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Anti fuzzy Bi-ideal of a Near Algebra

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Abstract. In this paper we introduce the notion of anti fuzzy bi-ideals of a near algebra and obtain some of the important properties related to the notion. We also study product of anti fuzzy bi-ideals in a near algebra and explore certain characterizations.

Keywords: Anti fuzzy set, bi-ideal, homomorphic image of a Near-algebra.

I. INTRODUCTION

Near-algebra is a near ring which admits a field as a right operator domain. Brown [2], Irish [4], Srinivas and Narasimha Swamy [13] have studied certain properties of a near-algebra.

Zadeh[15] has introduced the notion of fuzzy set. The fuzzy set theory developed by researchers has found many applications in the domain of mathematical sciences. The study of fuzzy algebraic structures started with the introduction of fuzzy subgroups (fuzzy groupoid) and fuzzy (left, right) ideals in the pioneering paper of Rosenfeld[12]. Biswas[1] introduced the notion of anti fuzzy subgroups. In near rings the notion of anti fuzzy ideals has initiated by Kim, Jun and Yon[5]. Kuroki[6] introduced and studied fuzzy(left, right) ideals and fuzzy bi-ideals in semigroups. Manikantan [8] has developed the notion of fuzzy bi-ideals in near ring. Later some basic concepts of fuzzy algebra such as fuzzy ideals and fuzzy bi-ideals in a semigroup, using a new approach of fuzzy spaces and fuzzy groups was introduced by Dib[3] in 1994. In this paper, we consider an anti-fuzzification to the concept bi-ideal in near-algebra. Further established homomorphic image and pre image to *Y* forms a semi-group under multiplication.
this notion. Throughout this paper we refer to right near-algebra as near-algebra.

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II. PRELIMINARIES

Definition 2.1[4]: A (RIGHT) near algebra *Y* over a field *X* is a linear space *Y* over *X* on which multiplication is defined such that

i. Multiplication is right distributive over addition
i e $(x+y)z = xz + yz$ for every $x, y, z \in Y$.
ii. $a(xy) = (ax)y$ for every $x, y \in Y$ and $a \in X$.

Definition 2.2[13]: A near algebra *Y* is said to be a *zero symmetric near algebra* or near c-algebra if $n.0 = 0$ *for every* $n \in Y$ where 0 is the additive identity in *Y*.

Definition 2.3[13]: Let *Y, Y'* be two near algebras over a field *X*. A mapping $f : Y \to Y'$ is called a *near algebra homomorphism* if

i. $f(x+y) = f(x) + f(y)$ for every $x, y \in Y$.
ii. $f(xy) = f(x) f(y)$ for every $x, y \in Y$.
iii. $f(ax) = a f(x)$ for every $x \in Y$, $a \in X$.

Definition 2.4[15]: Let *X* be a nonempty set. A *fuzzy subset* λ of *X* is a function $\lambda: X \to [0,1]$.

Definition 2.5: Let F be a fuzzy subset of X. Then F is called an anti fuzzy field of X if for $l, m \in X$.

i. $F(l+m) \le \max(F(l), F(m)) = F(l) \vee F(m)$
ii. $F(-l) \le F(l)$
iii. $F(lm) \le \max(F(l), F(m)) = F(l) \vee F(m)$
iv. $F(l^{-1}) \le F(l)$ for all $l(\neq 0) \in X$.

Definition 2.6: A fuzzy subset μ of Y is said to be an anti fuzzy near algebra over an anti fuzzy field *F* of X if,

i. $\mu(l+m) \le \max(\mu(l), \mu(m)) = \mu(l) \vee \mu(m)$
ii. $\mu(al) \le \max(F(a), \mu(l)) = F(a) \vee \mu(l)$,
iii. $\mu(lm) \le \max(\mu(l), \mu(m)) = \mu(l) \vee \mu(m)$,
iv. $F(1) \le \mu(l)$ for all $l, m \in Y$ *and* $a \in X$.

