

An Extended Artificial Neural Network Assisted Hybrid Harris Hawks and Whale Optimizer to Find Optimal Solution for Engineering Design Problems

Someet Singh, Anuj Jain, Sunil Kumar Mahla



Abstract: The algorithms that have been developed recently have decorous behavior to solve and find optimum solution to various optimization problems in search space. Withal such calculations stuck in issues nearby quest space for compelled engineering problems. In succession to achieve an optimal solution a hybrid algorithmic approach is proffered. Artificial Neural Network (ANN) is considered as better solution for the known outputs. A hybrid variant of applying ANN on Harris Hawks and Whale Optimization Algorithm (ANNHHOWOA) is proposed to achieve effective solution for engineering problems. The effectiveness of proposed algorithm is tested for various non-linear, non-convex and standard engineering problems and to approve consequences of proposed algorithm standard benchmarks and multidisciplinary design problems have been considered. The validation endorsed that the results shown by ANNHHOWOA showed much better results than individual ANN, HHO and WOA and its effectiveness on multidisciplinary engineering problems.

Keywords: meta-heuristic algorithms, artificial neural network.

I. INTRODUCTION

Implementation and use of machine learning and artificial intelligence has become popular and has been accepted as best techniques over the last decade. Machine learning and artificial intelligence are being widely used over the years and provide efficient solutions to solve real world, discrete, constrained or unconstrained, linear or non linear engineering problems. The continuous research and the results produced from the experiments done in this field has resulted that the available methods such as sequential quadratic programming, quasi-Newton method has not shown significant and effective behavior to find solutions for non-continuous, multi-model problems. So, meta-heuristic algorithms have been taken into consideration to get accurate and efficient result of the defined problems.

Meta-algorithms have been implemented to achieve efficient results for the natural, multi-modal and engineering problems. Machine learning is one of the most exciting recent technologies in Artificial Intelligence. Heuristics are strategies to discover great ideal arrangements in a computational expense without ensuring achievability or optimality. Neural systems are demonstrating strategies equipped for displaying complex functions. Artificial neural networks are naturally propelled simulations performed on the workstation to perform particular tasks such as clustering, classification, pattern recognition, statistical analysis and data modeling. A neural system is an interrelated assemblage of basic processing components or nodes, whose purpose is inexactly in view of human neuron. Those transforming capacity of the organize may be saved in the inter-unit association strengths, alternately weights, gotten toward a transform from claiming adjustment to, or taking in from, an set about preparation designs. Neural Networks present an efficient approach to compute and understand the working of human brain. It takes numerous sources of info having various weightings and has one yield which relies upon the characterized inputs. Neural network is broadly utilized as a result of its capacity to sum up and to react to unpredicted inputs. A neural network does not need to be reprogrammed as it learns itself. Throughout training, neurons are taught to distinguish different particular designs what's more if on start alternately not at that design may be gained.

A. Feedforward Neural Network

A feed forward neural network is an artificial neural system wherein associations among the hubs don't prompt development of cycle. In feed forward network the progression of data is unidirectional, forward from the information hubs to the yield nodes through concealed layers.

B. Basic Architecture

Neural Network layers are autonomous and can have any number of hubs. Count of concealed hubs must be greater than all input hubs. The network must contain at least one hidden layer. Value of bias nodes is always set to 1.

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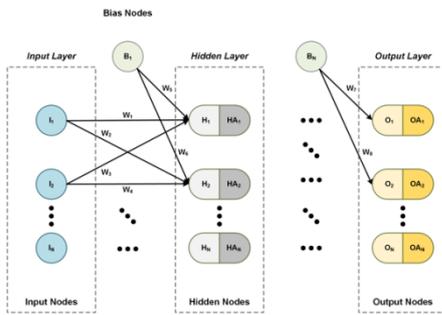


Figure 1: General Architecture

C. Basic Steps

- 1. Initialization of a neural network**
 - a) Initialize weights with a random number
 - b) Set bias nodes and initialize value to 1.
- 2. Feed Forward Network**
 - a) Define values for all input nodes.
 - b) Compute values of hidden nodes
 - c) Decide on an activation function for concealed layer
 - Compute values for creating hidden nodes.
 - Determine values for output node.
 - Choose an activation function for the output layer.
 - Compute activation values for output node.
 - Compute significant error.

D. Back propagation

In back propagation the loads are balanced steadily to deliver values near anticipated qualities from the training data. Meta-heuristic algorithms are collection of bright and responsive methodologies to upgrade the effectiveness of heuristic techniques. In optimization, population based methods are used to find the significant results of problems which is nearly similar to the exact solution. The process of optimization for the population based algorithm starts by generating the population based sets. The population corresponds to the candidate solution based upon the optimization method used. In each iteration, the values of population will change iteratively by replacing and updating the new population generated by the current set of values. This process continues until best possible optimal solution is found. Artificial neural system (ANN) has multilayer feed-forward neural system arrangement and it uses back-propagation (BP) to amend and upgrade the weight over the network. The for the most part utilized methodology is gradient descent (GD). Meta heuristic algorithms have been effectively executed for customary neural network to

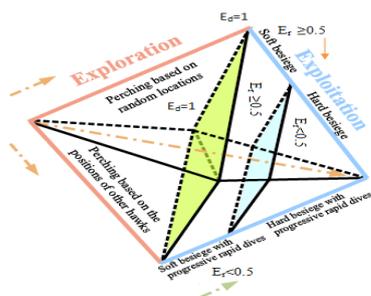


Figure 2: Different phases of HHO

For several hours, Harris Hawks observe and monitor the site of detecting attacking the prey. Let C be the number of

accelerate the preparation procedure by subbing the GD methodology with Harris Hawks and Whale optimization algorithms. The proposed research is a new artificial neural network based hybrid meta-heuristic algorithm ANHHOWOA, which is based on nature inspired Harris Hawks and Whale Optimization Algorithms. The primary notion for proposing this hybrid algorithm is to encourage and popular the natural hunting style of Harris Hawks and Whales to find and hunt its prey in a very efficient manner. This algorithm also aims to reflect the energy, escaping sequence of the prey. Consequently, a original mathematical algorithm is developed to embark upon different types of optimization problems.

