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Implementation of Graph Colouring Technique in Crime Science

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***Abstract*: This paper deals with the new method of graph colouring technique implemented in crime science. The graphical investigation uses the elements of graph theory to identify the suspects involved in unlawful act. The research paper focuses on the idea of graph colouring technique in crime problem which provides mathematical clarity for investigation in identifying the suspects. This method helps the crime scientist to expedite the investigation process to reduce time factor in decision making.**

***Keywords*: Proper vertex colouring, Proper edge colouring, Crime investigation problem.**

I. INTRODUCTION

Graph theory is a device used to inspect the relationship between the different entities. A graph G(V, E) is a collection of vertex set V(G) and edge set E(G). Assignment of colours to vertices or edges of the graph with a label called graph colouring. Assignment of colours to the vertices of the graph such that adjacent vertices are assigned distinct colours is called as vertex colouring. Assignment of colours to the edges of the graph such that adjacent vertices are assigned distinct colours is called as edge colouring. If any of the k-colours are assigned to the vertices of the graph then graph is said to be k-colourable. If the graph is k-colourable then k colours are required to colour the vertices of such that no two adjacent vertices share the common colour which is called the chromatic number of the graph (Bollobas, 1979; Bondy & Murty, 2008; Trudeau, 1994).

Graph theory is used to inspect the unstructured data in social network where the datasets are represented as vertices and their relationship is represented as edges. In the investigation of crime, graph theory is used in forensic examination to identify the suspects where the suspects are represented as vertices and their acquaintances are represented as edges. Investigation of crime rest on eyewitness, backtalk, hearsay, declaration or confessions and progress through the method of data collection and identification of suspects through analysis. The research paper describes the methodology to identify the suspect by examining the entire crime through graph theoretic approach which acts a model to investigation of offence. Proper understanding of crime graphs and colouring techniques are required to solve the conflicts in the real world in the process of obtaining the solution. Graph colouring techniques are applied in the various fields like Traffic analysis, forensic investigation, Register allocation, Aircraft scheduling, GSM mobile phone network (Deo, 2016; Easttom, 2017a; Easttom, 2017b).

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II. APPLICATION OF GRAPH COLOURING TECHNIQUE TO INVESTIGATE THE OFFENCE

A. Graph Theory in the Scene of Crime

In crime science, mathematical model is used for investigation. With the advent of computers, the modelling techniques represent the aspect of criminal behaviour and the analysis results in numerical solutions. These models helps the authorities to expedite the investigation process aimed at reducing time in decision making or to develop various strategies. In this paper, the method deals with the evidence in question and estimating the connections between individual evidence suspects, victims, and other entities connected with investigation (Haggerty et al., 2011; Hamed et al., 2014; Jennifer & Hsinchun, 2004).

This method analyses three elements of the graph such as the vertex set, the edge set, and the incidence function that relates edges to vertices. The mathematics of graph theory explains that an edge can be incident from one vertex to another if there is any connection between the entities. The first step in applying graph theory to any investigation is to identify the various elements involved in the incident. These elements will be represented as vertices. When any two elements have any connection that connection is represented as an edge. The edge not only connects the vertices but acts as the initiator of that connection which results in graph with parallel edges. For example, if the investigation includes confidential data that was robbed from firm $F1$ and subsequently found in firm $F2$ then consider each firm as a vertex and the connections between these employees from firm $F1$ to firm $F2$ would be represented as edges. The key acts as a modelling tool to ensure that all entities and connections are represented in a graph for various classes of crime (Hamed et al. 2011; Jennifer & Hsinchun, 2004; Roderic et al. 2014; Trudeau, 1994).

B. Methodology to Construct the Graph for Investigation

To illustrate the methodology of graph colouring technique in investigations of crime here we provide the algorithm which clearly depicts the formation of graph. Consider the scenario involving a crime where a jewellery shop has been subjected to a robbery. Let us assume the suspects U, V and W as vertices.

Assign the colours blue, red and green to the vertices. Suspect U says he is not the robber then lines to be drawn from U to V and W and assign colour blue. Suspect V says U is the robber then there is a line connecting from V to U and assign colour red. Suspect W says he is not the robber then lines to be drawn from W to U and V and assign colour green.