Definition 2.7: Let (*F, X*) be an anti fuzzy field and λ be an anti fuzzy subset of a linear space *Y* over the field *X*. Then λ is said to be an anti fuzzy subspace of *Y* if

i. $\lambda(x+y) \le \max(\lambda(x), \lambda(y))$.
ii. $\lambda(ax) \le \max(F(a), \lambda(x))$.

Definition 2.8: Let μ be an anti fuzzy sub set of near algebra *Y* over an anti fuzzy field (*F,X*). Then μ is called an anti fuzzy ideal of Y, if

i. $\mu(lm) \le \mu(l)$ and
ii. $\mu(m(l+e) - ml) \le \mu(e)$ for ever $l, m, e \in Y$.

Then μ is an anti fuzzy right ideal of Y if (i) is satisfied.
And μ is an anti fuzzy left ideal of Y if (ii) is satisfied.

Definition 2.9: Let f be a mapping from set *A* to *B*. Let μ and υ be anti fuzzy subsets of *A* and *B* respectively. Then

$f(\mu)$, the *image* of μ under f, is an anti fuzzy subset of B defined by

$$f(\mu)(y)=\begin{cases}\inf_{x\in f^{-1}(y)}\mu(x) \text{ if } f^{-1}(y)\neq\emptyset\\ 1 \quad otherwise\end{cases}$$

And the *pre-image* of υ under f is an anti fuzzy subset of A defined by $(f^{-1}(\upsilon))(x)=\upsilon(f(x))$ for all $x\in A$.

Definition 2.10[14]: Suppose μ and η be two anti fuzzy subsets of Y. Then μ∩ η, μ∪ η, anti -product $\mu o_a \eta$, and anti $*_a$product $\mu *_a \eta$, are anti fuzzy subsets of Y defined as,

i. $(\mu\cap\eta)(l)=\min\{\mu(l),\eta(l)\}$.
ii. $(\mu\cup\eta)(l)=\max\{\mu(l),\eta(l)\}$.
iii. $(\mu o_a\eta)(l)=\begin{cases}\inf_{l=mn}\max\{\mu(m),\eta(n)\}\\ 1 \; otherwise\end{cases}$
iv. $(\mu *_a\eta)(l)=\begin{cases}\inf_{l=a(b+c)-ab}\max\{\mu(a),\eta(c)\}\\ 1 \; otherwise\end{cases}$

Definition 2.11: An anti fuzzy subgroup λ of a near ring N is said to be an anti fuzzy bi-ideal, if
$(\lambda o_a\boldsymbol{\chi} o_a\lambda)\cup\left((\lambda o_a\boldsymbol{\chi})*_a\lambda\right)\supseteq\lambda$.

Definition 2.12: The anti-characteristic function of Y is denoted by $\boldsymbol{\chi}: Y\to[0,1]$ mapping every element of Y to 0. Throughout this paper Y will denote zero symmetric near-algebra over a fuzzy field (F, X).

III. ANTI FUZZY BI-IDEALS OF A NEAR ALGEBRA

Definition 3.1: An anti fuzzy subset λ of a near algebra Y over an anti fuzzy field (F,X) is an anti fuzzy bi-ideal, if
$\lambda(l+m)\leq\max\{\lambda(l),\lambda(m)\}$, $\lambda(al)\leq\max\{F(a),\lambda(l)\}$
and $(\lambda o_a\boldsymbol{\chi} o_a\lambda)\cup\left((\lambda o_a\boldsymbol{\chi})*_a\lambda\right)\supseteq\lambda$.

