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PSO Based Grey Bass Model for Simulating New Product Diffusion

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***Abstract:** The Bass model is one of the basic models for new product diffusion analysis. But the Bass model requires a large quantity of raw data to determine parameters of Bass model and this model also uses the potential capacity of market based on the subjective experience. To solve the problem of necessity of large raw data of Bass model, Wang put forward the Grey Bass model. Wang used non-linear least square (NLS) method to find the parameters of Grey Bass model and to assess the potential capacity of market. In the present paper a more appropriate method for Grey Bass equation is offered which estimates potential capacity of market even if the sample size is small. The proposed model is based on the minimization of sum of square of error between actual and predicted data using Particle Swarm Optimization (PSO) technique. Using the case study data, as used by Wang, the accuracy of the improved method is investigated. The results show that the mean absolute percentage error (MAPE) in the present case is 6.52 % compared to 7.93% reported by Wang.*

***Keywords:** Grey Bass, PSO, mean absolute percentage error, peak sale, Bass model.*

I. INTRODUCTION

For diffusion analysis of a new product generally Bass model [1] is considered as the most basic model. Bass Model can be expressed as:

$$\frac{dX(t)}{dt} = r\big(U - X(t)\big) + \frac{s}{U}X(t)\big(U - X(t)\big) \qquad (1)$$

Where $X(t)$ is the cumulative number of consumers using a new product at time t, U is the maximum capacity of the market for a new product. The constant r is called as the external influence coefficient. A high value of constant r indicates that earlier advantages were available in the market for new product diffusion. External influence coefficient directly depends on mass media communication. The constant s is called as the internal influence coefficient. It reflects the communications of earlier adopters with potential adopters. A high value of constant s indicates a high rate of diffusion of product which means high number of adopters. The best possible time in which peak sales (t*) occur is called point of inflection and is obtained by taking first derivatives of the Bass model. It is given by equation (2)

$$t^* = \frac{\ln\left(\frac{r}{s}\right)}{(r+s)} \qquad (2)$$

And the amount of peak sales is computed as:

$$s^* = \frac{U(r+s)^2}{4s} \qquad (3)$$

Bass model is employed on almost all new products and new technologies. In the case of large amount of data, ordinary least squares method [2], maximum likelihood estimation method [3], non-linear least squares method [4] etc. are used for parameters estimation of the Bass model. It is generally difficult to get sufficient amount of data for a new innovation or product. This lack of an adequate sample size causes to an inequitable estimation of the Bass parameter. In the Bass model the potential market capacity is obtained from subjective experience which relies on extra information so it is very difficult to estimate potential market capacity.

Grey model [3] requires only small amount of raw data as compare to conventional statistical method, to estimate the behavior of the unknown system. Deng [5-6] put forward that differential equations which are difficult to solve, can be greyed and approximate solution can be found out. For a new product diffusion having very limited data, Wang et al [7] suggested greying of differential equation of Bass Model. This model is known as Grey Bass model.

II. GREY BASS MODEL

The greying of Bass model is as follows:

Let $x^0(k)$ = Market diffusion sequence of a new product

$x^1(k)$ = Cumulative market diffusion sequence of a new product

$$x^1(k) = \text{AGO}\, x^0(k) \qquad (4)$$

Where AGO is Accumulating Generation Operator, which converts $x^0(k)$ into cumulative sum $x^1(k)$. It smoothen the randomness of primitive market diffusion sequence.

The mean value of adjacent AGO data $x^1(k)$ is given by

$$z^1(k) = 0.5\big(x^1(k) + x^1(k-1)\big) \qquad (5)$$

Through the basic mapping of the white differential equation, the following equation gives greyed Bass equation, called as Grey-Bass Model, in which the market diffusion of new product $\hat{x}^{(0)}(k)$ is computed as:

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$$\hat{x}^{(0)}(k) = r\left(U - z^{(1)}(k)\right) + s\, z^{(1)}(k)\left(1 - \frac{z^{(1)}(k)}{U}\right) \quad (6)$$

Where *r*, *s* are called Grey-Bass parameters.