II. MATHEMATICAL APPROACH TO ARTIFICIAL NEURAL NETWORK BASED HYBRID HARRIS HAWKS AND WHALE OPTIMIZATION ALGORITHM (ANHHOWOA)

The problems for which optimal solution has to be found are neither linear problems nor polynomial. So it becomes inevitable to use heuristic algorithm to extract possible solution. Few algorithms resolve restrictions by using derivatives, other evolutionary algorithms. It leads to the need of applying Artificial Neural Network to fairly accurate object functions. The objective of the optimization algorithm is to determine the best possible solution values for the parameters.

The learning algorithm can be described as follows:

- a) Initialize all weight w and bias b randomly.
- b) Iterate over all observations in data to find predicted distribution of data from neural network:
- c) Compute y
- d) Compute $L(w,b)$
 - $w_1 = w_1 - \eta \Delta w_1$
 - $w_2 = w_2 - \eta \Delta w_2$
 -
 - $w_n = w_n - \eta \Delta w_n$

Till conditions are satisfied.

To achieve optimum solution values, the algorithms based on natural behavior of animals are used. The Harris Hawks Algorithm (HHO) represents an efficient attacking strategy encouraged by the phenomenon of surprise attack. This approach used by Harris Hawks to hunt on prey (rabbit) is discussed. The algorithm works on two phases: Exploration and Exploitation. Harris Hawks Optimizer (HHO) is a population-based, nature-inspired optimization. The inspiration that leads to use of HHO algorithm is the behavior of Harris Hawks to attack surprisingly by following different ways to shock the prey. chances for all Harris Hawks to attack the prey as shown in equation (1). When the value of C is < 0.5 , all Harris Hawks are near to each other so that all can attack the prey. As shown in equation (2) Harris Hawks try to be stable on some random positions. The average position and location of Harris Hawks is calculated by using equation (3).

$$A(t+1) = Arand(t) - er1 | Arand(t) - 2er2A(t)j \quad C \geq 0.5 \quad (1)$$

$$A(t + 1) = Arabbit(t) - Am(t) - er3(LB + er4(UB - LB)) C > 0:5 \quad (2)$$

where $A(t + 1)$ represents location vector of hawks in the subsequent rotation t , $Arabbit(t)$ is the location of prey, $A(t)$ is the recent location vector of hawks, $er1, er2, er3, er4$, and C which are random numbers in $(0,1)$, and are modified in each iteration, LB and UB is the top and bottom limit of variables, $Arand(t)$ is a picked hawk from the existing hawks, and Am represents position of the recent number of available hawks.

The average location of hawks is given by: $Am(t) = 1/N \sum_{i=1}^N Ai(t) \quad (3)$

Where $Ai(t)$ refers to the position of every hawk in rotation t and N is number of hawks.

A. Conversion of Exploration to the Exploitation

B. Exploration Phase

The exploration phase refers to an action of exploring an unfamiliar area. The Harris Hawks search for prey by occupying random locations. The base of exploration is the evasion energy of rabbit, due to which energy of the prey is decreased.

The escape energy of a rabbit is calculated by: $EP = 2EP_0 (1 - \frac{t}{T}) \quad (4)$

Where EP is the escape energy of the rabbit, T refers to the most number of rotations, t is current iteration and EP_0 represents the starting level of energy of rabbit. For every iteration, EP_0 varies randomly from $(-1,1)$. If the energy of the prey reduces from 0 to -1, it indicates that target prey is running out of steam, losing the energy. When energy increases from 0 to 1, it means rabbit is strengthening. If the escape energy $|EP| \geq 1$, it means Harris Hawks discover the position of prey. When value $|EP| < 1$, it means algorithm tries to find the possible solution to catch prey.

C. Mathematical modeling of soft and hard encircles to attack the prey by Harris Hawks

Suppose E_r is the chance of a prey to escape. When $E_r < 0.5$, it indicates that the prey escapes successfully. When $E_r \geq 0.5$ indicates it is not a successful escape for the prey. The Harris Hawks can do surprise pounce. As per the activities of prey, the Harris Hawks will decide and perform hard and

F. Whale Optimization Algorithm

Whales are extravagant creatures which are supreme and evolved creatures on planet. Humpback whales can perceive the area of prey and en-circle them. After the best search specialist is characterized, the other hunt operators will consequently attempt to refresh their situations towards the best search specialist. Whale optimization algorithm is one of the swarm based meta-heuristic algorithms developed by being inspired by the hunting strategies of humpback whales. The unique hunting strategies of these whales are modeled in three major steps:

1. Encircling the prey
2. Moving towards the prey
3. Searching for the prey

G. Encircling the prey

When the whale identifies the location of the prey, it circles the target. In the beginning it is considered as the best solution where prey has been found. After sometime when

soft surround around the prey to catch it. The decision of Harris Hawks to attack prey depends upon the retained energy of the prey. To indicate the switch among the decision to attack prey, an additional parameter E_d is considered. When $|E_d| \geq 0.5$, indicates soft encircle whereas $|E_d| < 0.5$ indicates hard encircle.

D. Soft Encircle

When values of $E_r > 0.5$ and $|E_d| \geq 0.5$, it indicates rabbit has sufficient energy to make escape attempts. Harris Hawks encircle rabbit to exhaust it and then make a surprise attack. The complete attempt is shown using the following equations:

$$XE(t + 1) = \Delta XE - EX |R_j Arabbit(t) - XE(t)| \quad (5)$$

$$\Delta XE(t) = Arabbit(t) - XE(t) \quad (6)$$

Where, ΔX_E refers to the position vector of prey and present location of prey in each iteration. $R_j = 2(1 - r_s)$ refers to random jumps made by rabbit to escape. r_s is random number that varies in between $(0,1)$.

