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Effects of Fuzziness on an MHD Fluid Flow over an Exponentially Accelerated Inclined Plate

Palash Dutta, G. C. Hazarika, Joydeep Borah

Abstract— ***In most of the time, the real life phenomena are uncertain. Due to the uncertain nature, consideration of some exact values or definite conditions may cause errors in the solution process. The fluid flow problems are also uncertain due to the presence of various imprecise parameters, variables and conditions. Here, an MHD fluid flow over an exponentially accelerated inclined plate is considered in fuzzy environment. Fuzzy set theory (FST) may help to overcome the uncertain nature of the fluid flow problems. The governing equations and the boundary conditions of the considered problem are fuzzified using the extension principle. The fuzzified governing equations along with the fuzzified boundary conditions are solved using the finite difference method (FDM) by developing suitable computer programming code in Python. The values of the different parameters and variables (initial values) are taken as triangular fuzzy number (TFN).*** $\alpha-$***cut technique is used to find the results. The effects of the various involved parameters and*** α ***on the velocity, temperature and concentration profile are presented graphically and discussed.***

Index Terms— $\alpha-$***cut, exponentially accelerated inclined plate, extension principle, fuzziness, MHD flow.***

I. INTRODUCTION

Study of hydromagnetic free convection flow with coupled heat and mass transfer has been done in wide range due to its various utilizations in engineering and mechanical processes. It is a nature of fluid flow, in which the motions of the fluid are not generated by any foreign source, but by some parts of the fluid being heavier than other parts. Several analysts interrogated MHD natural or free convection flow in porous as well as non-porous medium ([3], [9], [20]).

Several researchers have done some notable works related to MHD free convection flow over inclined plate in existence of thermal radiation, heat source/sink and chemical reaction. Thermal radiation has a lot of extensive applications in many industrial processes. On the other hand, heat source/sink have strong domination on heat transfer attributes of several real life problems of practical interest. Coupled heat and mass transfer flow with chemical reaction is also momentous in many engineering and industrial processes. Therefore, it has achieved considerable attention of researchers in the last few years ([1], [16], [19]).

In all of the above investigations, the viscosity and the thermal conductivity of the fluid have been considered as constant. But, these two fluid properties are not constant in reality, they vary with temperature ([13], [11]). Some researchers have studied the fluid flow problems considering the viscosity and thermal conductivity as variable ([6], [7], [12]). All the fluid flow problems cited above as well as the other fluid flow problems also are uncertain and imprecise.

More generally, all the real life phenomena are uncertain. So, solutions of those problems by taking the parameters and conditions as some definite values may contain errors. To overcome these errors, FST may help. With the help of FST, we may find a zone in which the solutions of the problems lie. Fuzzy differential equation (FDE) is the bridge to the fuzzy initial value problem (FIVP)s. Kaleva[10] is the pioneer in the field of FDE. After his contribution, another several researchers have studied about FDE ([8], [14], [17]). Seikkala[15] developed the idea of FIVP. Then, the researchers have started to do works in the field of FIVP and FBVP ([2], [5], [18]). They have opened a new wonderful path towards the present research field.

As the fluid flow problems are uncertain and imprecise, application of FST there may be a better option in finding the solutions of those problems. In all of the above fluid flow problems, only crisp values are considered. Since the boundary conditions and the various parameters involved in a particular problem are not exact, so the crisp solutions may not be correct. Keeping the conditions of uncertainty in mind, in this paper, we are considering a BVP of fluid mechanics in fuzzy environment. The values of the parameters and the boundary conditions are considered as TFN. Concept of $\boldsymbol{\alpha}-$cut technique is applied to attain the intervals containing the solutions. Also, the viscosity and the thermal conductivity of the fluid are considered as variable. The governing equations along with the boundary conditions are solved by FDM by developing suitable computer programming code in Python.

II. MATHEMATICAL FORMULATION

An MHD unsteady fluid flow over an inclined plate in a fluid saturated porous medium with combined heat and mass transfer is considered. The fluid considered here is viscous- incompressible, heat absorbing, optically thick radiating and chemically reactive. A 2-dimensional cartesian co-ordinate system is considered with the following assumptions:

- The $\boldsymbol{x'}-$axis is taken along the inclined plate that makes an angle ω with the vertical direction and the $\boldsymbol{y'}-$axis is taken in the perpendicular direction to the plate.
- A uniform magnetic field with strength $\boldsymbol{B_0}$ is applied in the $\boldsymbol{y'}-$direction.

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* Correspondence Author
Palash Dutta, Department of Mathematics, Dibrugarh University, Dibrugarh, Assam, India. E-Mail: palashdtt@gmail.com
G. C. Hazarika, Department of Mathematics, Dibrugarh University, Dibrugarh, Assam, India, E-Mail: gchazarika@gmail.com
Joydeep Borah, Department of Mathematics, Dibrugarh University, Dibrugarh, Assam, India, E-Mail: joydeepborah8@rediffmail.com

- Initially, both the fluid and the plate are rest and are kept at uniform temperature T'_∞ and uniform concentration C'_∞.
- As time starts, the plate starts to accelerate exponentially in the x'–direction with a velocity $U_0 e^{a't'}$ where U_0 and a' are uniform velocity and plate acceleration coefficient respectively and the temperature and the concentration are kept at uniform level T'_w and C'_w respectively.
- At $0 < t' \le t_0$, the temperature and the concentration of the plate are changed to $T'_\infty + \frac{(T'_w - T'_\infty)t'}{t_0}$ and $C'_\infty + \frac{(C'_w - C'_\infty)t'}{t_0}$ respectively.
- A uniform first order homogeneous chemical reaction Ch exists between the diffusing species and the fluid.
- All the fluid properties except the viscosity and the thermal conductivity are considered as constant.

