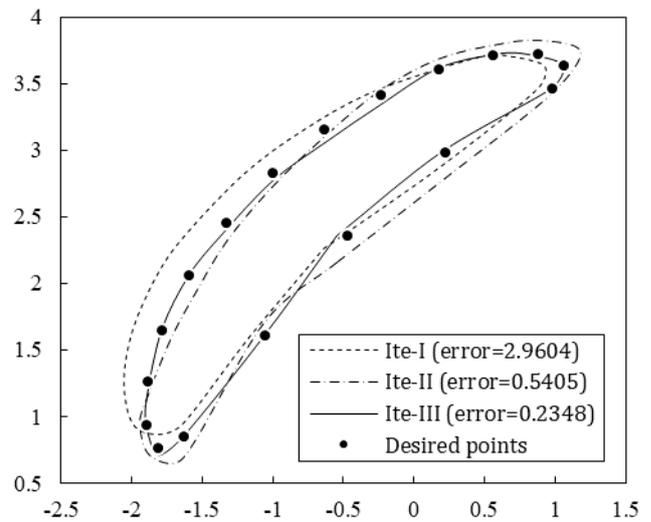


Fig. 2. The best path traced by the coupler obtained in three iterations with desired points

VI. CONCLUSION

In this paper, we presented the synthesis of six bar Watt-II mechanism using a heuristic optimization method, PSO which is earlier used for synthesis of four bar mechanism. As PSO is first time applied for six bar mechanism synthesis, the results are very exciting and took very little time to converge. The respective technique is used for this study because PSO requires very few parameters to run the algorithm and has low convergence time over other heuristic algorithms. This study was carried out by a program, developed MATLAB environment on intel Core i5 6th Gen 2.20 GHz processor.

From the above study, it can be considered that PSO is also a competitive technique for synthesis of six bar mechanism. The applicability of PSO may be examined in future by applying on other members of 1-F six bar mechanism family with greater number of precision points.

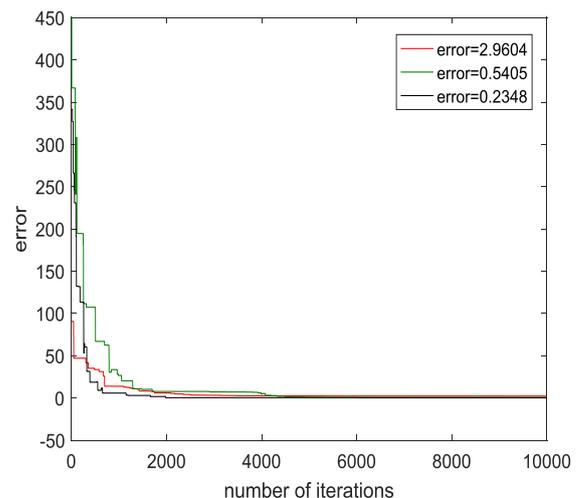


Fig. 3. Typical error curve for three different iterations

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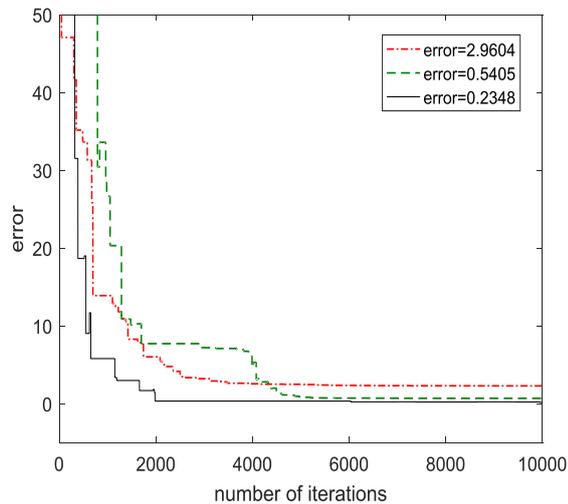


Fig. 4. Zoomed view of Fig 3

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