E. Hard Encircle

When values of $E_r \geq 0.5$ and $|E_d| < 0.5$. It indicates prey is exhausted and has low energy to escape from the attack of Harris Hawks. This activity is modeled by:

$$XE(t + 1) = Arabbit - EX |\Delta XE(t)| \quad (7)$$

Considering all the possibilities of fast dives in soft and hard encircle, in soft encircle of prey by Harris Hawks, final position of prey can be achieved by:

$$A(t + 1) = \{Mn, \text{if } F(Mn) < F(A(t))\} \quad (8)$$

$$A(t + 1) = \{P, \text{if } F(P) < F(A(t))\} \quad (9)$$

Considering possibility of actual position in hard encircle, in this situation the prey do not have enough energy to escape so values of equation (8) and (9) can be updated to find next position by:

$$Mn = Arabbit(t) - EP |K Arabbit(t) - Am(t)| \quad (10)$$

$$P = Mn + RS \times LFT(DIM) \quad (11)$$

Value of $A_m(t)$ can be obtained from equation (3).

whale actually captures and identify the actual location of the prey, the other whales upgrade their positions using following equations.

$$W = |P \cdot S * (n) - S(n)| \quad (12)$$

$$S(n + 1) = S * (n) - A \cdot W \quad (13)$$

where n is present rotation, A and P are coefficient vectors, S refers to the location of the best solution obtained, S is the position vector, $| |$ is the absolute value, and \cdot refers to element-by-element multiplication.

The vectors A and P are calculated as follows:

$$A = 2a \cdot r - a \quad (14)$$

$$P = 2 \cdot r \quad (15)$$

Where, a is linearly decreased from 2 to 0 over the course of iterations and r is a random vector in $[0,1]$.



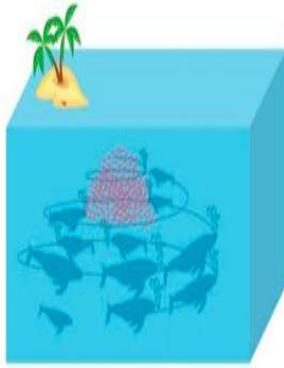


Figure 3: Encircling the prey

H. Moving towards the prey

Humpback whales approach their prey with a strategy called bubble-net feeding method. In this method, whales move towards their prey by blocking the view of their prey with the bubbles they create. Whales move towards their prey using 2 methods:

- a) Narrowing the circle
- b) Spiral movement method

The method of narrowing the circle can be achieved by decreasing the value of ‘a’ in equation (14).

In the spiral movement method, the distance between the whale and the prey is calculated in equation (12). This value is then used to calculate the spiral movement method in equation (13).

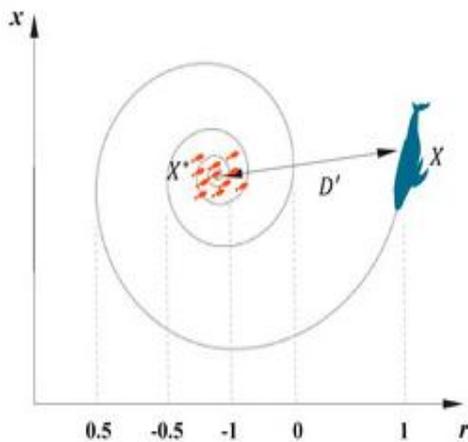


Figure 4: Spiral Movement

$$X(t + 1) = D' \cdot e^{bl} \cdot \cos(2\pi l) + X^*(t) \quad (16)$$

$$D' = X^*(t) - X(t) \quad (17)$$

‘b’ represents the log spiral constant. l represent random number in the range of [-1,1]. In the algorithm, which one of them, spiral movement or linear movement, will be employed is determined by 50% probability as shown:

$$X(t + 1) = X^x(t) - A \cdot D, p < 0,5 \quad (18)$$

$$X(t + 1) = D' \cdot e^{bl} \cdot \cos(2\pi l) + X^*(t), p \geq 0,5 \quad (19)$$

‘p’ represents random numbers in the range of [0,1].

I. Searching of the prey

The whales search for the prey randomly and change their position according to the position of other whales. The mathematical model of this is represented as:

$$D' = C \cdot Xrand - X \quad (20)$$

$$X(t + 1) = Xrand - A \cdot D \quad (21)$$

Xrand represents randomly selected solution vector.

III. PROPOSED ANN-HYBRID HARRIS HAWKS AND WHALE OPTIMIZATION ALGORITHM PSEUDOCODE (ANNHHOWOA)

Inputs:

Initialize Input parameters of ANN, HHO and WOA i.e. Search agents, EG, current iteration iter, maximum number of iterations T, population size N

Output: Position of prey and its best value of fitness.

Compute inputs using feed forward network.

Compute the nearest target values.

Initialize random population Pi (i=1,2,3.....,N)

while(t < T)

For every iteration calculate fitness value of Harris Hawk.

Set the parameter A as the best position for prey.

For each Harris Hawk (Pi)

Do

EP0 = 2rand()-1

K=2(1-rand()) to update energy at initial condition EP0

Update energy EP using equation (4)

First phase of exploitation of Exploration and Exploitation if |EP|≥1 then

Update position using equation (5) Exploration Phase

Update position vector using equation. (1) and equation. (2)

if |EP|<1 then Exploitation Phase

if (Er ≥0.5 and |EP|≥0.5) then //Soft Encircle

Update location vector using equation (5)

elseif (Er ≥0.5 and |EP|<0.5) then //Hard Encircle

Update location vector using equation (7)

elseif (Er <0.5 and |EP|≥0.5) then // Soft Encircle with advanced fast dives

Update location vector using equations (8) and (9)

elseif (Er <0.5 and |EP|<0.5) then // Hard Encircle with advanced fast dives

Update location vector using equations (10) and (11)

end end end

Initialize starting position of search agents using final position obtained through Harris Hawk Optimization Algorithm

Initialize t=1

Do

Evaluate each search agent using objective functions.

