Mathematical Simulation of the Movement of a Machine Unit of a Saw Cylinder with Distributed Parameters

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Abstract: Dynamic characteristics of a saw gin are considered as the subsystems with concentrated and distributed parameters. The graphs plotted as a result of studying the machine units made it possible to determine the maximum values of the angle of relative rotation and the rotation angle of the saw cylinder shaft under torsion.

Keywords: fiber extracting machines, gin, saw cylinder, machine unit with concentrated and distributed parameters, electric motor, torsional vibrations, mathematical model, shaft rotation angle.

I. INTRODUCTION

Studying machines in the form of machine units allows us to more accurately assess the dynamic processes occurring in the drive – gear-actuator system under technological loads [1].

In [2], a dynamic model of a machine unit was developed, including an operating device, an electric drive, and an elastic-dissipative clutch. A mathematical model of a machine unit was made up. The implementation of the model in a computer program made it possible to conduct a computer experiment varying mechanical parameters of dynamic system, to substantiate energy-saving parameters and operating modes of machine units.

N.V. Loshchinin in [3], has derived a differential equation of motion of a machine unit with a variator at a gear ratio \(\omega_1/\omega_2 = \omega(t)/\omega_2\), which depends on time \(t\) and angular velocity \(\omega_2\) of the driven shaft of the variator. Corresponding formulas are stated and the dynamic essence for certain generalized parameters of a unit is revealed: the moment of inertia of the unit, the drive moment, as well as the total moment of the drive forces and useful resistance forces reduced to the driven shaft of the variator.

In [4], the authors theoretically investigated multi-mass torsion systems ad joint with a large amount of computation.

Theoretical issues of free and forced multi-mass torsional vibrations were discussed in the paper in the presence of clearances in machine systems.

The correct choice of electric drive parameters is a necessary condition for high-performance and economical operation of a machine unit [5]. In dynamic synthesis of an electric drive, it was proposed to determine not only the moment of inertia of the fly mass, but also the gear ratio of the gear mechanism under condition to ensure the maximum engine efficiency and the possibility of starting the drive. In accordance with the proposed method, a dynamic analysis was carried out and the synthesis of a plunger pump was considered.

Yu.S. Korneev et al. in [6] have determined the acceleration time of the driven half-coupling along with the technological machine when studying a machine unit with a start-up clutch.

In [7], the authors proposed a systematic approach to modeling engine units. The absence of general agreement on the definition, structure and classification of subsystems of engine units has been established. The paper outlines the general principles of modeling machine units. The machine unit is treated as a complete integration of the electronic control subsystem, the electric drive subsystem and the mechanical working subsystem.

In [8], I.I. Wolfson has analyzed some unexplored factors that affect oscillations in machines drives with cyclic mechanisms when taking into account the characteristics of the electric motor. Engineering methods for calculating such systems is proposed, based on the use of transition matrices that are well adapted to computer procedures. A specific effect has been identified associated with a high equivalent “compliance” of the engine and a change in the reduced inertial and elastic-dissipative characteristics of the drive. The efficiency of the implementation of quasi-stationary conditions in the optimization of parameters is shown.

The monographs [9, 10] set forth modern methods for calculating machines vibrations, including mechanisms of cyclic action (lever, cam, step ones, etc.). The methods of schematization and correct mathematical description of oscillatory systems are given taking into account the variability of parameters and nonlinearities, in particular, the derivation of a system of differential equations for dynamic models of mechanisms including the elements with distributed parameters. To reduce the complexity of the calculation, a less idealized calculation scheme is proposed in...
which the corresponding element is displayed as a subsystem with distributed parameters.

In the second volume of N.S. Piskunov’s book [11], in the section “Equations of Mathematical Physics”, the equation of torsional vibrations of a homogeneous cylindrical rod in the form of the Laplace equation is given.