Example 3.2: Let $X=Z_2$ & F is an anti fuzzy field of X defined by

$$F(m)=\begin{cases}0.1 & if\ m=0\\ 0.3 & if\ m=1\end{cases}$$

Let $Y=\{0,p,q,r\}$ be a set with two binary operations "+" and "." as

+	0	p	q	r
0	0	p	q	r
p	p	0	r	q
q	q	r	0	p
r	r	q	p	0

.	0	p	q	r
0	0	0	0	0
p	0	q	0	q
q	0	0	0	0
r	0	q	0	q

And scalar multiplication on Y is defined as $0.l=0$, $1.l=l$ for every $l\in Y$, where $0,1\in X$. Clearly Y is a near algebra over a field X. Let μ be a fuzzy sub set of Y defined as $\mu(0)=0.5$, $\mu(p)=0.7$, $\mu(q)=\mu(r)=0$. Very easily one can verify that μ is an anti fuzzy bi-ideal of Y. At the same time μ is not a fuzzy bi-ideal of Y since $\mu(0)=\mu(p-p)\not\supseteq$ $\min\{\mu(p),\mu(p)\}$

Theorem 3.3: Union of family of anti fuzzy bi-ideals of a near algebra Y over a fuzzy field (F,X) is an anti fuzzy bi-ideal.

proof: $\{\lambda_i(l)\}_{i\in\Omega}$ is a group of anti fuzzy bi-ideals of Y
Where $\lambda(l)=\cup\lambda_i(l)=sup\,\lambda_i(l), i\epsilon\Omega$.
For any l, m, $n\in$Y and $a\in$X we have
$\cup\lambda_i(l+m)=\sup\{\lambda_i(l+m), i\epsilon\Omega\}$
$\leq\sup\{\max\{\lambda_i(l),\lambda_i(m)\}\}, i\epsilon\Omega$
$=\max\{sup\lambda_i(l), sup\lambda_i(m)\}, i\epsilon\Omega$
$=\max\{\lambda(l),\lambda(m)\}$.
$\cup\lambda_i(al)=\sup\{\lambda_i(al), i\epsilon\Omega\}$
$\leq\sup\{\max\{F(a),\lambda_i(l)\}\}, i\epsilon\Omega$
$=\max\{F(a), sup\lambda_i(l)\}, i\epsilon\Omega=\max\{F(a),\lambda(l)\}$.
$\lambda=\cup\lambda(l_i)=\lambda_i(l), i\epsilon\Omega$. We have
$(\lambda o_a\boldsymbol{\chi} o_a\lambda)\cup\left((\lambda o_a\boldsymbol{\chi})*_a\lambda\right)(l)$
$\geq(\lambda_i o_a\boldsymbol{\chi} o_a\lambda_i)\cup((\lambda_i o_a\boldsymbol{\chi})*_a\lambda_i))(l)$
$\geq\lambda_i(l),\ \forall i\epsilon\Omega$. Since λ_i is anti -fuzzy bi-ideal.
$(\lambda o_a\boldsymbol{\chi} o_a\lambda)\cup((\lambda o_a\boldsymbol{\chi})*_a\lambda)(l)$
$\geq\sup\{\lambda_i(l)/i\epsilon\Omega\}=(\cup\lambda_i)(l)=\lambda(l)$.
Thus $(\lambda o_a\boldsymbol{\chi} o_a\lambda)\cup\left((\lambda o_a\boldsymbol{\chi})*_a\lambda\right)\supseteq\lambda$
Hence $\lambda=\cup\lambda_i(l), i\epsilon\Omega$ is an anti fuzzy bi-ideal.

Theorem 3.4 If η is any anti fuzzy subspace of a zero symmetric near algebra Y and λ is an anti fuzzy bi-ideal of Y then $\lambda o\eta$ is an anti fuzzy bi-ideal of Y.

