Vector Generation of Reflection and Shearing of 2D Objects: Sequencing in Column Major and Row Major Pattern

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***Abstract:** Transformations are the best and effective techniques of producing changed objects in terms of Displacement, Enlargements and Orientations. In other words the Transformation are the most challenging way of shifting or changing the dimensions and orientations of images in the most effective way.But This modifying and repositikoning of the existing images is calculated by a standard convention of Matrix Calculations.The usual practice of doing so is straight forward. The transformed object can be obtained by coupling (Matrix Multiplication) original object Matrix with the transformation vectors. The main challenge is how to evaluate it. The usual practice is standard Column Major Pattern. The alternative Row Major Pattern is also known approach but what matters is the sequence of operations that make these both approaches worth mentioning.Visualization can be enhanced if either of the approach is adopted. But what makes it more exciting is the calculations that come with both of these approaches. A normal practice is to use the standard Column Major Pattern for processing objects .An Alternative to this Technique is Row Major Pattern which equates to the same result only if sequence of operations are not compromised. This paper describes the transformation (Reflection and Shearing) in both Column and Row Major Pattern and at the same time aims in putting down clear semantics in justifying the sequence of operations.*

***Keywords :** 2D Transformations, Homogenous Coordinate System, Reflection, Shearing, Row Major Pattern , Column Major Pattern, Scaling, Translation*

I. INTRODUCTION

Computer Graphics needs strong visualization of 2D and 3D objects. Consider a analogy of an a graphic designer who analyzes Objects. It becomes important for the designer to study objects be in 2D or 3D from different angles. i.e. visualizing front Dimensions, Side Dimensions and Top Dimension, All such Visualizations require a complete analysis of displacement vectors ,zoom in parameters zoom out parameters and orientations. If such visuals are converted into numbers, the numbers can be saved and calculated for detailed analysis.

This calculation could be done effectively by Matrix Operations. All of these transformations can be efficiently and succinctly handled using some simple matrix representations My paper aims to compare the mathematical techniques laid down for such transformations (Reflection and Shearing) and at the same time explores the distinct sequence of calculations by this comparison

II. REVIEWING TRANSFORMATIONS

A. Definition of transformations

The process of changing of sizes orientations or positions of an object by a mathematical formulae is called Transformation. This can be accomplished by by two methods Geometric Transformation: The Object itself is transformed relative to stationary coordinate system. It is applied to each point of the object Coordinate Transformation: The object is held stationary while the coordinate system is transformed relative to the object

B. Classification of transformations

There are basically five transformations which are listed as below:

- Translation
- Scaling
- Rotation
- Reflection
- Shearing

The Objectives for laying this analytical methodology is important as far as transformations of 2D objects are concerned .They aim at

- Developing a clear semantics in laying down the mathematical formulae and
- Laying the proper sequencing of operation while performing the concatenation of the transformations

C. Scope of the work

This work helps in analyzing the underlying formulae and comparison in two approaches towards transforming a 2D object

III. METHODOLOGY

The methods in all the listed 2D transformations involve two different approaches:

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Column Major Pattern: In the Approach we multiply Column Vector Transformation matrix with the Original Object matrix to get the Transformed Object Matrix

- Row Major Pattern: In this Approach we multiply the Original Object Matrix with the Row Vector Transformation Matrix to get the Original Object Matrix

Before analysing the Column Major Pattern first followed by Row Vector pattern for 2D Transformations, we need to understand why we need to use **Homogenous Coordinate System**.To perform Concatenation of Transformations like Translation followed by rotation and shearing, we need to follow a sequential process −

- Shift the coordinates(additive operation)
- Perform Rotation of the displaced coordinates
- Shear the rotated coordinates to reach to concatenated transformation.

To get the operations into the uniform pattern, we introduce a homogenous system which means adding a dummy variable in 2x2 Matrix to make it 3x3 Matrix. Thus all the matrices discussed here will be using homogenous coordinate system. Let's explore Shearing first followed by Reflection

A. Shearing

Shearing is the transformation that distorts the object.it would not be wrong to mention that non-uniform scaling can result in shearing.This transformation can be along x-axis of 2D plane or Y-axis of 2D plane or it can be simultaneously on both X-axis and Y-axis.