After obtaining the parameters *r*, *s* and *M* the whitenisation equation of the Grey-Bass Model according to the theory of differential equations is as

$$\frac{dx^1(k)}{dt} = r\left(U - x^1(k)\right) + \frac{s}{U}\, x^1(k)\left(U - x^1(k)\right) \quad (7)$$

The solution of the equation (7) is given by

$$\hat{x}^{(1)}(k) = U\left(\frac{1 - e^{-(r+s)k}}{1 + \frac{s}{r} e^{-(r+s)k}}\right) k=1,2,3\ldots n \quad (8)$$

III. PARTICLE SWARM OPTIMIZATION

Particle Swarm Optimization (PSO) was established by Kennedy J. et.al [8]. It is based on the behavior of bird flocking. Imagine a group of birds smell a concealed source of food when flying around an area. The bird having a better condition i.e. closeness to food will inform it to its group and the others will move in unison to that place. This continues till one of the birds find the food.

In PSO, every single solution is a "bird" which is called particle. PSO is commenced with a group of random particles. The fitness of every particle is first assessed by the fitness function which is to be optimized. Then individual and global best fitnesses and positions are updated. Finally the velocity of every particle is updated. These steps are continued till final condition is met.

In each iteration, the best fitness achieved by a particle so far, is called 'pbst'. The best value, obtained up to now by any particle in the population is called global best and denoted as 'gbst'. Every particle is updated by following these two 'best' values. The 'gbst' value changes when any particle's 'pbst' value comes closer to the target than 'gbst'. In every iteration 'gbst' gradually changes its position nearer and nearer to the target until one of the particles achieves the target and optimum value is obtained. The updated velocity of particle is given by following equation

$$v_i\ (t+1) = iw\ v_i(t) + A_1 r_1\{\hat{x}_i(t) - x_i(t)\} + A_2 r_2\ \{gb(t) - x_i(t)\} \quad (9)$$

Where

v_i = Velocity of i^{th} particle at time t and

$x_i(t)$ = Position of i^{th} particle at time t

$\hat{x}_i(t)$ = The individual best solution for i^{th} particle at time t

gb(t) = Global best solution at time t.

iw= Inertia weight parameter

A_1 , A_2 = Acceleration parameters. r_1 and r_2 ($0\leq r1\leq 1$ and $0\leq r2\leq 1$) are random values regenerated for each velocity update.

After calculating the velocity of every particle, position of every particle is updated by applying the new velocity to the preceding position of the particle.

$$x_i(t+1) = x_i(t) + v_i\ (t+1) \quad (10)$$

This process is continued till final condition is met.

IV. PARAMETER OPTIMIZATION OF GREY BASS MODEL USING PSO

The Gauss Newton non-linear least squares method was utilized by Wang et al [9] to find the parameters of Grey-Bass model. It is found out by many researchers [10, 11] that non-linear regression with conventional methods like Gauss-Newton and Levenberg-Marquardt methods involve a lot of mathematical operations such as Matrix operations and Jacobean matrix calculation. Gauss-Newton method can also be trapped nearer to local minima and the convergence to the global minima may be too slow which is time-consuming. Gauss-Newton method also requires starting values of the unknown parameters. Inappropriate values at the beginning can result in convergence nearer to the local minimum. So with the purpose of overcoming these difficulties, in this paper, the use of Particle Swarm Optimization (PSO) algorithm is proposed. It is reported by Cui et al. [10] that PSO is more accurate than Gauss-Newton method.

In the present paper a more accurate method for estimating the parameter of Grey Bass equation is proposed which is appropriate for the small sample data and also finds the possible capacity of market. The purposed Model is based on the minimization of sum of square of error using PSO as given below:

Let $x^{(0)}\ (k)$ = Actual value of the market diffusion sequence of new product

$\hat{x}^{(0)}(k)$ = Predicted market diffusion sequence of new product computed using Grey Bass Model (6)

The error $\epsilon\,(k)$ between actual and predicted market diffusion sequence is given by

$$\epsilon(k) = \left(\hat{x}^{(0)}(k) - x^{(0)}\ (k)\right) k= 2,3\ldots\ldots n \quad (11)$$

Here it is assumed that first year data for both actual and predicted value are same i.e. $x^{(0)}\ (1) = \hat{x}^{(0)}(1)$

Therefore the objective function of optimization is to minimize the sum of square of error is

$$PSOFUN= \min[sum\left(\epsilon(k)\right)^2]$$

$$PSOFUN= \min[\ \sum_{k=2}^{n} \left(\hat{x}^{(0)}(k) - x^{(0)}\ (k)\right)^2] \quad (12)$$

In the above minimization function (6) we assumes that three parameters external influence coefficient (*r*), internal influence coefficient (*s*) and maximum market potential (*U*) are unknown and the sum of square of error between actual and predicted sequence of the product diffusion are minimized.