This represents the graph with three vertices and connected by edges to identify the robber (Easttom, 2017a; Easttom, 2017b; Haggerty et al. 2011; Hamed et al. 2011; Jennifer & Hsinchun, 2004).

Algorithm for Construction of Graph G

Step 1: Consider the suspects as vertices U,V and W.

Step 2: Assign distinct colour to the vertices blue, red and green to U,V and W.

Step 3: Represent the connection between the vertices U, V and W as edges and assign the respective colour to the edges.

Step 4: Suspect U says he is not the robber then draw lines from U to V and W. Now assign the colour blue to the edges.

Step 5: Suspect V says U is the robber then draw lines connecting from V to U. Now assign colour red to the edges.

Step 6: Suspect W says he is not the robber then draw lines from W to U and V. Now assign colour green to the edges.

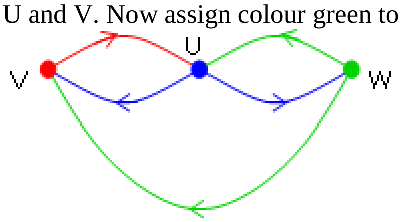

Fig. 1.Crime graph G for three suspects

III. RESULT AND DISCUSSION

C. Discussion of crime problem and role of colouring technique in crime investigation

Proposition 1:

In the scene of crime, police arrested three suspects for investigation. Suspect U says he is not robber. Suspect V says U is the robber. Suspect W says he is not the robber where one of the suspects says truth out of the three. The information of the suspects are given in the following table. Find out the robber in this crime scene?.

Table I: Information of suspects for the graph G

SUSPECTS	INVESTIGATION
U	I am not the robber
V	U is the robber
W	I am not the robber

Proof:

Consider the crime graph G where the vertices U,V,W are suspects and connection between is represented as edges. Assign the colours blue, red and green to the vertices. Suspect U says he is not robber then lines to be drawn from U to V and W and assign colour blue. Suspect V says U is the robber then there is a line connecting from V to U and assign colour red. Suspect W says he is not the robber then lines to be drawn from W to U and V and assign colour green. This represents the graph with three vertices and connected by edges to identify the robber.

Case 1: Police suspects U to be the robber

Suppose if U is the robber then V or W are not the robbers. Here V is telling truth as he says U is the robber. Also V and W become liars. If so W says he is not the robber which is a contradiction which violates one person says truth. Hence U is not the robber.

Implementation of colouring technique for case 1:

Suppose if U is the robber then redraw the graph G by removing all the lines coming out from U. Here V is telling truth as he says U is the robber so there is a line from V to U. Here W says he is not the robber so remove the lines coming out from W which is a contradiction to assumption which is clearly depicted in graph G1. Graphically, two lines pointing towards A which is a contradiction to one person says truth. Hence U is not the robber.

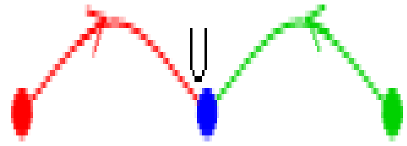

Fig. 2.1st level investigation graph G1

Case 2: Police suspects *V* to be the robber

Suppose if V is the robber then U or W are not the robbers. Here U is telling truth as he says he is not the robber. Also V and W become liars. Here V becomes liar as he says U is the robber. But W says he is not the robber which is a contradiction which violates one person says truth. Hence V is not the robber.

Implementation of colouring technique for case 2:

Suppose if V is the robber then redraw the graph G by removing all the lines coming out from V. Here U is telling truth as he says he is not the robber so remove the line from W to U. But W says he is not the robber which is a contradiction which violates one person says truth which is clearly explained in the graph G2. Also graphically, two lines pointing towards V which is a contradiction. Hence V is not the robber.

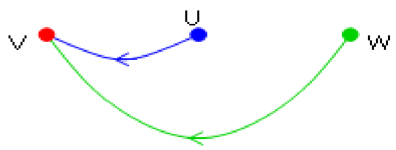

Fig. 3.2nd level investigation graph G2

Case 3: Police suspects *W* to be the robber

Suppose if W is the robber then V or U are not the robbers. Here U is telling truth as he says he is not the robber. Clearly V become liar from his statement. Also W become liar as V says U is the robber. This case is logically consistent where one person telling truth. Hence W is the robber.