Proof: For l, m, $n\in$Y and $a\in$X,
$(\lambda o_a\eta)(l+m)=\max\{\lambda(l+m),\eta(l+m)\}$
$\leq\max\{\max\{\lambda(l),\lambda(m)\},\max\{\eta(l),\eta(m)\}\}$
$=\max\{\max\{\lambda(l)\,\eta(l)\},\max\{\lambda(m),\eta(m)\}$
$=\max\{(\lambda o_a\eta)(l),(\lambda o_a\eta)(m)\}$. Therefore
$(\lambda o_a\eta)(l+m)\leq\max\{(\lambda o_a\eta)(l),(\lambda o\eta)(m)\}$
$(\lambda o_a\eta)(al)=\max\{\lambda(al),\eta(al)\}$
$\leq\max\{\max\{F(a),\lambda(l)\},\max\{F(a),\eta(l)\}\}$
$=\max\{maxF(a),\max(\lambda(l),\eta(l))\}$
$=\max\{F(a),(\lambda o_a\eta)(l)\}$
Hence $(\lambda o_a\eta)$ is an anti fuzzy subspace of Y.
$((\lambda o_a\eta)o_a\boldsymbol{\chi} o_a(\lambda o_a\eta))\cup\left(((\lambda o_a\eta)o_a\boldsymbol{\chi})*_a(\lambda o_a\eta)\right)$
$=[\lambda o\eta o_a\boldsymbol{\chi} o_a\lambda\cup\lambda o\eta o_a\boldsymbol{\chi}*_a\lambda]o_a$
$\geq[\lambda o_a\boldsymbol{\chi} o_a\boldsymbol{\chi} o_a\lambda\cup\lambda o_a\boldsymbol{\chi}*_a\lambda]o_a\eta$
$\geq[\lambda o_a\boldsymbol{\chi} o_a\lambda\cup\lambda o_a\boldsymbol{\chi}*_a\lambda]o_a\eta\ \geq\lambda o_a\eta$
(since λ is anti fuzzy bi-ideal)
Hence $(\lambda o_a\eta)$ is an anti fuzzy bi-ideal.

Theorem 3.5: Every anti fuzzy left ideal of Y is an anti fuzzy bi-ideal of Y.

Proof: Assume that λ is an anti fuzzy left ideal of Y. Let $l,m,n,a,b,c,i,p,q\in Y$ such that $l=pq=(km)n=a(b+i)-ab$, Then we have
$\left((\lambda o_a\chi o_a\lambda)\cup\left((\lambda o_a\chi)*_a\lambda\right)\right)(l)$
$\geq\max((\lambda o_a\chi o_a\lambda)(l),(\lambda o_a\chi)*_a\lambda)(l))$
$=\max\{\inf_{l=pq}\max\{(\lambda o_a\chi)(p),\lambda(q)\},$
$\inf_{l=a(b+i)-ab}\max\{(\lambda o_a\chi)(a),\lambda(i)\}\}$
(Since $\lambda o_a\chi\supseteq\chi$ & $since\ \lambda$ is an anti fuzzy left ideal of Y $\lambda(a(b+i)-ab)\leq\lambda(i))\}$
$\geq\min\{\sup_{l=pq}\min\{\chi(km),\chi(q)\},$
$\sup_{l=a(b+i)-ab}\min\{\chi(a),\lambda(a(b+i)-ab)\}\}$
$=\lambda(a(b+i)-ab)=\lambda(l)$
Hence $(\lambda o_a\chi o_a\lambda)\cup\left((\lambda o_a\chi)*_a\lambda\right)\supseteq\lambda$.
Hence λ is an anti fuzzy bi-ideal of near algebra Y.

Theorem 3.6: Every anti fuzzy right ideal of near algebra Y is an anti fuzzy bi-ideal of Y.

Proof: Assume that λ is an anti fuzzy right ideal of Y. Let $l,m,n,a,b,c,i,p,q\in Y$ such that $l=pq=(km)n=a(b+i)-ab$, Then we have