Following the above assumptions we have the flow governing equations as follows:

$$\frac{\partial u'}{\partial t'} = \nu \frac{\partial^2 u'}{\partial y'^2} + \frac{1}{\rho}\frac{\partial \mu}{\partial y'}\frac{\partial u'}{\partial y'} - \frac{\sigma B_0^2 u'}{\rho} - \frac{\nu}{K_p'}u' + g\beta_T(T' - T'_\infty)\cos\omega + g\beta_C(C' - C'_\infty)\cos\omega$$

(1)

$$0 = -\frac{1}{\rho}\frac{\partial p}{\partial y} - g\beta_T(T' - T'_\infty)\sin\omega - g\beta_C(C' - C'_\infty)\sin\omega$$

(2)

$$\rho C_p \frac{\partial T'}{\partial t'} = \lambda \frac{\partial^2 T'}{\partial y'^2} + \frac{\partial \lambda}{\partial y'}\frac{\partial T'}{\partial y'} - \frac{\partial q_r}{\partial y'} - Q_0(T' - T'_\infty)$$

(3)

$$\frac{\partial C'}{\partial t'} = D\frac{\partial^2 C'}{\partial y'^2} + \frac{\partial D}{\partial y'}\frac{\partial C'}{\partial y'} - Ch(C' - C'_\infty)$$

(4)

Boundary conditions are-

$$\left.\begin{array}{l} y' \ge 0, t' \le 0: u' = 0, T' = T'_\infty, C' = C'_\infty \\ y' = 0: \begin{cases} \text{for } t' > 0: u' = U_0 e^{a't'} \\ \text{for } 0 < t' \le t_0: T' = T'_\infty + \frac{(T'_w - T'_\infty)t'}{t_0}, \\ \qquad C' = C'_\infty + \frac{(C'_w - C'_\infty)t'}{t_0} \\ \text{for } t' > t_0: T' = T'_w, C' = C'_w \end{cases} \end{array}\right\}$$

(5)

By Rosseland diffusion model, we have

$$q_r = -\frac{4\bar{\sigma}}{3\bar{a}}\frac{\partial T'^4}{\partial y'}$$

(6)

Expanding T'^4 in Taylor's series about T'_∞ we find

$$T'^4 \cong 4T'^3_\infty T' - 3T'^3_\infty$$

(7)

Using Eq.(6) and (7) in Eq.(3) we get the equation of conservation of energy as

$$\frac{\partial T'}{\partial t'} = \frac{\lambda}{\rho C_p}\frac{\partial^2 T'}{\partial y'^2} + \frac{1}{\rho C_p}\frac{\partial \lambda}{\partial y'}\frac{\partial T'}{\partial y'} + \frac{16\bar{\sigma}T'^3_\infty}{3\bar{a}\rho C_p}\frac{\partial^2 T'}{\partial y'^2} - \frac{Q_0}{\rho C_p}(T' - T'_\infty)$$

(8)

Now, the following non-dimensional parameters are introduced to make the equations along with the boundary conditions dimensionless.

$$y = \frac{y'}{U_0 t_0}, u = \frac{u'}{U_0}, t = \frac{t'}{t_0}, a = \frac{a'\nu_\infty}{U_0^2}, \theta = \frac{T' - T'_\infty}{T'_w - T'_\infty}, \phi = \frac{C' - C'_\infty}{C'_w - C'_\infty} \quad (9)$$

Also, following Lai and Kulacki[13] and Khound and Hazarika[11], we have-

$$\frac{1}{\mu} = \frac{1}{\mu_\infty}[1 + \alpha(T' - T'_\infty)]$$

(10)

$$\frac{1}{\lambda} = \frac{1}{\lambda_\infty}[1 + \beta(T' - T'_\infty)]$$

(11)

Let us define two parameters called viscosity parameter and thermal conductivity parameter respectively as $\theta_r = \frac{T'_r - T'_\infty}{T'_w - T'_\infty}$ and $\theta_c = \frac{T'_c - T'_\infty}{T'_w - T'_\infty}$.

After using the above parameters, Eqs.(1), (8) and (4) respectively transformed into the following forms.

$$\frac{\partial u}{\partial t} = -\frac{\theta_r}{\theta - \theta_r}\frac{\partial^2 u}{\partial y^2} + \frac{\theta_r}{(\theta - \theta_r)^2}\frac{\partial \theta}{\partial y}\frac{\partial u}{\partial y} - Mu + \frac{\theta_r}{\theta - \theta_r}\frac{u}{K_p} + Gr\theta\cos\omega + Gm\phi\cos\omega$$

(12)

$$\frac{\partial \theta}{\partial t} = \frac{1}{Pr}\left[-\frac{\theta_c}{\theta - \theta_c} + Kr\right]\frac{\partial^2 \theta}{\partial y^2} + \frac{\theta_c}{\Pr(\theta - \theta_c)^2}\left(\frac{\partial \theta}{\partial y}\right)^2 - S\theta$$

(13)

$$\frac{\partial \phi}{\partial t} = -\frac{\theta_r}{Sc(\theta - \theta_r)}\frac{\partial^2 \phi}{\partial y^2} + \frac{\theta_r}{Sc(\theta - \theta_r)^2}\frac{\partial \theta}{\partial y}\frac{\partial \phi}{\partial y} - \xi\phi$$