Update best fitness solution X*obtained so far.

Update random numbers re1,re2,re3,re4.

if(p<0.5) then

if(A≤1) then

Update position of present hunt agent using equation (12)

else

Update position of present hunt agent using equation (21)

else

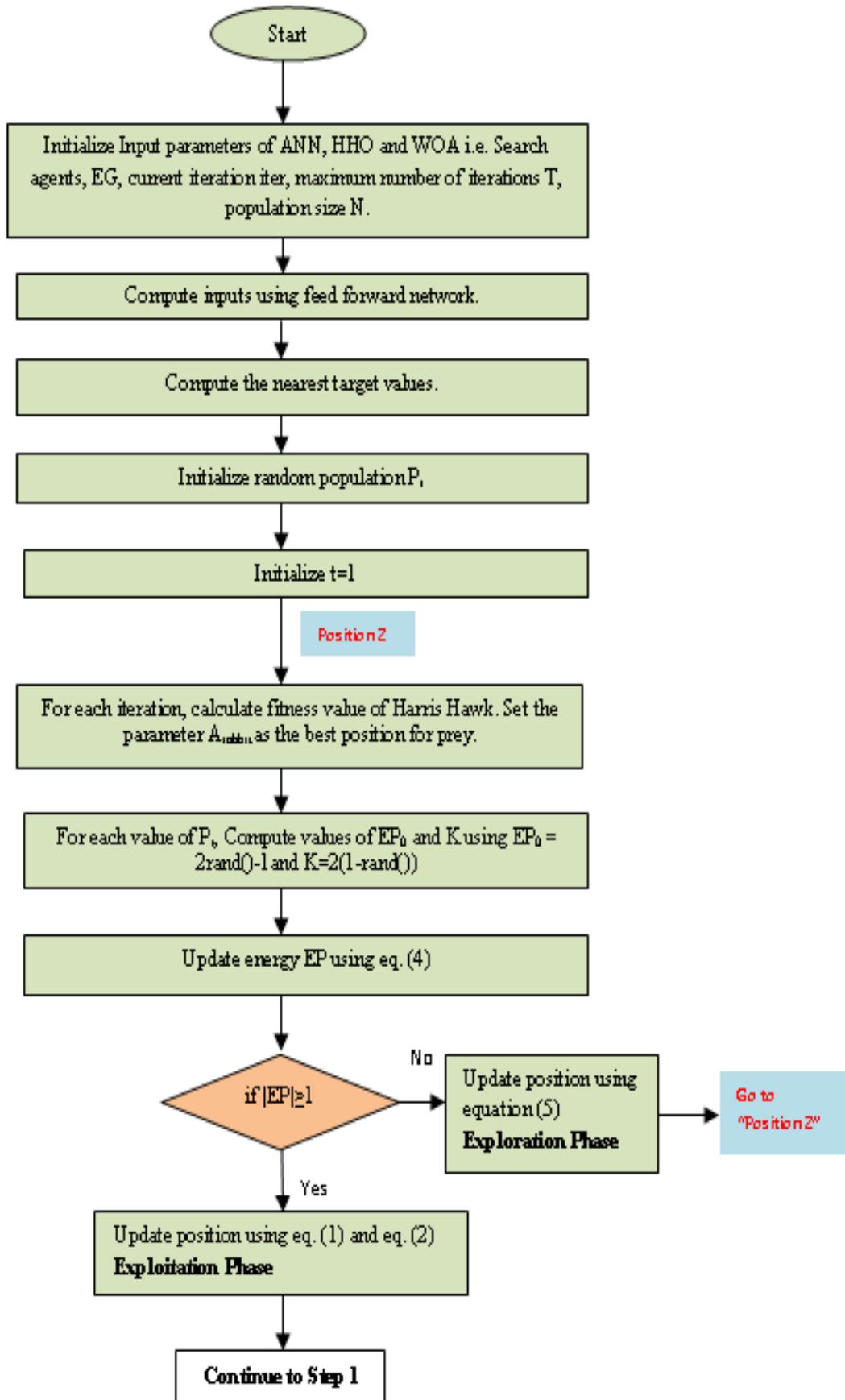
Update position of present hunt agent using equation (16)

while(t < T)