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$\big((\lambda o_a \chi o_a \lambda) \cup ((\lambda o_a \chi) *_a \lambda)\big)(l)$
$\geq \max((\lambda o_a \chi o_a \lambda)(l), (\lambda o_a \chi) *_a \lambda)(l))$
$= \max\{\inf_{l=pq} \max\{(\lambda o_a \chi)(p), \lambda(q)\},$
$(\lambda o_a \chi) *_a \lambda)(a(b+i) - ab)\}$
$= \max\{\inf_{l=pq} \max\{\{\inf_{p=km} \max\{\lambda(k), \chi(m)\}, \lambda(q)\},$
$(\lambda o_a \chi) *_a \lambda)(a(b+i) - ab)\}$
$= \max\{\inf_{l=pq} \max\{\lambda(k), \lambda(q)\}, (\lambda o \chi) *_a \lambda)(a(b+i) - ab)\}$
(Since λ is a fuzzy right ideal of Y we have
$\lambda(pq) = \lambda((km)q) = \lambda(k(mq)) \leq \lambda(k))$
$\geq \max\{\inf_{l=pq} \max\{\lambda(pq), \chi(q), \chi(a(b+i) - ab)\}$
$= \max\{\lambda(pq), \chi(a(b+i) - ab)\} = \lambda(pq) = \lambda(l)$
Thus $(\lambda o_a \chi o_a \lambda) \cup ((\lambda o_a \chi) *_a \lambda) \supseteq \lambda$.
Hence λ is an anti fuzzy bi-ideal of Y.

Theorem 3.7: Let λ is an anti fuzzy bi-ideal of a zero symmetric near algebra Y. Then $\lambda(lmn) \leq \max\{\lambda(l), \lambda(m)\}$ for all *l*, *m*, *n* $\in$Y.
Proof: Assume that λ is an anti fuzzy bi-ideal of a zero symmetric near algebra Y. Then we have $\lambda o_a \chi o_a \lambda \supseteq \lambda$. Let *l*, *m*, *n* $\in$Y then for $lmn = (lm)n = pq$ we have
$\lambda(lmn) \leq \lambda o_a \chi o_a \lambda(lmn)$
$= \inf_{lmn=pq} \max\{(\lambda o_a \chi)(p), \lambda(q)\}$
$= \max\{(\lambda o_a \chi)(p), \lambda(q)\}$
$= \max\{\inf_{p=lm} \max\{(\lambda(l), \chi(m)\}, \lambda(n)\}$
$= \max\{\max\{(\lambda(l), \chi(m)\}, \lambda(n)\}$
$= \max\{\lambda(l), \lambda(n)\}$
Hence $\lambda(lmn) \leq \max\{\lambda(l), \lambda(n)\}$