It was stated in [12, 13] that non-uniform rotation of the saw cylinder can worsen the ginning process and damage the fibers. Therefore, in [14], to find the law of change in the frequency and non-uniformity of rotor rotation of electric motor and saw cylinder, depending on the elastic-dissipative parameters of the clutch, the moment of inertia of the electric motor, the moment of inertia and resistance of the saw cylinder at various values using the Lagrange equation of the second kind, the equation movement of the machine unit of 156-saw gin cylinder was derived.

The value of multiplicity of the starting torque relative to the nominal one is 1.5–6. For the asynchronous electric motor that we used, it equals to 2 [15]. From this it follows that the maximum load on the electric motor comes at the time of start-up.

II. SUBSTANTIATION OF THE RELEVANCE OF THE PROBLEM

In this paper, we have studied the dynamics of electric motor start-up and torsional vibrations of the shaft of a gin saw cylinder with concentrated and distributed parameters, which is characterized by a sufficiently large length and has considerable flexibility.

To study dynamic characteristics, consider a system consisting of subsystems with concentrated and distributed parameters. A mathematical model of the first subsystem is built according to the data from [12–14], and of the second subsystem - according to the data from [8–11].

III. THE ESSENCE OF THE PROBLEM SOLUTION

Machine unit subsystem with concentrated parameters

As follows from the dynamic model of the saw cylinder (Fig. 1), the angular movement of the electric motor (D) is transmitted via the clutch to the long shaft of the saw cylinder (SC); its torsional vibrations can be very significant. In the adopted dynamic model of the saw cylinder, shown in Fig. 1, the following notations are used: \( \mathbf{3}_d\mathbf{3}_c \) - are the concentrated moments of inertia of the motor and the saw cylinder, \( \mathbf{kg}\mathbf{m}^2 \); \( \mathbf{3} \) - is the distributed moment of inertia of the shaft of the saw cylinder and the parts rigidly connected with it, \( \mathbf{kg}\mathbf{m}^2 \); \( c, s \) - are the coefficients of rigidity coefficients (N·m/rad) and dissipation (N·m·s/rad) (H·kg·m/s·rad) of the clutch; \( \varphi_d, \varphi_c(x) \) are the absolute coordinates of the corresponding sections, rad; \( Q(x) \) is the distributed generalized force applied to the shaft of the saw cylinder.

\[ \varphi_d \text{ and } \varphi_c(x) \text{ are taken as the generalized coordinates.} \]

The cross section \( x = 0 \), divides the dynamic model (Fig. 1) into subsystems with concentrated and distributed parameters. It is obvious that in the indicated section, two reactive moments \( M_c, M_s \) should be applied to the cut off parts, equal in magnitude and opposite in direction, i.e. \( M_s = M_c \) (Fig. 2). However, to avoid possible errors when choosing the sign of the reactive moment, it is advisable to be guided by the following rule: the reactive moment at the "output" of the element (in our case, to the right) is considered positive if its direction coincides with the chosen positive direction of the reference angle \( \varphi_c(x) \); for the reactive moment at the "input" of an element (in our case, to the left), the sign rule is the opposite.

**Fig. 1. Dynamic model of the saw cylinder.**

When deriving differential equations of the saw cylinder, the Lagrange equation of the second kind is used:

\[ \frac{d}{dt} \left( \frac{\partial T}{\partial \dot{\varphi}_i} \right) - \frac{\partial T}{\partial \varphi_i} + \frac{\partial \Pi}{\partial \dot{\varphi}_i} - \frac{\partial \Phi}{\partial \varphi_i} = Q(\varphi), \quad (1) \]

where \( T \) is the kinetic energy of the system; \( \Pi \) is the potential energy of the system; \( \Phi \) is the dissipative function of the system; \( \varphi_i \) is the generalized coordinate; \( \dot{\varphi}_i \) is the generalized velocity; \( Q(\varphi_i) \) is the generalized force.