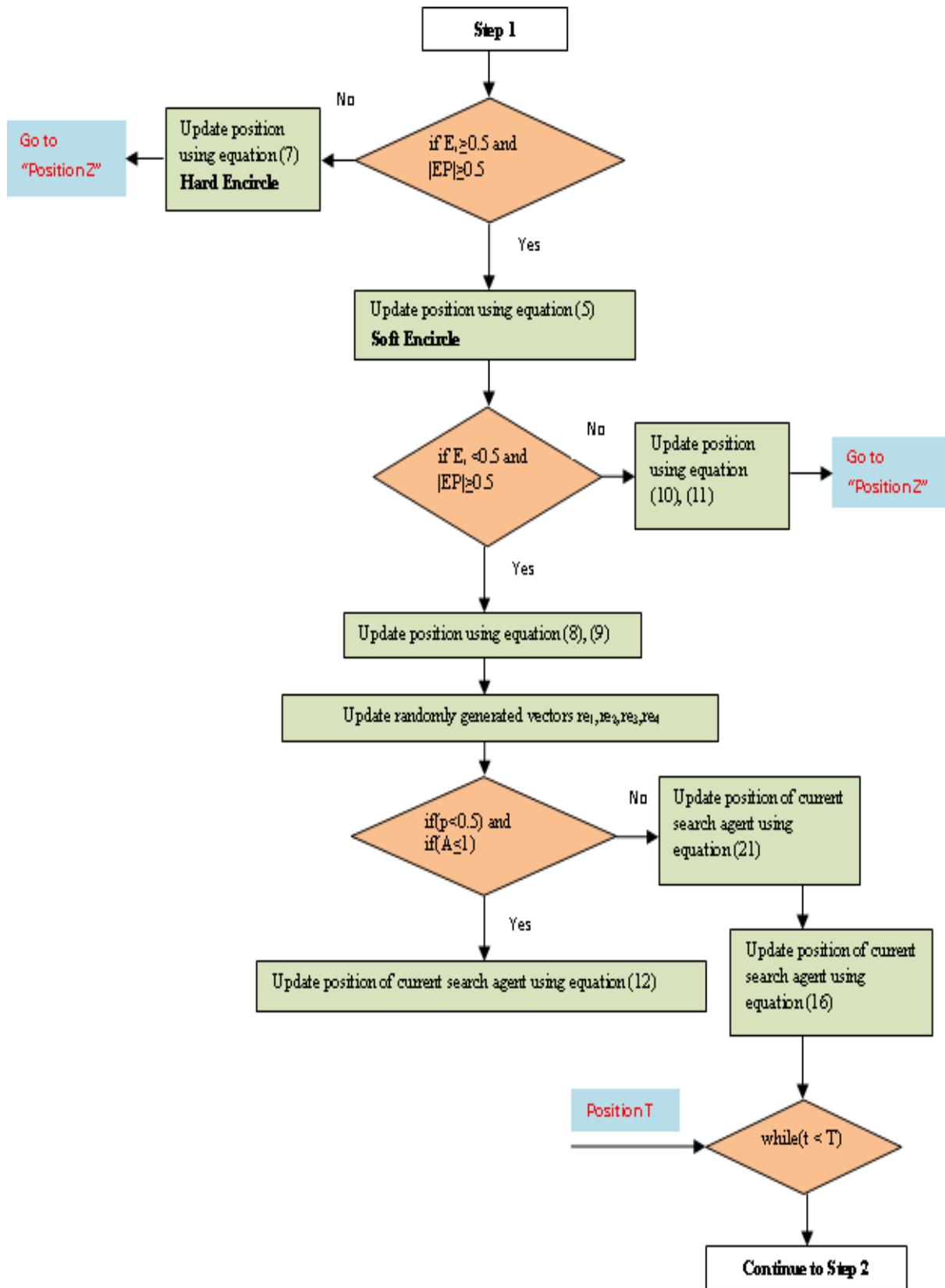
Return best optimal solution

Note down mean, standard deviation, best and worst fitness solutions.

Document and Save optimal value result for best solution obtained through successive trail iteration runs.



Flow Chart of Proposed Algorithm



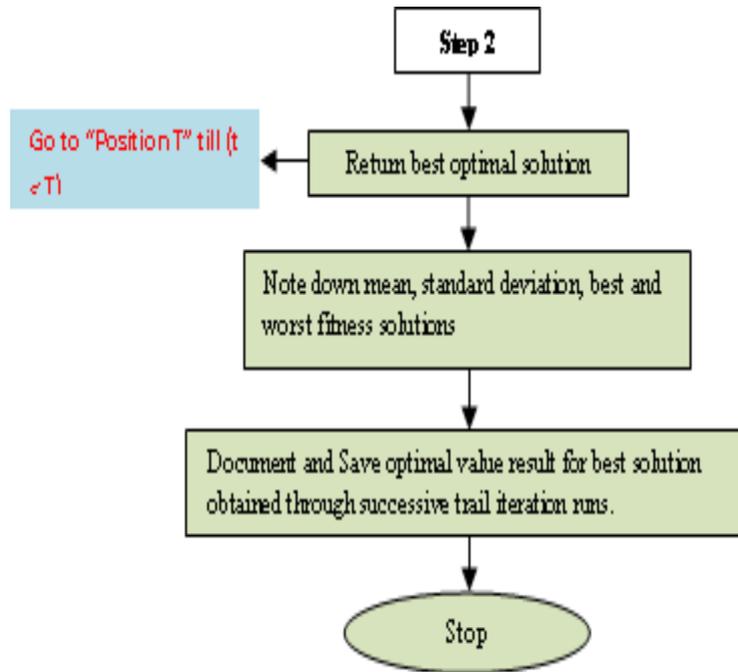


Figure 6: Multi-modal benchmark functions

IV. BENCHMARK FUNCTIONS

To observe the usefulness and efficiency of proposed ANHHOWOA algorithm, various unimodal, multimodal and fixed dimension benchmarks are considered [30]. In projected research, the performance of proposed hybrid algorithm is tested for unimodal, multi-modal, fixed dimension benchmarks functions. The mathematical formulae of unimodal, multi-modal and fixed dimension benchmarks is shown in Figure 5, Figure 6 and Figure 7 respectively.

C. Fixed dimension benchmark functions

A. Unimodal benchmark functions

Name	Function	Range	Dim	f_{min}
Sphere	$f_1(x) = \sum_{i=1}^n x_i^2$	[-100, 100]	200	0
Schwefel's 2.22	$f_2(x) = \sum_{i=1}^n x_i + \prod_{i=1}^n x_i $	[-10, 10]	200	0
Schwefel's 1.2	$f_3(x) = \sum_{i=1}^n \left(\sum_{j=1}^i x_j \right)^2$	[-100, 100]	200	0
Schwefel's 2.21	$f_4(x) = \max\{ x_i , 1 \leq i \leq n\}$	[-100, 100]	200	0
Rosenbrock	$f_5(x) = \sum_{i=1}^{n-1} [100(x_{i+1} - x_i^2)^2 + (x_i - 1)^2]$	[-30, 30]	200	0
Step	$f_6(x) = \sum_{i=1}^n (x_i + 0.5)^2$	[-100, 100]	200	0
Quartic	$f_7(x) = \sum_{i=1}^n (x_i^4 + \text{random}(0,1))$	[-1.28, 1.28]	200	0
X.S.Yang-7	$f_8(x) = \sum_{i=1}^n x_i - \frac{1}{i} , e_j \in U[0,1]$	[-5, 5]	200	0