Theorem 3.8: If λ is an anti fuzzy bi-ideal of near algebra Y if and only if λ^c is a fuzzy bi-ideal of Y.
Proof: Let Y is a near algebra and λ is an anti fuzzy bi-ideal of Y. For *l*, *m*, *n* $\in$ Y and *a* $\in$ X then,
$\lambda^c(l+m) = 1 - \lambda(l+m) \geq 1 - \max\{\lambda(l), \lambda(m)\}$
$= \min\{1 - \lambda(l), 1 - \lambda(m)\} = \min\{\lambda^c(l), \lambda^c(m)\}$.
Therefore$\lambda^c(l+m) \geq \max\{\lambda^c(l), \lambda^c(m)\}$.
$\lambda^c(al) = 1 - \lambda(al) \geq 1 - \max\{F(a), \lambda(l)\}$
$= \min\{F(a), 1 - \lambda(l) = \max\{F(a), \lambda^c(l)\}$.
$(\lambda^c o_a \boldsymbol{\chi} o_a \lambda^c) \cap ((\lambda^c o_a \boldsymbol{\chi}) *_a \lambda^c)$
$\leq \min\{(\lambda o_a \boldsymbol{\chi} o_a \lambda)^c(l), (\lambda o_a \boldsymbol{\chi} *_a \lambda)^c(l)$
$= \min\{(1 - \lambda o_a \boldsymbol{\chi} o_a \lambda)(l), (1 - \lambda o_a \boldsymbol{\chi} *_a \lambda)(l)$
$= 1 - \max\{(\lambda o_a \boldsymbol{\chi} o_a \lambda)(l), (\lambda o_a \boldsymbol{\chi} *_a \lambda)(l)\}$
$= 1 - [(\lambda o_a \boldsymbol{\chi} o_a \lambda)](l) = 1 - \lambda(l) = \lambda^c$.
Thus $(\lambda^c o_a \boldsymbol{\chi}\, o_a \lambda^c) \cap ((\lambda^c o_a \boldsymbol{\chi}) *_a \lambda^c) \subseteq \lambda^c$.
Hence λ^c is a fuzzy bi-ideal of near algebra Y.
conversely suppose that λ^c is a fuzzy bi-ideal
For *l*, *m*, *n* $\in$ Y and *a* $\in$ X then,
$\lambda(l+m) = 1 - \lambda^c(l+m) \leq 1 - \min\{\lambda^c(l), \lambda^c(m)\}$
$= \max\{1 - \lambda^c(l), 1 - \lambda^c(m)\} = \max\{\lambda(l), \lambda(m)\}$.
Therefore $\lambda(l+m) \leq \max\{\lambda(l), \lambda(m)\}$.
$\lambda(al) = 1 - \lambda^c(al) \leq 1 - \min\{F(a), \lambda^c(l)\}$
$= \max\{1 - F^c(a), 1 - \lambda^c(l)\} = \max\{F(a), \lambda(l)\}$.
$(\lambda o_a \boldsymbol{\chi} o_a \lambda) \cup ((\lambda o_a \boldsymbol{\chi}) *_a \lambda)(l)$
$= 1 - \{(\lambda o_a \boldsymbol{\chi} o_a \lambda)^c \cup (\lambda o_a \boldsymbol{\chi} *_a \lambda)^c\}(l)$
$\geq 1 - \max\{(\lambda o_a \boldsymbol{\chi} o_a \lambda)^c(l), (\lambda o_a \boldsymbol{\chi} *_a \lambda)^c(l)\}$
$= 1 - [(\lambda o_a \boldsymbol{\chi} o_a \lambda)^c](l) = 1 - \lambda^c(l) = \lambda(l)$.
Hence λ is an anti fuzzy bi-ideal of Y.

Theorem 3.9: If $f: Y_1 \to Y_2$ be a near algebra homomorphism. If λ is an anti fuzzy bi-ideal of Y_1 then the image $f(\lambda)$ of λ under f is an anti fuzzy bi-ideal of Y_2.
Proof: Given λ is an anti fuzzy bi-ideal of Y_1.
Therefore $(\lambda o_a \boldsymbol{\chi} o_a \lambda) \cup ((\lambda o_a \boldsymbol{\chi}) *_a \lambda) \supseteq \lambda$
Let $l_1, m_1, n_1 \in Y_1$ and $l_2, m_{m2}, n_2 \in Y_2$.
$f(\lambda)(l_1 + m_1) \leq \sup\{\lambda(l_2 + m_2)\}$
$\leq \sup\{\max\{\lambda(l_2), \lambda(m_2)\}$
$\leq \max\{\sup\{\lambda(l_2)\}, \sup\{\lambda(m_2)\}\}$
$= \max\{f(\lambda)(l_1), f(\lambda)(m_1)\}$
Therefore $f(\lambda)(l_1 + m_1) \leq \max\{f(\lambda)(l_1), f(\lambda)(m_1)\}$
$f(\lambda)(al_1) \leq \sup\{\lambda(al_2)\} \leq \sup\{\max\{F(a), \lambda(l_2)\}$
$\leq \max\{F(a), \sup\{\lambda(l_2)\}\}\} = \max\{F(a), f(\lambda)(l_1)\}$
Therefore $f(\lambda)(al_1) \leq \max\{F(a), f(\lambda)(l_1)\}$
Hence $f(\lambda)$ is an anti fuzzy subspace. Now consider
$[(f(\lambda) o_a \boldsymbol{\chi} o_a f(\lambda)) \cup ((f(\lambda) o_a \boldsymbol{\chi}) *_a f(\lambda))](l)$
$\geq \max\{(f(\lambda) o_a \boldsymbol{\chi} o_a f(\lambda))(l), ((f(\lambda) o_a \boldsymbol{\chi}) *_a f(\lambda))(l)\}$
$\geq (f(\lambda) o_a \boldsymbol{\chi} o_a f(\lambda))(l) \geq f(\lambda)(l)$
Thus $(f(\lambda) o_a \boldsymbol{\chi} o_a f(\lambda)) \cup ((f(\lambda) o_a \boldsymbol{\chi}) *_a f(\lambda)) \supseteq f(\lambda)$.
Hence $f(\lambda)$ is an anti fuzzy bi-ideal of Y_2.