(14)

Corresponding boundary conditions are-

$$\left.\begin{array}{l} y \ge 0, t \le 0: u = 0, \theta = 0, \phi = 0 \\ y = 0: \begin{cases} \text{for } t > 0: u = e^{at} \\ \text{for } 0 < t \le 1: \theta = t, \phi = t \\ \text{for } t > 1: \theta = 1, \phi = 1 \end{cases} \\ y \to \infty, t > 0: u \to 0, \theta \to 0, \phi \to 0 \end{array}\right\}$$

(15)

III. FUZZIFICATION OF THE PROBLEM

Fluid flow problems are uncertain due to the presence of various uncertain and imprecise parameters and conditions. Application of FST may be a better option in finding the solutions where uncertainty arises. When a problem is considered under fuzzy environment, the governing equations and the conditions must be fuzzified. Zadeh gave a principle with the help of which a function whose inputs and outputs are points can be extended into function whose inputs and outputs are sets.

If there is a classical function f and a classical set A as input, then the united extension of A is the union of the images of all elements of A. In cases of FST, if an element x of a set A has membership degree $\mu_A(x)$, then by means of an injective function, the image of x, say $y = f(x)$ has also the same membership degree, i.e., $\mu_{f(A)}(y) = \mu_A(x)$. If more than one element have same image, then the membership degree of that image is given by the supremum of the membership degrees of all possible pre-images. This process of extending a function is known as extension principle.

Let U and V be two universes and $f: U \to V$ be a classical function. For each $A \in \mathcal{F}(U)$ we define the extension of f as $\tilde{f}(A) \in \mathcal{F}(V)$ such that

$$\mu_{\tilde{f}(A)}(y) = \begin{cases} \sup_{s \in f^{-1}(y)} \mu_A(s), & if\ f^{-1}(y) \ne \phi \\ 0, & if\ f^{-1}(y) = \phi \end{cases}$$

Now, using the Zadeh's extension principle, we have the fuzzified form of the functions involved in the governing equations as follows.

1. $u(y)$

$$u(y) = \frac{u'}{U_0}$$

$$\Rightarrow z = \frac{u'}{U_0}$$

$$y=\frac{y'}{U_0t_0}\Rightarrow U_0=\frac{y'}{yt_0}$$
$$\therefore z=\frac{u'yt_o}{y'}$$
$$\Rightarrow y=\frac{zy'}{u't_0}$$

$$\Rightarrow u^{-1}(z)=\frac{zy'}{u't_0}$$

Now,

$$\mu_{\tilde{u}(Y)}(z)=\sup_y \mu_Y(y)$$
$$\Rightarrow \mu_{\tilde{u}(Y)}\left[\frac{u'yt_0}{y'}\right]=\mu_Y(y)$$
$$\Rightarrow \tilde{u}(Y)=\frac{u'Yt_0}{Y'}$$

2. $\frac{\partial u}{\partial t}$

$$\frac{\partial u}{\partial t}=\frac{t_0}{U_0}\frac{\partial u'}{\partial t'}$$
$$\Rightarrow f(t)=\frac{t_0}{U_0}\frac{\partial u'}{\partial t'}$$
$$\Rightarrow z=\frac{t_0}{U_0}\frac{\partial u'}{\partial t'}$$
$$t=\frac{t'}{t_0}$$
$$\Rightarrow f^{-1}(z)=\frac{t'}{t_0}$$

Now,

$$\mu_{\tilde{f}(T)}(z)=\sup_t \mu_T(t)$$
$$\Rightarrow \mu_{\tilde{f}(T)}\left[\frac{t_0}{U_0}\frac{\partial u'}{\partial t'}\right]=\mu_T(t)$$
$$\Rightarrow \tilde{f}(T)=\frac{t_0}{U_0}\frac{\partial u'}{\partial T'}$$
$$\Rightarrow \frac{\partial\tilde{u}}{\partial T}=\frac{t_0}{U_0}\frac{\partial u'}{\partial T'}$$

3. $\frac{\partial^2 u}{\partial y^2}$

$$\frac{\partial^2 u}{\partial y^2}=t_0^2\frac{\partial^2 u'}{\partial y'^2}$$
$$\Rightarrow f(y)=t_0^2\frac{\partial^2 u'}{\partial y'^2}$$
$$\Rightarrow z=t_0^2\frac{\partial^2 u'}{\partial y'^2}$$

Now,

$$\mu_{\tilde{f}(Y)}(z)=\sup_y \mu_Y(y)$$
$$\Rightarrow \mu_{\tilde{f}(Y)}\left[t_0^2\frac{\partial^2 u'}{\partial y'^2}\right]=\mu_Y(y)$$
$$\Rightarrow \tilde{f}(Y)=t_0^2\frac{\partial^2 u'}{\partial Y'^2}$$
$$\Rightarrow \frac{\partial^2\tilde{u}}{\partial Y^2}=t_0^2\frac{\partial^2 u'}{\partial Y'^2}$$

Similarly, the other functions which are involved in the equations and the boundary conditions can be extended into fuzzy forms.

Also, the fuzzy forms of dimensionless physical quantities can be preserved as the non-fuzzy forms. A further condition is the numerical equality of all the corresponding non-dimensional variables figuring in both equations [4].