Figure 5: Unimodal benchmark functions

B. Multi-modal benchmark functions

Function	Dim	Range	f_{min}
$F_8(x) = \sum_{i=1}^n -x_i \sin(\sqrt{ x_i })$	30	[-500, 500]	-418.9829 × 5
$F_9(x) = \sum_{i=1}^n [x_i^2 - 10 \cos(2\pi x_i) + 10]$	30	[-5.12, 5.12]	0
$F_{10}(x) = -20 \exp\left(-0.2 \sqrt{\frac{1}{n} \sum_{i=1}^n x_i^2}\right) - \exp\left(\frac{1}{n} \sum_{i=1}^n \cos(2\pi x_i)\right) + 20 + e$	30	[-32, 32]	0
$F_{11}(x) = \frac{1}{4000} \sum_{i=1}^n x_i^2 - \frac{n}{i-1} \cos\left(\frac{x_i}{\sqrt{i}}\right) + 1$	30	[-600, 600]	0
$F_{12}(x) = \frac{\pi}{n} \left\{ 10 \sin(\pi y_i) + \sum_{i=1}^{n-1} (y_i - 1)^2 \left[1 + 10 \sin^2\left(\frac{30y_{i+1}}{\pi}\right) + (y_{n-1}^{50} - 5)^{\beta} \right] \right\} + \sum_{i=1}^n u(x_i, 10, 100, 4)$ $y_i = 1 + \frac{x_i + 1}{4}$ $u(x_i, a, k, m) = \begin{cases} k(x_i - a)^m & x_i > a \\ 0 & -a < x_i < a \\ k(-x_i - a)^m & x_i < -a \end{cases}$			0

Fixed dimension multi modal functions	Range	f_{min}
$F_{14}(x) = \left(\frac{1}{500} + \sum_{j=1}^{20} \frac{1}{\sum_{i=1}^n (x_i - a_i)^6} \right)^{-1}$	[-65, 65] ²	1
$F_{15}(x) = \sum_{i=1}^{10} \left[a_i - \frac{x_i (b_i^2 + b_i x_i)}{b_i^2 + b_i x_i + x_i} \right]^2$	[-5, 5] ⁴	0.00030
$F_{16}(x) = 4x_1^2 - 2.1x_1^4 + \frac{1}{5}x_1^6 + x_1x_2 - 4x_2^2 + 4x_2^4$	[-5, 5] ²	-1.0316
$F_{17}(x) = (x_2 - \frac{5.1}{4\pi^2}x_1^2 + \frac{5}{\pi}x_1 - 6)^2 + 10(1 - \frac{1}{8\pi})\cos x_1 + 10$	[-5, 5] ²	0.398
$F_{18}(x) = [1 + (x_1 + x_2 + 1)^2(19 - 14x_1 + 3x_1^2 - 14x_2 + 6x_1x_2 + 3x_2^2)] \times [30 + (2x_1 - 3x_2)^2 \times (18 - 32x_1 + 12x_1^2 + 48x_2 - 36x_1x_2 + 27x_2^2)]$	[-2, 2] ²	3
$F_{19}(x) = -\sum_{i=1}^n c_i \exp\left(-\sum_{j=1}^n a_j (x_j - p_j)^2\right)$	[1, 3] ³	-3.86
$F_{20}(x) = -\sum_{i=1}^n c_i \exp\left(-\sum_{j=1}^n a_j (x_j - p_j)^2\right)$	[0, 1] ⁴	-3.32
$F_{21}(x) = -\sum_{i=1}^n [(X - a_i)(X - a_i)^2 + c_i]^{-1}$	[0, 10] ⁴	-10.1532
$F_{22}(x) = -\sum_{i=1}^n [(X - a_i)(X - a_i)^3 + c_i]^{-1}$	[0, 10] ⁴	-10.4028
$F_{23}(x) = -\sum_{i=1}^n [(X - a_i)(X - a_i)^7 + c_i]^{-1}$	[0, 10] ⁴	-10.5363

Figure 7: Fixed dimension benchmark functions

V. RESULTS AND ANALYSIS

To authenticate the results, total thirty trial runs are made to find the optimal solutions generated by ANHHOWOA algorithm. Average value, best values, standard deviation and worst values are calculated for every objective function. To endorse the exploitation stage by the proposed algorithm, unimodal benchmark functions from F1-F7, multimodal benchmark functions from F8-F11 and fixed benchmark functions from F12-F20 are taken into consideration. These results are compared with HHO [20] which is considered as an algorithm that provides better solutions when compared with others recently established meta-heuristics algorithms [30]. ANHHOWOA algorithm gave remarkable results in comparison to HHO which shows even better results than HHO.