For the purpose of optimization we have used the Particle Swam Optimization (PSO) technique. The schematic flow chart of PSO algorithm is shown in Fig. 1.

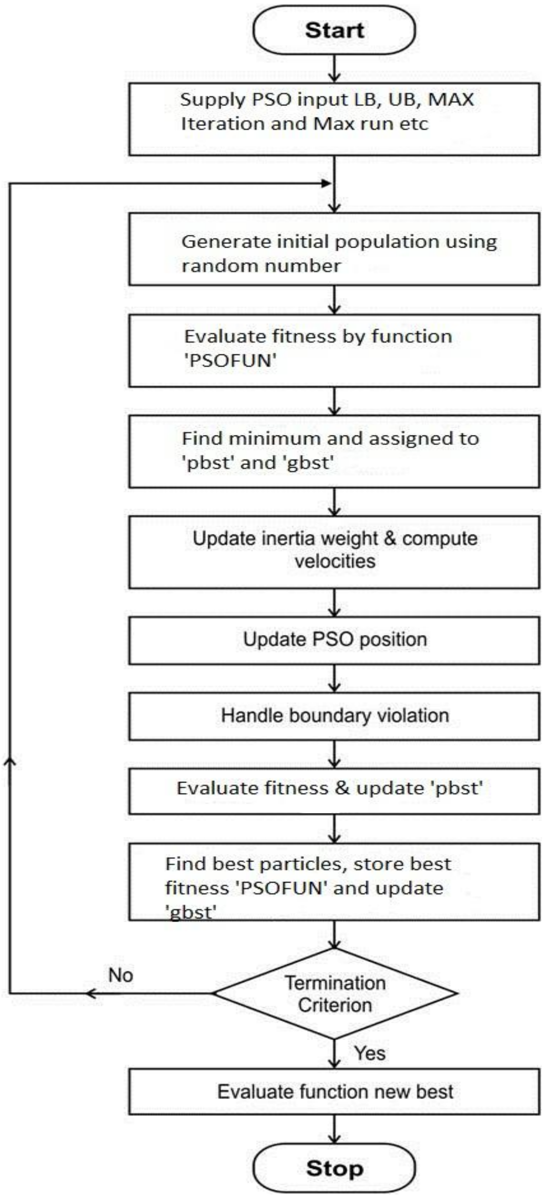


Fig. 1. Schematic flow chart of Particle Swarm Optimization

V. CASE STUDY

In this section, the superiority of the proposed PSO based Grey Bass model over the NLS-based Grey Bass model is demonstrated by actual example of 'WeChat' diffusion in the Chinese internet market. The data were collected from Wang et al. [9] paper. Data contained consumers from June 2011 to October 2013, and the time interval was two months. August 2011was used as a starting point (t = 0). The original data are as shown in Fig. 2.

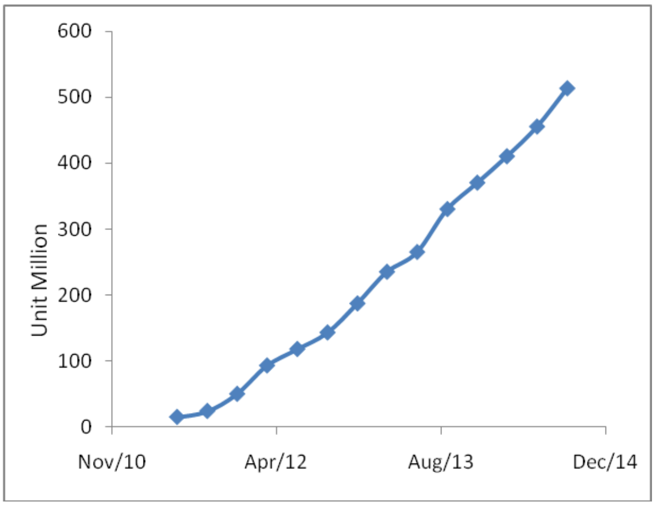


Fig. 2. Numbers of consumers from June 2011 to October 2013[9]

In order to implement the procedure of proposed Grey-Bass Model a computer program based on the minimization of the sum of square of error is developed in MATLAB 14.2 programming environment. The MATLAB environment is a natural choice as it require a lot of matrix manipulation (like inverse, transpose and multiplication) to compute Grey Bass parameters. The parameters obtained from proposed PSO based Grey-Bass Model and that from Wang et al. [9] are reported in Table 1. This is obvious from Table I that parameters obtained from PSO are 0.0194, 0.1888 and 832.1 Million which is nearly equal to that obtained from Wang et al. [9].