The drive of the saw cylinder consists of a clutch, for which the following kinematic ratio is appropriate:

\[ i = \frac{\varphi_d}{\varphi_2} \quad (2) \]

Angular displacements of the rotating masses of the saw cylinder \( \dot{\varphi}_d, \dot{\varphi}_2 \) are taken as the generalized coordinates. Kinetic energy of the feeder is:

\[ T = \frac{1}{2} \left[ \mathbf{3}_d \dot{\varphi}_d^2 + \mathbf{3}_c \dot{\varphi}_c^2 \right] \quad (3) \]

Potential energy of the saw cylinder is a homogeneous quadratic form of the generalized coordinates and is written in the following form:

\[ \Pi = \frac{1}{2} \left[ c (\varphi_d - i \varphi_2)^2 \right] \quad (4) \]

Dissipative function of the system is expressed as

\[ \Phi = \frac{1}{2} \left[ \theta (\dot{\varphi}_d - i \dot{\varphi}_2)^2 \right] \quad (5) \]

**Fig. 2. Schematic representation of the acting moments of the moments acting on an elementary section of a shaft.**

**Fig. 1. Dynamic model of the saw cylinder.**
Define the terms of the Lagrange equations:

a) partial derivatives of displacements from potential energy
$$\frac{\partial P}{\partial \Phi_d} = c (\Phi_d - i \Phi_2); \quad \frac{\partial P}{\partial \Phi_2} = -c i (\Phi_d - i \Phi_2)$$  \hspace{1cm} (6)

b) partial derivatives of displacements from dissipative function
$$\frac{\partial \Phi}{\partial \Phi_d} = \Phi (\Phi_d - i \Phi_2); \quad \frac{\partial \Phi}{\partial \Phi_2} = -\Phi (\Phi_d - i \Phi_2)$$ \hspace{1cm} (7)

c) partial derivatives of velocities from generalized coordinates
$$\frac{\partial \mathcal{T}}{\partial \Phi_d} = \mathcal{I}_d \Phi_d, \quad \frac{\partial \mathcal{T}}{\partial \Phi_2} = \mathcal{I}_2 \Phi_2$$ \hspace{1cm} (8)

d) time derivatives
$$\frac{d}{dt} \left( \frac{\partial \mathcal{T}}{\partial \Phi_d} \right) = \mathcal{I}_d \Phi_d, \quad \frac{d}{dt} \left( \frac{\partial \mathcal{T}}{\partial \Phi_2} \right) = \mathcal{I}_2 \Phi_2$$ \hspace{1cm} (9)

i) generalized forces
$$Q_d(\Phi_d) = M_d, \quad Q_2(\Phi_2) = -\mathcal{M}.$$ \hspace{1cm} (10)

Substituting certain terms into the Lagrange equations (1), the differential equations of motion of machine unit of the feeder is obtained in the general form
$$\mathcal{I}_d \Phi_d = M_d - c (\Phi_d - i \Phi_2) - \Phi (\Phi_d - i \Phi_2), \quad \mathcal{I}_2 \Phi_2 = c i (\Phi_d - i \Phi_2) + \Phi (\Phi_d - i \Phi_2) - \mathcal{M}.$$ \hspace{1cm} (11)

With i=1 and disturbing moment $M=M_\psi + M_0 \sin(\omega t + \phi_0)$ acting on the system, equation (11) is written in the form
$$\mathcal{I}_d \Phi_d = M_d - c (\Phi_d - \Phi_2) - \Phi (\Phi_d - \Phi_2), \quad \mathcal{I}_2 \Phi_2 = c (\Phi_d - \Phi_2) + \Phi (\Phi_d - \Phi_2) - M_0 \sin(\omega t + \phi_0).$$ \hspace{1cm} (11')

When examining machine units, it is important to choose the right engine characteristics. Currently, static, linearized dynamic, refined dynamic and mechanical-dynamic characteristics of asynchronous electric motors are used. One of the most promising areas is the approximate consideration of electromotive transients in the engine and their mathematical description by a system of differential equations.