A. Convergence Graphs using unimodal benchmark functions

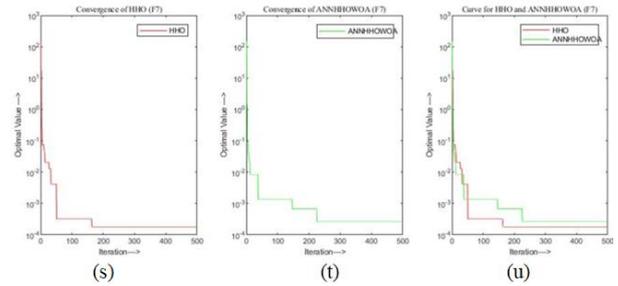
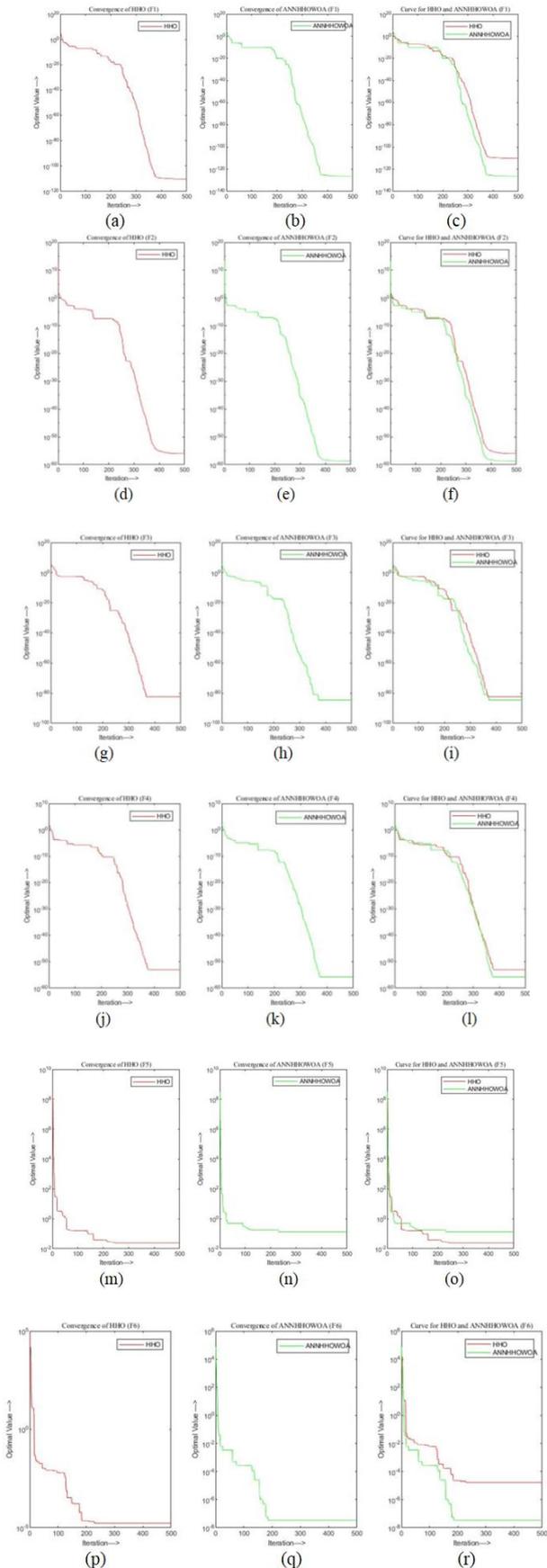
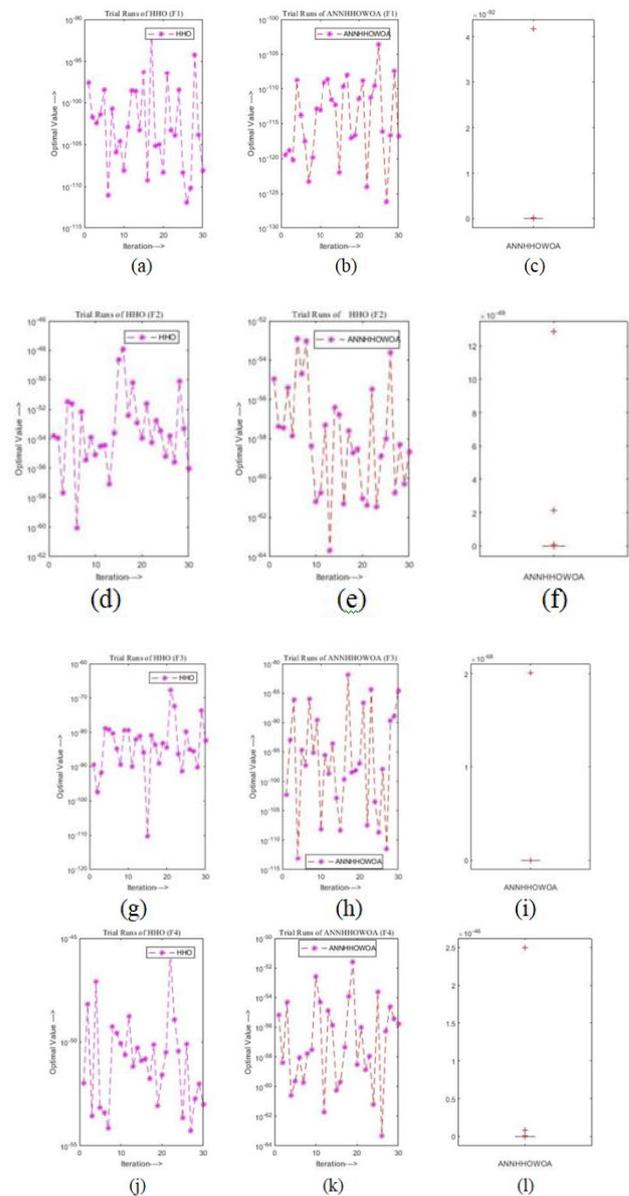


Figure 8: Graphs (a)-(u) represents convergence graphs of HHO, ANHHOWOA and comparative graph plotted by taking number of iterations along x-axis and an optimal value generated along y-axis for unimodal, benchmark functions. It can be clearly depicted that results generated by ANHHOWOA are better than HHO.