Table I Comparison of Grey-Bass Parameters

SN	Grey-Bass Parameter	Parameter value by NLS based Model [9]	Parameter value by PSO based Model
1	p	0.0156	0.0194
2	q	0.191	0.1888
3	M	887.6 Million	832.1 Million

The result of simulated value of raw input data along with error are compared with the data available in Wang et al. [9] and reported in Table II. It is evident from
Table II that that computer program developed has predicted the simulated values which are in excellent agreement with the publish results. Further the mean absolute percentage error (MAPE) in the present case is 6.52 % compared to 7.93% reported Wang et al. [9]. Fig. 3 shows the graphical representation of original data, simulated data reported in literature and simulated data computed through the program developed in the present case using PSO algorithm. Hence it can be concluded that the proposed improvement in Grey-Bass Model is superior to NLS based Grey-Bass Model. Using the estimated parameters the sample prediction is done from August 2011 to December 2020 as shown in Fig. 4.

Table II Comparison of PSO Based Model with NLS Based Model

SN	Raw input data (Million)	Simulated data by Wang et al. [14] (Million)	Absolute Percentage Error Wang et al. [9]	Simulated data by present computation (Million)	Absolute Percentage Error
1	15	15.12	0.79	15.00	0.00
2	24	33.02	37.57	35.14	46.40
3	50	54.04	8.08	57.67	15.34
4	93	78.51	15.58	84.64	8.98
5	118	106.70	9.58	115.60	2.03
6	143	138.77	2.96	149.24	4.36
7	187	174.77	6.54	186.23	0.41
8	235	214.55	8.70	227.08	3.37
9	265	257.76	2.73	270.66	2.13
10	330	303.84	7.93	316.86	3.98
11	370	352.02	4.86	365.08	1.33
12	410	401.36	2.11	414.23	1.03
13	455	450.83	0.92	463.78	1.93
14	513	499.40	2.65	513.00	0.00
			MAPE= 7.93%		MAPE=6.52%