Hence, the crisp governing equations and the crisp boundary conditions are fuzzified and obtained as-

$$\frac{\partial\tilde{u}}{\partial t}=-\frac{\tilde{\theta_r}}{\tilde{\theta}-\tilde{\theta_r}}\frac{\partial^2\tilde{u}}{\partial y^2}+\frac{\tilde{\theta_r}}{(\tilde{\theta}-\tilde{\theta_r})^2}\frac{\partial\tilde{\theta}}{\partial y}\frac{\partial\tilde{u}}{\partial y}-\tilde{M}\tilde{u}+\frac{\tilde{\theta_r}}{\tilde{\theta}-\tilde{\theta_r}}\frac{\tilde{u}}{\tilde{K_p}}+\tilde{Gr}\tilde{\theta}\cos\omega+\tilde{Gm}\tilde{\phi}\cos\omega$$

(16)

$$\frac{\partial\tilde{\theta}}{\partial t}=\frac{1}{\tilde{Pr}}\left[-\frac{\tilde{\theta_c}}{\tilde{\theta}-\tilde{\theta_c}}+\tilde{Kr}\right]\frac{\partial^2\tilde{\theta}}{\partial y^2}+\frac{\tilde{\theta_c}}{\tilde{Pr}(\tilde{\theta}-\tilde{\theta_c})^2}\left(\frac{\partial\tilde{\theta}}{\partial y}\right)^2-\tilde{S}\tilde{\theta}$$

(17)

$$\frac{\partial\tilde{\phi}}{\partial t}=-\frac{\tilde{\theta_r}}{\tilde{Sc}(\tilde{\theta}-\tilde{\theta_r})}\frac{\partial^2\tilde{\phi}}{\partial y^2}+\frac{\tilde{\theta_r}}{\tilde{Sc}(\tilde{\theta}-\tilde{\theta_r})^2}\frac{\partial\tilde{\theta}}{\partial y}\frac{\partial\tilde{\phi}}{\partial y}-\tilde{\xi}\tilde{\phi}$$

(18)

Corresponding boundary conditions are-

$$\left.\begin{array}{l} y\ge 0, t\le 0: \tilde{u}=0, \tilde{\theta}=0, \tilde{\phi}=0 \\ y=0: \begin{cases} for\ t>0: \tilde{u}=e^{\tilde{a}t} \\ for\ 0<t\le 1: \tilde{\theta}=t, \tilde{\phi}=t \\ for\ t>1: \tilde{\theta}=1, \tilde{\phi}=1 \end{cases} \\ y\to\infty, t>0: \tilde{u}\to 0, \tilde{\theta}\to 0, \tilde{\phi}\to 0 \end{array}\right\}$$

(19)

IV. METHOD OF SOLUTION

Here, to solve the fuzzified governing equations along with the fuzzified boundary conditions, FDM can be used. By FDM, we have-

$$\frac{\partial u}{\partial t}=\frac{u(i+1,j)-u(i,j)}{\Delta t}$$
$$\frac{\partial u}{\partial y}=\frac{u(i,j+1)-u(i,j)}{\Delta y}$$
$$\frac{\partial^2 u}{\partial y^2}=\frac{u(i,j+1)-2u(i,j)+u(i,j-1)}{\Delta y^2}$$

By dropping the ˜ sign from Eq.(16)-Eq.(19) and applying the finite difference scheme there, the following expressions are obtained.

$$u(i,j)=\frac{1}{-\frac{1}{\Delta t}+\frac{2}{\Delta y^2}+M+\frac{1}{K_p}}\left[-\frac{u(i+1,j)}{\Delta t}+\frac{u(i+1,j)+u(i-1,j)}{\Delta y^2}+Gr\theta i,j\cos\omega\quad +Gm\phi\cos\omega\right]$$

(20)

$$\theta(i,j)=\frac{1}{-\frac{1}{\Delta t}+\frac{2(Kr+1)}{Pr\cdot\Delta y^2}+S}\left[-\frac{\theta(i+1,j)}{\Delta t}+\frac{Kr+1}{Pr}\frac{\theta(i+1,j)+\theta(i-1,j)}{\Delta y^2}\right]$$

(21)

$$\phi(i,j)=\frac{1}{-\frac{1}{\Delta t}+\frac{2}{Sc\cdot\Delta y^2}+\xi}\left[-\frac{\phi(i+1,j)}{\Delta t}+\frac{1}{Sc}\frac{\phi(i+1,j)+\phi(i-1,j)}{\Delta y^2}\right]$$

(22)

Corresponding boundary conditions are-

$$\left.\begin{array}{l} y\ge 0, t\le 0: u(0,j)=0, \theta(0,j)=0, \phi(0,j)=0 \\ y=0: \begin{cases} for\ t>0: u(i,0)=e^{at} \\ for\ 0<t\le 1: \theta(i,0)=t, \phi(i,0)=t \\ for\ t>1: \theta(i,0)=1, \phi(i,0)=1 \end{cases} \\ y\to\infty, t>0: u(i,N)\to 0, \theta(i,N)\to 0, \phi(i,N)\to 0 \end{array}\right\}$$

(23)

$\forall i,j$ and N is the total number of sub-divisions in the interval $[0,\infty)$.

In case of $\alpha-$cut set theory, the results with maximum possibilities, i.e., the grade of membership 1 is given by $\alpha=1$. So here, we first find the solutions for various values of α between 0 and 1.

Then, the distributions of the velocity, temperature and species concentration against the various involved parameters are investigated by taking $\boldsymbol{\alpha = 1}$ and they are shown graphically.