Therefore, when studying the dynamic parameters of the feeder, we have used the dynamic mechanical characteristic of an asynchronous electric motor. This characteristic takes into account electromotive transient processes of the start-up and the steady-state motion described by a system of differential equations containing the vector components of the stator and rotor inter-linkages at a synchronous speed of rotation of the coordinate axes, and has the following form [12–14]:

$$M_\psi = \frac{3 P K_\omega}{2 \pi x_2} (\psi_{x1}, \psi_{r1} - \psi_{x1}, \psi_{r2}),$$
$$\psi_{x1} = U_m \cos \gamma - a \psi_{x1} + a \psi_{r1}, \quad \psi_{x2} = a \psi_{x1} + a \psi_{r1},$$
$$\psi_{r1} = U_m \sin \gamma - a \psi_{x1} + a \psi_{r1}, \quad \psi_{r2} = a \psi_{x1} + a \psi_{r1},$$

where $\psi_{x1}, \psi_{r1}$ are the components of the generalized vector of stator inter-linkage along the $x$ and $y$ axes, rotating at a synchronous speed; $\psi_{x2}, \psi_{r2}$ are the components of the generalized vector of rotor inter-linkage along the $x$ and $y$ axes; $K_\omega = x_2/\omega_0 = 0.9558824, \quad K_\omega = x_2/\omega_0 = 0.9523809,$ are the coefficients equal to the ratios of the total reactance of the mutual induction $x_p = 3.957668 \Omega$ to the total reactance of the stator $x_s$ and rotor $x_r$, respectively; $\alpha_0, \alpha_0$ are the coefficients equal to the ratios of the total active resistance of the stator phase $r_1 = 0.042621 \Omega$ and the rotor $r_2 = 0.031996 \Omega$, respectively, to the total reactance of the stator $x_s$ and rotor $x_r, (\alpha_0 = r_1/x_s = 0.01029412; \alpha_0 = r_2/x_r = 0.0076923)$.

Next, determine the passport parameters and coefficients of the asynchronous motor 4A280M8UZ [12, 13, 15]: $N_p = 75 \text{ kW}$ is the engine power rating; $n_0 = 735 \text{ rpm}$ - nominal speed of the engine rotor; $M_{DK} = 1948.8 \text{ H} \cdot \text{m}$ - critical moment on the rotor shaft of the engine; $M_{ON} = M_{DK} / 2 = 974.4 \text{ H} \cdot \text{m}$ - nominal torque on the rotor shaft of the engine; $f_c = 50 \text{ Hz}$ - network frequency; $U_m = 220 \text{ V}$ - voltage phase rating; $\eta = 0.925$ - engine efficiency; $\cos \varphi = 0.85$ is the nominal power factor of the engine; $\alpha_{00} = 78.54 \text{ s}^{-1}$ - synchronous frequency of engine rotor rotation; $\beta_{00} = 76.97 \text{ s}^{-1}$ is the nominal rotation speed of the engine rotor; $S_0 = 0.02$

- nominal value of engine slip; $S_{x0} = 0.07464$ - critical value of engine slip; $P = 4$ - the number of pairs of poles; $I_{D,0} = 144.53 \text{ A}$ - rated phase current; $J_0 = 4.20 \text{ kg m}^2$ is the total moment of inertia of the engine rotor.

To study the machine unit of the gin saw cylinder SDP-156, the moment of inertia of the saw cylinder was experimentally determined by the acceleration method $J_2 = 1.244 \text{ kg m}^2$, a technological load acting on the rotating shaft of the saw cylinder $M=M_{ip}+M_0 \sin(\omega t + \phi_0)$ (here $M_{ip} = 843.72 \text{ N m}$; $M_0 = 78.78 \text{ N m}$; $\omega_0 = \pi 735 / 30 \text{ rad/s}$; $t$ is time; $\phi_0$ is the initial phase) is determined; the rigidity $c = 23065.2 \text{ N m} / \text{ rad}$ and the dissipation coefficient

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\( \sigma = 128.5346 \) N/rad of the clutch are determined by calculations [12, 13].