Theorem 3.10: If $f: Y_1 \to Y_2$ be a near algebra homomorphism. If λ is an anti fuzzy bi-ideal of Y_2 then the preimage $f^{-1}(\lambda)$ of λ under f is an anti -fuzzy bi-ideal of Y_1
Proof: Since λ is an anti fuzzy bi-ideal of Y_2,
then $(\lambda o_a \boldsymbol{\chi} o_a \lambda) \cup ((\lambda o_a \boldsymbol{\chi}) *_a \lambda)(l) \supseteq \lambda$.
And $f^{-1}(\lambda)(l) = \lambda f(l)$, If *l*, *m* ,*n* $\in Y_1$ and a $\in$ X
subsequently$f^{-1}(\lambda)(l+m) = \lambda(f(l+m))$
$= \lambda(f(l) + f(m)) = \max\{\lambda(f(l)), \lambda(f(m))\}$
$= \max\{f^{-1}(\lambda(l)), f^{-1}(\lambda(m))\}$.
Hence$f^{-1}(\lambda)(l+m) \leq \max\{f^{-1}(\lambda(l)), f^{-1}(\lambda(m))\}$.
$f^{-1}(\lambda)(al) = \lambda(f(al)) = \lambda(F(a), f(l))$
$= \max\{(F(a), \lambda(f(l))\} = \max\{F(a), f^{-1}(\lambda(l))\}$.
Therefore$f^{-1}(\lambda)(al) \leq \max\{F(a), f^{-1}(\lambda(l))\}$.
Thus $f^{-1}(\lambda)$ is an anti fuzzy subspace of Y_1.
$[(f^{-1}(\lambda) o_a \boldsymbol{\chi}\, o_a f^{-1}(\lambda)) \cup ((f^{-1}(\lambda) o_a \boldsymbol{\chi}) *_a f^{-1}(\lambda))](l)$
$= [(\lambda f(x) o_a \boldsymbol{\chi}\, o_a \lambda f(x)) \cup ((\lambda f(x) o_a \boldsymbol{\chi}) *_a \lambda f(x))]$
$= \max\{(\lambda o_a \boldsymbol{\chi}\, o_a \lambda) f(l), ((\lambda\, o_a \boldsymbol{\chi}) *_a \lambda) f(l)\}$.
$\geq (\lambda\, o_a \boldsymbol{\chi}\, o_a \lambda) f(l) \geq \lambda f(l)$ (since λ is a fuzzy bi-ideal)
$= f^{-1}(\lambda)(l)$.
$[(f^{-1}(\lambda) o_a \boldsymbol{\chi} o_a f^{-1}(\lambda)) \cup ((f^{-1}(\lambda) o_a \boldsymbol{\chi}) *_a f^{-1}(\lambda))](l)$
$\geq f^{-1}(\lambda)(l)$.
Hence $f^{-1}(\lambda)$ is an anti fuzzy bi-ideal of Y_1.

IV. CONCLUSION

In this paper, we introduced the notion of anti fuzzy bi-ideals of a near algebra and also studied few properties of it. We have studied union of anti fuzzy bi-ideals is an anti fuzzy bi-ideal, we proved that the complement of anti fuzzy bi-ideals is a fuzzy bi-ideal the homomorphic image is an anti fuzzy bi-ideal then its preimage also is an anti fuzzy bi-ideal of a near algebra.

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