Types of Shearing

- Shearing along X-axis
- Shearing along Y-axis
- Shearing along X-Yaxis

Shearing in Column Major Pattern

Shearing along X-axis(First matrix of Fig[1]),Y-axis(Second matrix of Fig[1]) and simultaneous X-Y axis(Third matrix of Fig[1]) in a Column Major Pattern can be calculated by multiplying the either of the shearing matrix S with the original object P1 to get the sheared object P2

The matrices for all can be written as

$$\begin{bmatrix} 1 & sh_x & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ sh_y & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & sh_x & 0 \\ sh_y & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Fig [1]

Thus P2=S·P1

Where S is the Shearing Matrix. It is important to note that if S is the Composite Transformation that contains Translation followed by Rotation and Shearing then the sequence of operations is taken from RIGHT->LEFT and the final transformation is multiplied by original coordinates to get the transformed coordinate

Shearing in Row Major Pattern

The Row Major Pattern takes the different form. Columns of all matrices shown in Fig[1] will become rows in Row major Pattern

$$\begin{bmatrix} 1 & 0 & 0 \\ Sh_x & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & Sh_y & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & Sh_y & 0 \\ Sh_x & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Fig[2]

In Row major Pattern the sequence of operation in case of composite transformations is that all operations will be laid down from LEFT->RIGHT and the final transformation is multiplied by original object to get the sheared object.

Thus P2=P1.S

B. Reflection

It is a change of the object.This change is nothing but the mirror image of the original object.The image resembles in symmetry and can be projected either about x-axis or y-axis. The object is rotated by 180°.

Types of Reflection

- Reflection through x-axis [Fig 3]
- Reflection through y-axis [Fig 4]
- Reflection about an axis perpendicular to xy plane and passing through the origin [Fig 5]
- Reflection about the line y=x [Fig 6]

Our major concern is the underlying semantics of matrices on reflection which differ in both Column Major Pattern as well as Row Major Pattern.This becomes more challenging when composite transformations come in play as in case of reflection about the line y=x. Lets now see the Reflection listed above in both Column and Row major Pattern

Reflection in Column Major Pattern

x-axis $(y = 0)$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

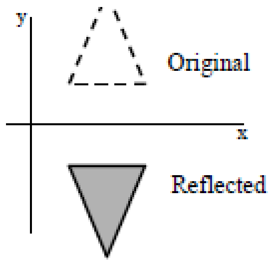

Fig [3]

In this Reflection value of x will remain same whereas the value of y will flip with it sign

y-axis $(x = 0)$

$$\begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

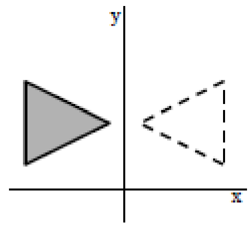

Fig [4]

In this Reflection, the value of y will remain the same and the value of x flips on its sign. The object will lie another side of the y-axis.

xy-plane

$$\begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

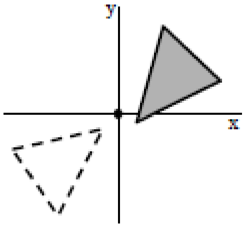

Fig [5]

In this Reflection value of x and y both will be reversed. This is also called as half revolution about the origin or simply reflection about origin.

xy-plane

$$\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

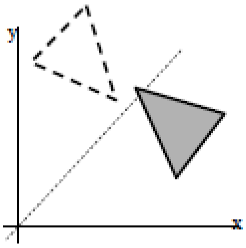

Fig [6]

First of all, the object is rotated at 45°. The direction of rotation is clockwise. After it reflection is done concerning x-axis. The last step is the rotation of y=x back to its original position that is counterclockwise at 45°.

Thus P2=R·P1

Where R is the Reflection Matrix which can be any of the matrices from Fig[3],Fig[4],[Fig[5]orFig[6]. If R is the composite transformation(as in case of Reflection about line y=x) and contains translation followed by rotation and finally Reflection then the sequence of operations is taken from RIGHT->LEFT and the final transformation is multiplied by original object to get the reflected object

Reflection in Row Major Pattern

The Reflection matrix takes the same form both in Column Major Pattern as well as in Row Major Approach. We can confirm it by rewriting the Reflection Matrix by arranging the elements in column as row. Importantly transformed Object with a Row Major can be calculated by multiplying the Original Object with the either of the Reflection Matrix.

IV. RESULT AND DISCUSSION

We can set up a Matrix for any number or sequence of Transformations as a Composite Transformation Matrix by calculating the matrix product for the individual transformations. It is often referred as concatenation or composition of matrices.In concatenation of transformations , the sequence of the transformations play a vital role .This sequence is written from Right to Left(R->L)in column Approach of synthesis.But in Row Approach the sequence takes the order from Left to Right(L->R). [T1]*[T2] is not equal to the [T2]*[T1].

V. CONCLUSION

It can be concluded that if as an graphics analyst we don't draw the clear comparisons between the both approaches we might compromise on the issues of having simple ,consistent matrix notation using Homogenous Coordinate System and finally land up in misinterpretation of window modeling. Thus it is important to have a clear understanding between the sequencing of operations in both the approaches .Concatenation of Transformations multiplied with the Original Object in Column Major Pattern will equate same with Row Major Pattern only if Original Object is multiplied with Concatenation of Transformations

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