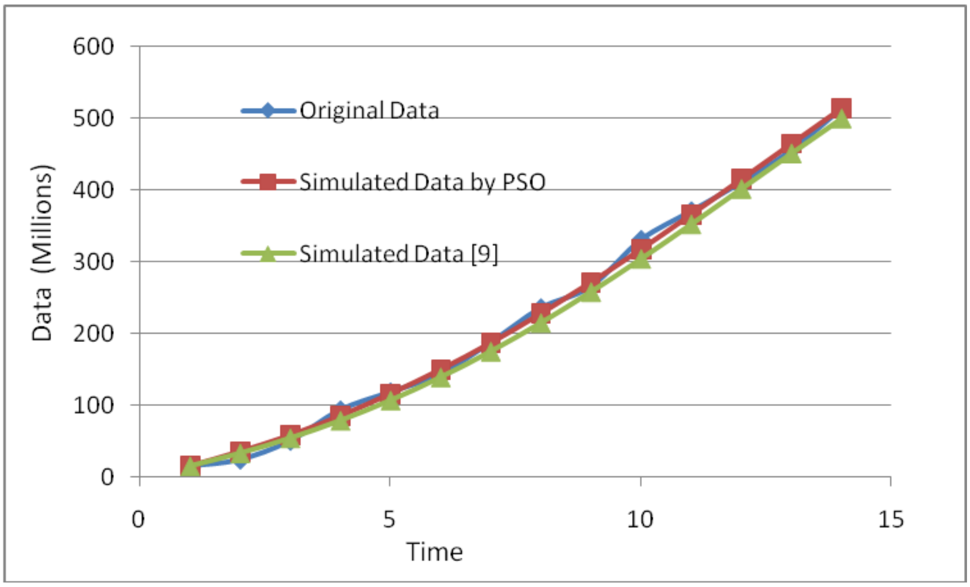
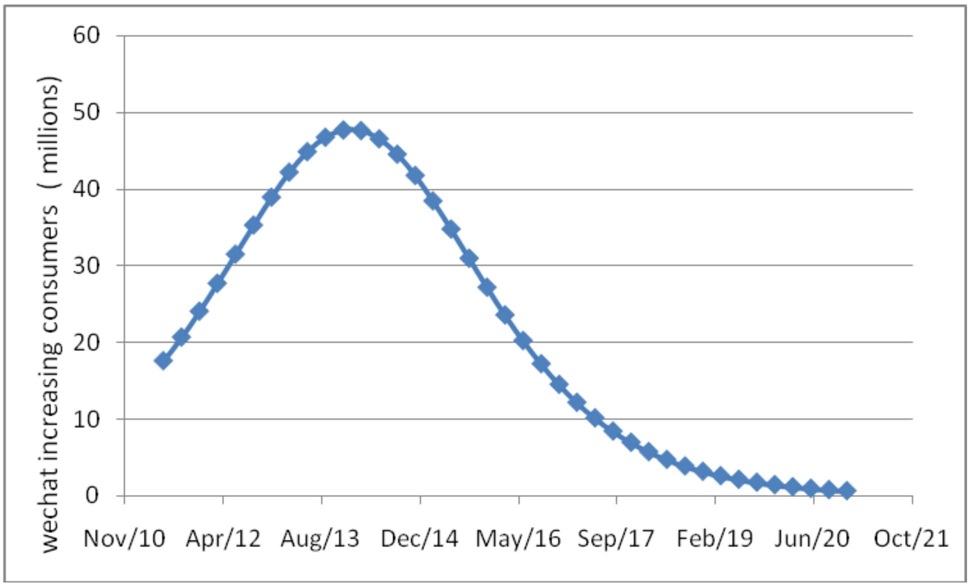
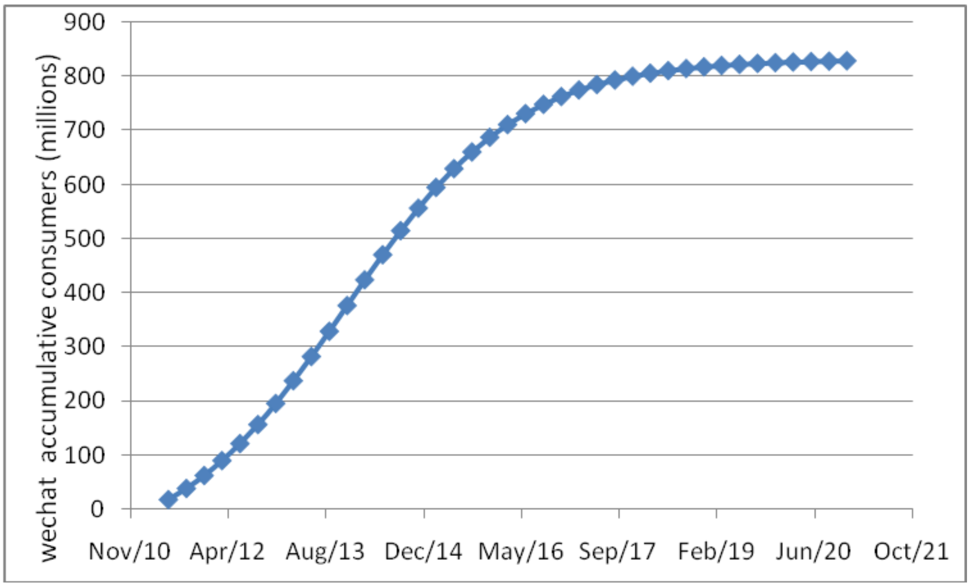


Fig. 3. Graphical representation of original data, simulated data by NLS based Model and simulated data using PSO algorithm.



(a) Increasing Consumers



(b) Accumulative Consumers

Fig. 4 o

As per official website of Tencent, the number of active user in March 2016 had reached more than 711 million, which is close to our predicted result by the same time and approaches the 829.45 million in Dec 2020. It is also found that the number of consumers reached its peak at t^* = 47.62 in December 2013. Additionally, we predict that the saturation point will happen at the starting of 2016.

VI. CONCLUSION

In this paper a PSO based Grey Bass Model is developed. The mean absolute percentage error using Grey Bass PSO model is 6.52 % compared to 7.93% obtained from NLS based Grey Bass model. The proposed model is appropriate for the small sample data and also estimates possible market capacity. Further it is found that Grey-Bass-PSO Model not only offers excellent simulation and prediction result but also demonstrates a good result of future forecasting of new product.

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