V. RESULTS AND DISCUSSION

The system of the fuzzified algebraic Eqs.(20), (21) and (22) along with Eq.(23) is solved numerically using an iterative scheme based on Gauss Seidal iterative method by developing code in Python. TFNs have been considered throughout.

Here, the parameter values are taken as $\boldsymbol{\theta_r} = [\mathbf{-11, -10, -9}]$, $\boldsymbol{\theta_c} = [\mathbf{-11, -10, -9}]$, $\boldsymbol{M} = [\mathbf{0.2, 0.25, 0.3}]$, $\boldsymbol{K_p} = [\mathbf{0.1, 0.15, 0.2}]$, $\boldsymbol{Gr} = [\mathbf{4, 5, 6}]$, $\boldsymbol{Gm} = [\mathbf{0.4, 0.5, 0.6}]$, $\boldsymbol{Pr} = [\mathbf{0.6, 0.71, 0.8}]$, $\boldsymbol{Kr} = [\mathbf{0.04, 0.05, 0.06}]$, $\boldsymbol{S} = [\mathbf{0.9, 1.0, 1.1}]$, $\boldsymbol{Sc} = [\mathbf{0.20, 0.22, 0.24}]$, $\boldsymbol{\xi} = [\mathbf{0.09, 1, 1.1}]$ and $\boldsymbol{\omega} = [\mathbf{30°, 30°, 30°}]$, unless otherwise stated. The velocity, temperature and concentration variations are observed in time $\boldsymbol{t} = \mathbf{0.4, 0.8}$ and $\mathbf{1.2}$.

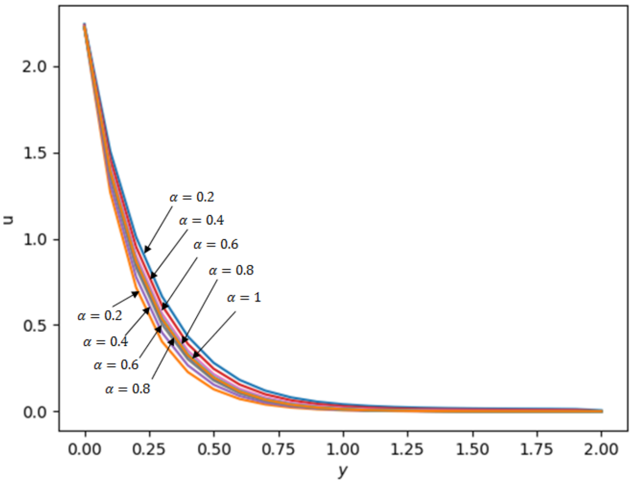

Fig. 1. Effect of $\boldsymbol{\alpha}$ on $\boldsymbol{u}$

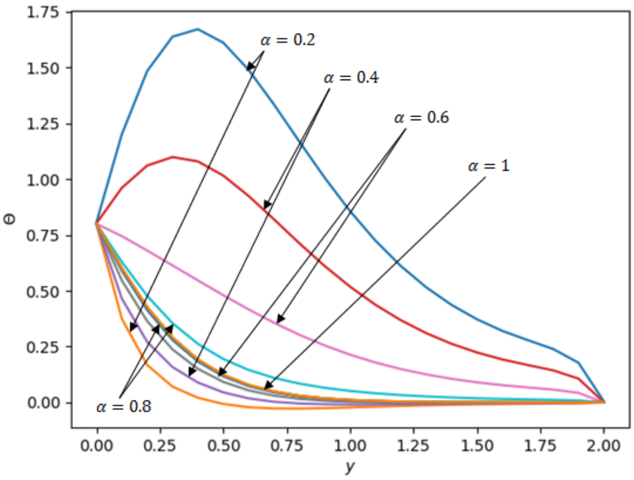

Fig. 2. Effect of α on θ

Fig.1 - Fig.3 depict the variation of velocity(u), temperature(θ) and concentration($\boldsymbol{\phi}$) profile due to the change of the value of $\boldsymbol{\alpha}$, i.e. for $\boldsymbol{\alpha} = \mathbf{0.2, 0.4, 0.6, 0.8}$ and 1. In all of these three figures, it is seen that for any value of $\boldsymbol{\alpha}$ except 1, we have two graphs which indicates the interval between in which the actual graphs will live. This actual graph will have membership grade 1 (i.e. $\boldsymbol{\alpha} = \mathbf{1}$). It is also true from the $\boldsymbol{\alpha}$−cut set theory. In the figures also, for $\boldsymbol{\alpha} = \mathbf{1}$, there is only one graph, which is of membership grade 1. These are displayed in Fig.4 to Fig.13.