To study on a computer the dynamics of the electric motor start-up and torsional vibrations of the saw cylinder shaft with concentrated parameters, the equations of motion of the saw cylinder machine unit (11’) with the characteristic of the drive motor (12) are solved. The numerical Runge Kutta method was used for the second-order differential equation \( S = \frac{d^2 \varphi d\tau^2}{dt^2} = F(t, \varphi, \varphi’), \) with an error of \( \Delta t^\alpha \).

The implementation of the equations of motion of the machine unit of the saw cylinder (11’) with the drive motor performance (12) made it possible to establish the pattern of change in the angular acceleration of the saw cylinder as a function of \( t \) (Fig. 3).

![Fig. 3. Change in the angular acceleration of the saw cylinder as a function of time.](image)

The results of the analysis of Fig. 3 show that the critical driving moment of the electric motor is \( 40,000 \) Nm, and the transient takes 3 seconds; the maximum value of the angular acceleration of the saw cylinder reaches \( 8739.828 \) rad/s² at \( t = 1.844 \) s, and the pattern of change in the angular acceleration can be expressed as a function:

\[
\frac{d^2 \varphi}{dt^2} = \ddot{\varphi}(t) = 200 e^{0.105t} \cos 75t. \quad (12')
\]

**IV. RESULTS AND DISCUSSION**

Now proceed to the subsystem with distributed parameters [8–11]. We select an elementary section with a length \( dx \) on the shaft of the saw cylinder 2 (Fig. 1, 2); its moment of inertia is equal to \( I = \frac{d \mathcal{A}}{dx} dx \). In the general case, if \( \mathcal{A} \) is taken as a variable reduced moment of inertia, which is also uniformly distributed along the \( x \) axis, it may turn out that \( I = \rho(x, t) \); at \( \mathcal{A} = const \), we have \( \rho = \rho(x) \); at a uniform distribution of masses \( \rho = \mathcal{A}/l = const \), where \( l \) is the length of the shaft (Fig. 4).

![Fig. 4. Change in the angular acceleration of the saw cylinder as a function of time along the length of the shaft.](image)

For the selected element, the theorem on the change in the kinetic moment is used, according to which the derivative of the kinetic moment in time is equal to the sum of the applied external moments:

\[
\frac{\partial}{\partial t} \left( \rho \frac{\partial \varphi}{\partial x} \right) dx = -M + (M + dM) + Q dx. \quad (13)
\]

where \( dM = \frac{\partial M}{\partial x} dx \) – is the increment of the moment \( M \) in section \( dx \).

As is known, the elementary angular deformation \( d\varphi \) can be expressed as follows:

\[
d\varphi = -\frac{M}{G I(x)} \, dx, \quad (14)
\]

where \( G = 8 \cdot 10^{10} \) N/m² – is the shear modulus for steel shaft; \( I(x) \) is the polar moment of inertia of the shaft, which in the general case can also vary along the \( x \) axis. From dependence (14) we find the moment

\[
M = G I(x) \frac{d\varphi}{dx}, \quad (14')
\]

hence its differential. After substituting the last dependence in equation (13) and reducing it by \( dx \), we obtain

\[
\frac{\partial}{\partial t} \left( \rho \frac{\partial \varphi}{\partial x} \right) - G \frac{\partial}{\partial x} \left( I(x) \frac{\partial \varphi}{\partial x} \right) = Q(x). \quad (15)
\]

If \( \rho = const \) and \( I(x) = const \) = 9.817 \cdot 10^{-6} \) m⁴, equation (15) has the form

\[
\rho \frac{\partial^2 \varphi}{\partial t^2} - G I \frac{\partial^2 \varphi}{\partial x^2} = Q(x) \quad (15')
\]

the generalized force applied to the shaft of the saw cylinder, distributed over the length \( x \in [0; l] \), has the form

\[
Q(x) = \frac{M_{ep} + M_0 \cos(\pi \omega_2 t + \varphi_{02})}{R \Delta \varphi_1 l} x. \quad (16)
\]