Implementation of colouring technique for case 3:

Suppose if W is the robber then redraw the graph by removing all the lines coming out from W. Here U is telling truth as he says he is not the robber so remove all lines pointing towards U. Also V says U is the thief who is a liar hence remove the line from V to U which is clearly demonstrated in the graph G3.Graphically there is a single line from W to U which is logically consistent where one person telling truth. Hence W is the robber.

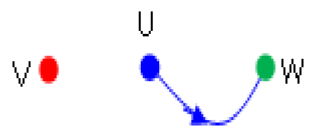

Fig. 4.3rd level investigation graph

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Thus we arrive at the **solution** for the crime problem using the graph colouring techniques during the investigation process which expedites the crime scientist to reduce time factor for detecting crime.

Observations:

1. In the proposition 1, we arrive at the following graphical sequence (2,2,1) is the solution set for the result of investigation of crime problem which indicates the number of lines coming towards (V,U,W).This sequence clearly detects the number of person telling truth if the person is the robber.

Algorithm to Construct Graphical Sequence

Step 1: Consider a vertex in the graph G.
Step 2: Count the number of lines going inward from the graph G. This count gives the number of person telling truth if the person is the thief.
Step 3: Repeat the iteration process.

2. In this investigation, we arrived at W is the robber for one person telling truth which is proved above.
3. If the investigation deals with two people telling truth, we arrive at choices as V or U. In this case, we cannot find the exact person as it leads to choice instead we can say definitely not W.

D. Extension of the Proposition 1

Here we discuss about the case of more suspects $X1,X2,X3...$ for a complicated set of acquisitions. For example, we extend the robbery case to four suspects in the crime scene. The information of the suspects are given in the following table. Find out the robber involved in this crime?.

Table II: Information of Suspects for the graph G*

SUSPECTS	INVESTIGATION
P	I am not the robber
Q	P is the robber
R	I am not the robber
S	Q is the robber

Proof:

Consider the crime graph G* where the vertices P,Q,R,S are suspects and connection between is represented as edges. Assign the colours blue, green, red and yellow to the vertices. Suspect P says he is not robber then lines to be drawn from P to Q, R and S and assign colour blue to the corresponding vertex and edges. Suspect Q says P is the robber then there is a line connecting from Q to P and assign colour green. Suspect R says he is not the robber then lines to be drawn from R to P,Q and S and also assign colour green. . Suspect S says Q is the robber then there is a line connecting from S to Q and assign colour yellow. Hence this graph clearly reveals the connection between the acquisitions. This represents the graph with four vertices and connected by edges to identify the robber.

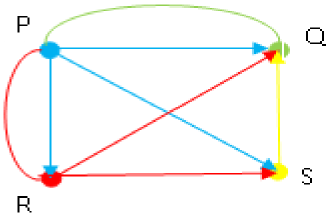

Fig. 5.Crime graph G* for four suspects

The proof is obtained by proceeding in the same method as proposition 1. Here we arrive at the following sequence (2,3,1,2) is the solution set for the result of investigation of crime problem which indicates the number of lines coming towards (P,Q,R,S). This sequence clearly detects the number of person telling truth if the person is the robber. we can find possible solutions directly by counting number of lines coming in from the vertices which detects the number of people telling truth.

Observations:

1. In this extension of proposition 1, we arrive at the following sequence (2,3,1,2) is the solution set for the result of investigation of crime problem which indicates the number of lines coming towards (P,Q,R,S). This sequence clearly detects the number of person telling truth if the person is the robber.
2. In this extension of proposition 1, from the solution set it is clear that R is the robber for one person telling truth which is proved above in the cases of proposition 1.
3. For two person telling truth we arrive at choices as P or S. In this case, we cannot find the exact person as it leads to choice instead we can say definitely not Q and R.

For three person telling truth we arrive at accurate solution that Q is the robber. These solutions helps the crime scientist to identify the criminal for lower number of suspects directly. In higher orders, this method helps to minimize the confusion over suspects as the case results in guesses either $X1$ or $X2$.

IV. CONCLUSION

This research paper presents a new methodology in identifying the suspect to solve a real time investigation problem from a mathematical perspective. In this paper, we provide the data for crime detection which helps authorities who are responsible for enquiry with a lack of information. In the future study, research work is based upon programming the algorithm in computer defined language. This research paper blends graph colouring techniques with crime scene for investigation which results in mathematics of crime.

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