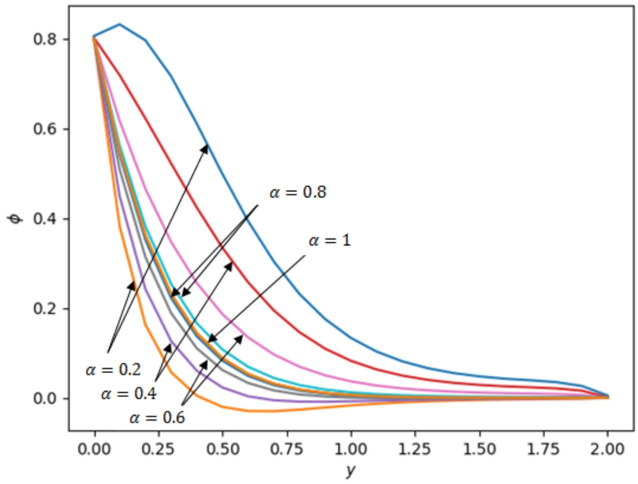

Fig. 3. Effect of α on ϕ

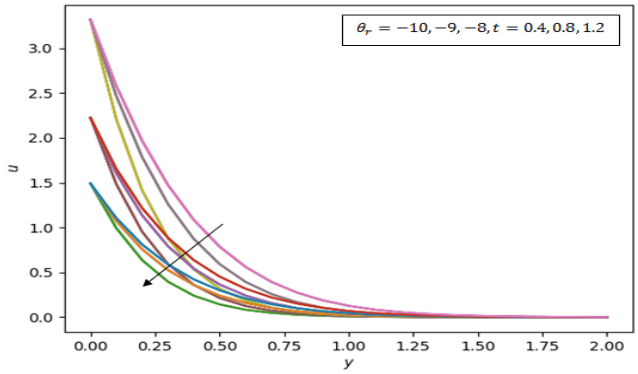

Fig. 4. Effect of $\boldsymbol{\theta_r}$ on u

Fig.4 gives the velocity distribution u against the variable viscosity parameter $\boldsymbol{\theta_r}$. Here, it is noticed that u decreases when $\boldsymbol{\theta_r}$ increases. As $\boldsymbol{\theta_r}$ increases, the resistance force between different layers of the fluid increases and as a result velocity of the fluid decreases. Concentration of the species also decreases with the increasing values of $\boldsymbol{\theta_r}$(Fig.5).

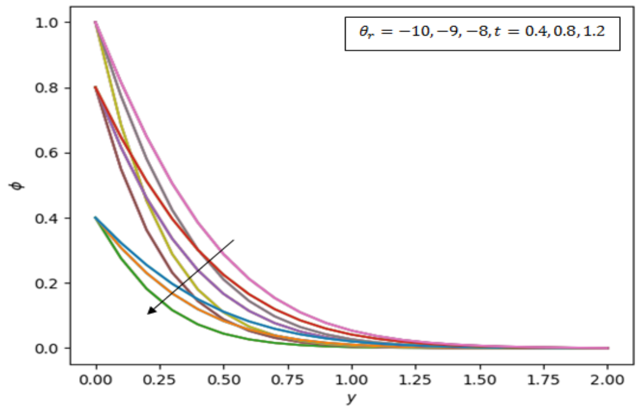

Fig. 5. Effect of $\boldsymbol{\theta_r}$ on ϕ

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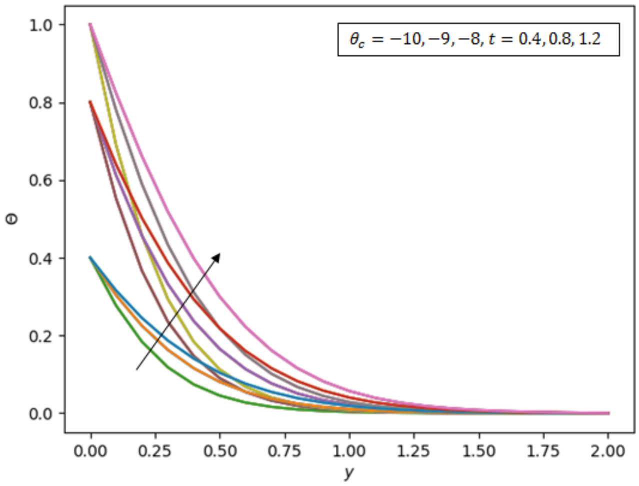

Fig. 6. Effect of θ_c on θ

Temperature varies directly with the variable thermal conductivity parameter $\boldsymbol{\theta_c}$. It is expected that the temperature within the fluid rises as a result of increase of thermal conductivity. So, θ enhances with $\boldsymbol{\theta_c}$(Fig.6).

Fig.7 and Fig.8 highlight the effect of magnetic field on *u* and $\boldsymbol{\theta}$. The presence of a magnetic field in normal direction fixes a resistive force, which prevents the flow. Due to this force, which increases the friction between the fluid layers, heat is generated and hence $\boldsymbol{\theta}$ increases.

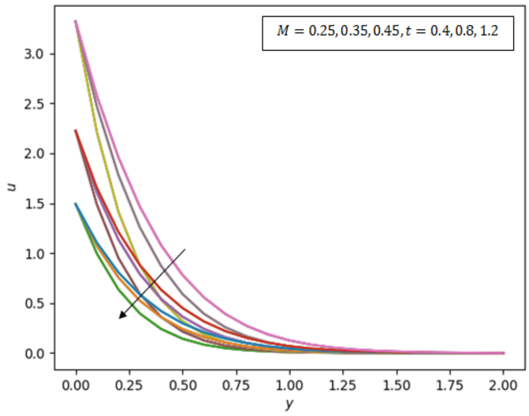

Fig. 7. Effect of M on u

Temperature is directly proportional to radiation (Fig.9). As radiation parameter Kr increases, the thermal buoyancy force gets stronger, as a result of which temperature increases. Heat absorption parameter S affects the fluid temperature adversely (Fig.10).

Species concentration is getting diminished with the enhancement of Schmidt number Sc (Fig.11). As Sc increases, the thickness of the concentration boundary layer decreases due to which the concentration gradient increases. As a result, the species concentration decreases. On the other hand, an increase in the positive value of the chemical reaction parameter $\boldsymbol{\xi}$ results in the utilization of species with a better rate and therefore the concentration boundary layer thickness decreases. Hence, the concentration decreases (Fig.12).