Where \( M_{ep} = 843.72 \) Hm; \( M_0 = 78.78 \) Hm; \( \omega_2 = \pi \cdot 735/30 \) rad/s; \( t – time; \varphi_{02} = 0 – initial phase; l = 2.322 \) m – shaft length...
of saw cylinder; \( R = 0.16 \) m is the radius of the saw blades; 
\( \Delta \theta_R = \pi / 3 \) – is the arc sector of the saw blade. Then equation (15) takes the form
\[
\frac{\partial^2 \phi_2}{\partial t^2} = \frac{1}{G1} \left( 200 \rho e^{200t} \cos 75t - \frac{M_0 + M_e \cos \pi \omega_1 t}{R \Delta \phi_1} \right),
\]
(17)

at \( \frac{\partial^2 \phi_1}{\partial t^2} = \ddot{\phi}_1 \), equation (17) \( \ddot{z} \) is written as
\[
\dot{\phi}_2 = \frac{1}{G1} \left( 200 \rho e^{200t} \cos 75t - \frac{M_0 + M_e \cos \pi \omega_1 t}{R \Delta \phi_1} \right),
\]
(18)

\[
\phi_2 = \frac{1}{G1} \left( 200 \rho e^{200t} \cos 75t \right) \frac{x}{2} \frac{M_0 + M_e \cos \pi \omega_1 t}{2 R \Delta \phi_1} + \frac{x}{2} + C_1 + C_2 t,
\]
(19)

\[
\phi_{2x} = \frac{1}{G1} \left( 200 \rho e^{200t} \cos 75t \right) \frac{x}{2} \frac{M_0 + M_e \cos \pi \omega_1 t}{6 R \Delta \phi_1} + \frac{x}{2} + C_1 + C_2 t,
\]
(20)

If \( x = 0 \), \( \frac{\partial \phi_2}{\partial x} = \phi_2, \phi_2 = 0 \), \( \phi_{2x} = 0 \), then \( C_2 = 0 \) and \( C_2 = 0 \) equations (19) and (20) have the form
\[
\phi_2 = \frac{1}{G1} \left( 200 \rho e^{200t} \cos 75t \right) \frac{x}{2} \frac{M_0 + M_e \cos \pi \omega_1 t}{2 R \Delta \phi_1},
\]
(21)

\[
\phi_{2x} = \frac{1}{G1} \left( 200 \rho e^{200t} \cos 75t \right) \frac{x}{2} \frac{M_0 + M_e \cos \pi \omega_1 t}{6 R \Delta \phi_1},
\]
(22)

The solution of equations (21) - (22) allowed us to study the dynamics of torsional vibrations of the shaft of a gin saw cylinder with distributed parameters.

The graphs of changes in the angle of relative rotation of the saw cylinder shaft (Fig. 5) and the angular rotation of the shaft under torsion (Fig. 6) depending on the shaft length \( l \) are plotted.

**Fig. 5. Change in the angle of relative rotation of the shaft of the saw cylinder as a function of time for different shaft lengths.**

The plotted graphs (Figs. 5 and 6) made it possible to determine the maximum values of the angle of relative rotation and the angle of rotation of the saw cylinder shaft under torsion, respectively, amounting to 1.89° / m, or 4.39°.

**V. CONCLUSION**

In general, the study of machines in the form of machine units made it possible to state the dynamics of electric motor start-up and torsional vibrations of the shaft of the saw cylinder gin with distributed parameters. The subsystems were used with concentrated parameters (Lagrange equations of the second kind) and distributed parameters (Laplace equation in cylindrical coordinates).

The study of machine unit of the saw cylinder with concentrated parameters showed that the critical driving moment of the electric motor is 40,000 N·m, the transition process takes 3 seconds, and the maximum angular acceleration of the saw cylinder reaches 9000 rad/s² at \( t = 1.8 \) s.

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