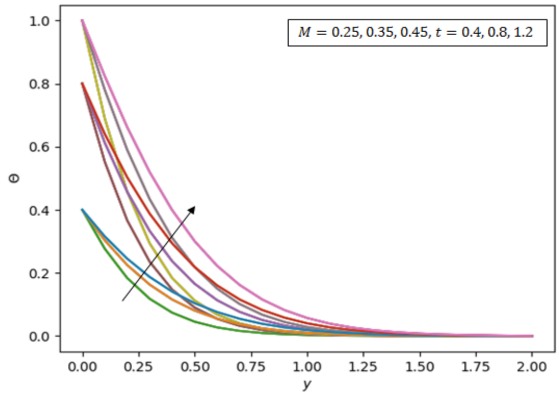

Fig. 8. Effect of M on θ

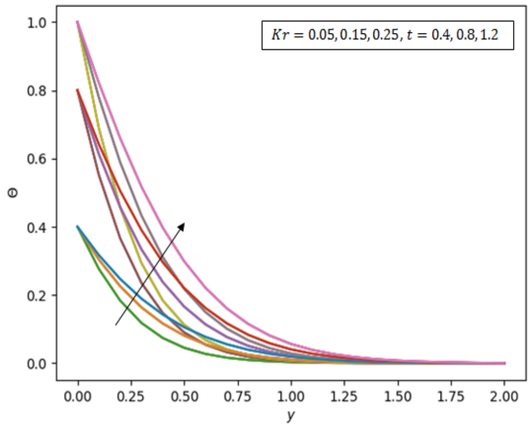

Fig. 9. Effect of Kr on θ

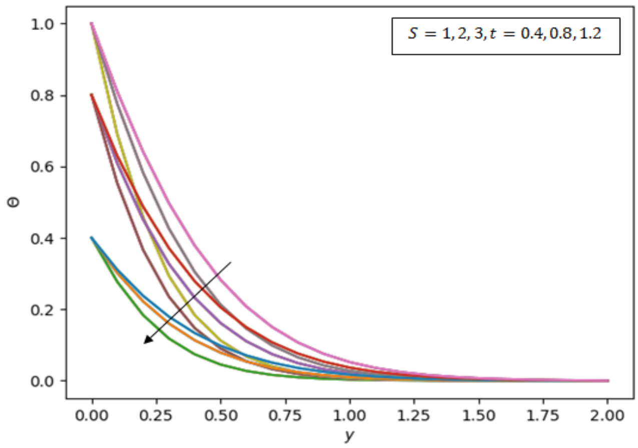

Fig. 10. Effect of S on θ

Fig.13 interprets the variation of *u* for the change of the angle of inclination $\boldsymbol{\omega}$. u is getting drop-off on enhancing $\boldsymbol{\omega}$. This is due to the fact that the effect of buoyancy force reduces due to the product of the term $\boldsymbol{cos\omega}$ in the buoyancy force term. Since $\boldsymbol{cos\omega}$ has maximum value for $\boldsymbol{\omega = 0}$ and it is receded as $\boldsymbol{\omega}$ increases. This figures out that the strength of buoyancy forces is getting diminished on increasing the angle of inclination of the plate.

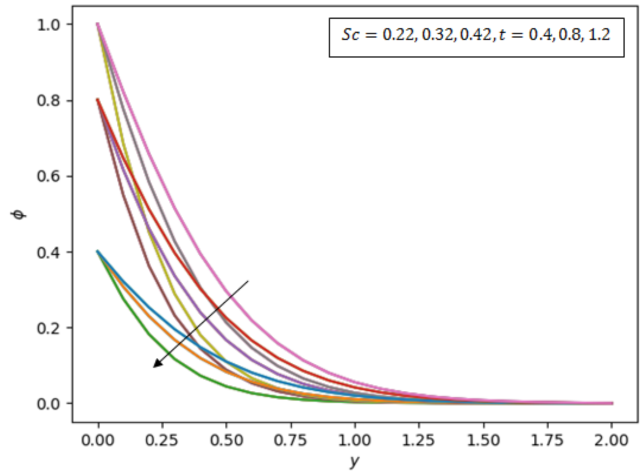

Fig. 11. Effect of Sc on ϕ

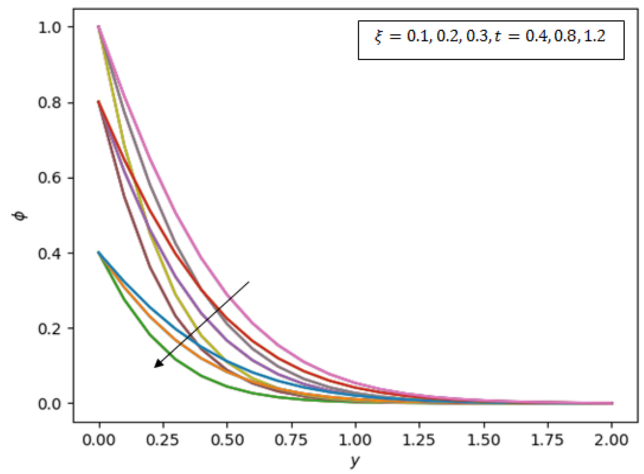

Fig. 12. Effect of ξ on ϕ

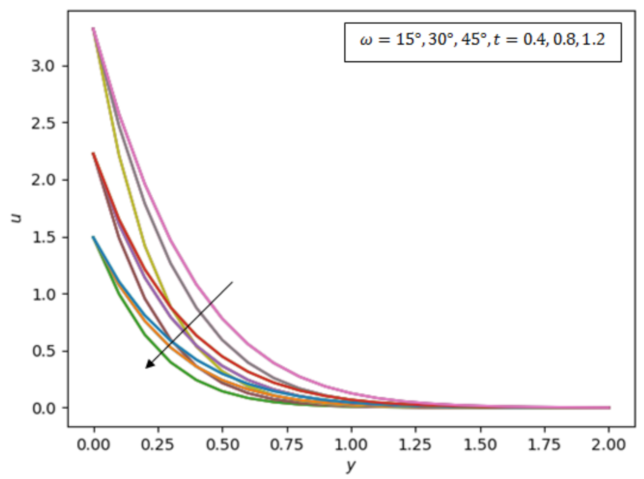

Fig. 13. Effect of ω on u

VI. CONCLUSION

From the above study, the following outcomes may be attempted.

1. The fuzzified forms of the functions as well as the physical parameters are of the similar forms as the crisp forms, and hence the equations and the conditions.
2. For any value of α except 1, we have two graphs which indicate the interval between in which the actual graphs will live. This actual graph will have membership grade 1 (i.e. $\alpha = 1$).
3. Velocity and species concentration vary inversely with the variable viscosity parameter. On the other hand, temperature is directly proportional to the variable thermal conductivity parameter.
4. Magnetic field parameter effects positively on temperature whereas adversely on the velocity distribution.
5. Temperature rises with the rising value of the radiation parameter and diminishes with the heat source parameter.
6. Species concentration distribution is inversely proportional to both Schmidt number and chemical reaction parameter.
7. The increasing of the inclination of the plate decreases the velocity of the fluid.
8. From the analysis it is observed that effects of fuzziness are prominent. This prominence will enhance further if we will use different left and right values of the parameters.

VII. LIMITATIONS AND FUTURE SCOPE

The study of fluid flow problems in fuzzy environment is still in nascent level. Only a few numbers of works related to it are obtained till now. This work is also one of the elementary steps towards fuzzification of the fluid flow problems. So, a lot of problems arise related to fuzzification of the equations, solving the BVPs or how to express the findings. Here, we have tried to overcome these limitations as far as possible. There exist so many future scopes related to it including different types of fluids using different types of numerical and analytical methods.

Nomenclature

t'/t	Dimensional/dimensionless time
u'/u	Dimensional/dimensionless velocity along x'-axis
μ	Dynamic viscosity
ρ	Density of the fluid
$\nu = \frac{\mu}{\rho}$	Kinematic viscosity
σ	Electrical conductivity
K_p'	Permeability
g	Acceleration due to gravity
β_T	Coefficient of thermal expansion
β_C	Coefficient of volumetric expansion
T'/T'_∞	Temperature/ temperature at free stream
C'/C'_∞	Species concentration/ species concentration at free stream
p	Pressure

C_p	Specific heat at constant pressure
λ	Thermal conductivity of the fluid
q_r	Radiating heat flux
Q_0	Heat absorption coefficient
D	Molecular diffusivity
Ch	Coefficient of chemical reaction
$\bar{\sigma}$	Stefan-Boltzmann constant
$\bar{a}$	Mean absorption coefficient

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μ_∞	Viscosity of the fluid at free stream
λ_∞	Thermal conductivity at free stream
θ/ϕ	Dimensionless temperature/dimensionless concentration
$M = \dfrac{\sigma B_0^2 \nu_\infty}{\rho U_0^2}$	Hartmann number
$K_p = \dfrac{K'_p U_0^2}{\nu_\infty^2}$	Permeability parameter
$Gr = \dfrac{g\beta_T \nu_\infty (T'_w - T'_\infty)}{U_0^3}$	Grashof number for heat transfer
$Gm = \dfrac{g\beta_C \nu_\infty (C'_w - C'_\infty)}{U_0^3}$	Grashof number for mass transfer
$Pr = \dfrac{\nu_\infty \rho C_p}{\lambda_\infty}$	Prandtl number
$Kr = \dfrac{16\bar{\sigma} {T'_\infty}^3}{3\lambda_\infty \bar{a}}$	Radiation parameter
$S = \dfrac{\nu_\infty Q_0}{\rho C_p U_0^2}$	Heat absorption parameter
$Sc = \dfrac{\nu}{D}$	Schmidt number
$\xi = \dfrac{\nu_\infty Ch}{U_0^2}$	Chemical reaction parameter

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AUTHOR'S PROFILE:

Dr. Palash Dutta, He has been working as an Assistant Professor in Department of Mathematics, Dibrugarh University since 1st February, 2013. His area of research is fuzzy set theory and evidence theory based on uncertainty modeling and decision making, medical diagnosis etc.

Dr. G. C. Hazarika, He is a retired Professor of Department of Mathematics, Dibrugarh University. He joined in the same Department as Lecturer in 1985 and has retired in 2019. His area of research is fluid dynamics, heat and mass transfer, computer oriented numerical methods, blood flow etc.

Joydeep Borah, He is a research scholar of Department of Mathematics, Dibrugarh University. He has done M. Phil. in 2018 and then has registered as a Ph.D. scholar in 2018 in the same University. His area of research is fluid dynamics using fuzzy concept.