B. Trail run Graphs using unimodal benchmark functions



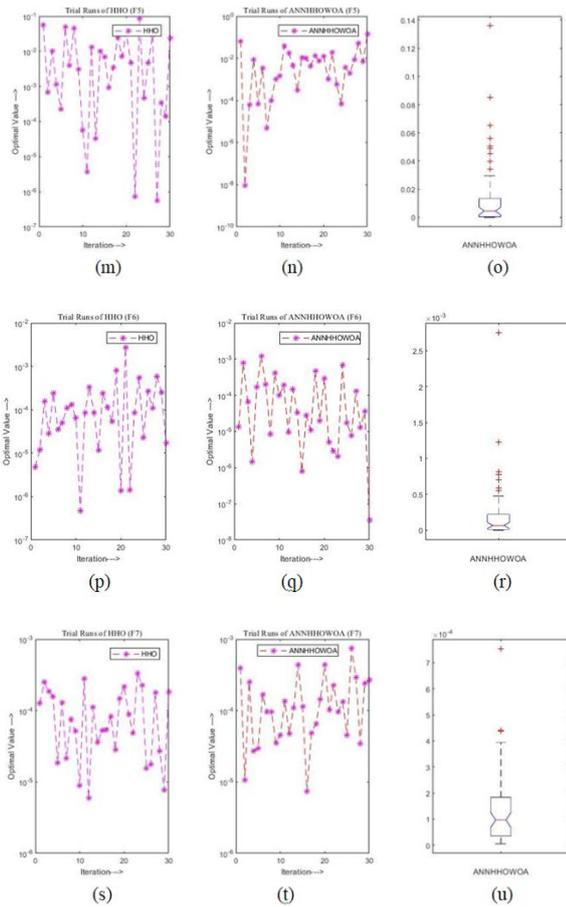


Figure 9: Graphs (a)-(u) represents trial runs done using HHO, ANNHOWOA and comparative graphs plotted by taking number of iterations along x-axis and an optimal value generated along y-axis for unimodal, benchmark functions. It can be clearly depicted that results generated by ANNHOWOA are better than HHO.

C. Convergence Graphs using multimodal benchmark functions

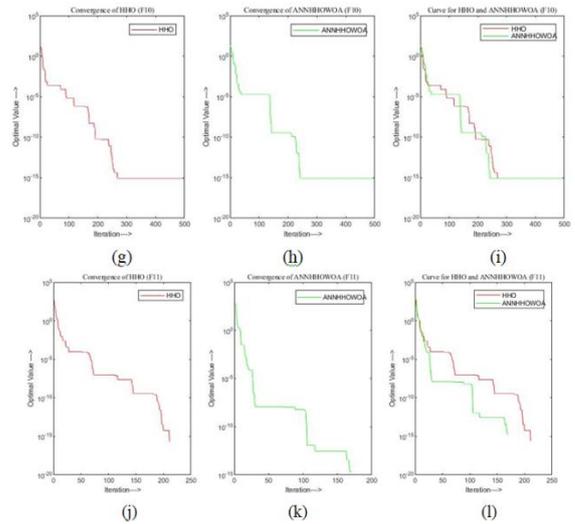
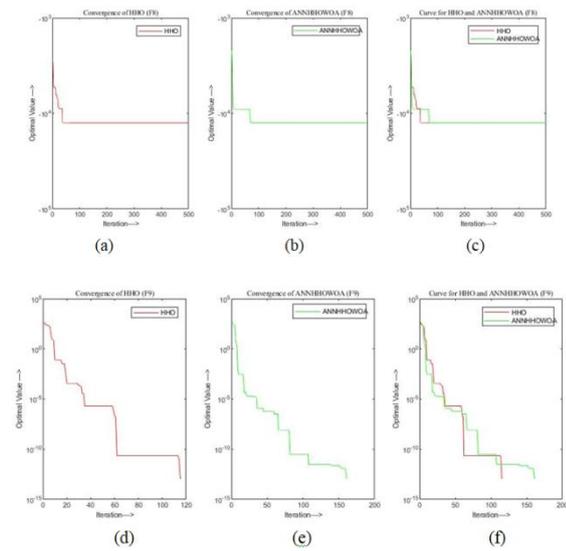
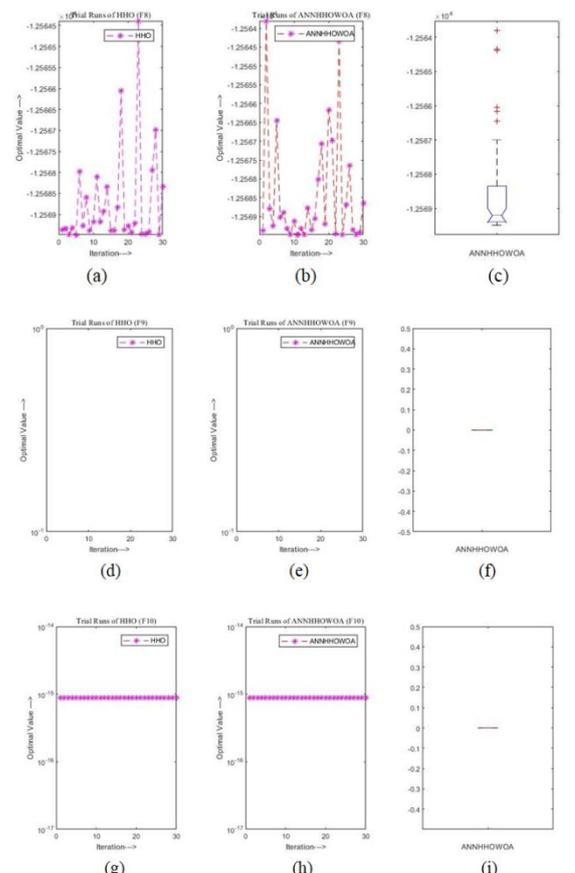


Figure 10: Graphs (a)-(u) represents convergence graphs of HHO, ANNHOWOA and comparative graph plotted by taking number of iterations along x-axis and an optimal value generated along y-axis for multimodal, benchmark functions. It can be clearly depicted that results generated by ANNHOWOA are better than HHO.

D. Trial runs Graphs using multimodal benchmark functions



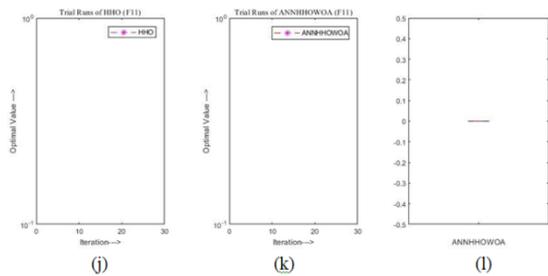


Figure 11: Graphs (a)-(u) represents trial runs done using HHO, ANHHOWOA and comparative graphs plotted by taking number of iterations along x-axis and an optimal value generated along y-axis for multimodal, benchmark functions. It can be clearly depicted that results generated by ANHHOWOA are better than HHO.

E. Convergence Graphs using fixed benchmark functions

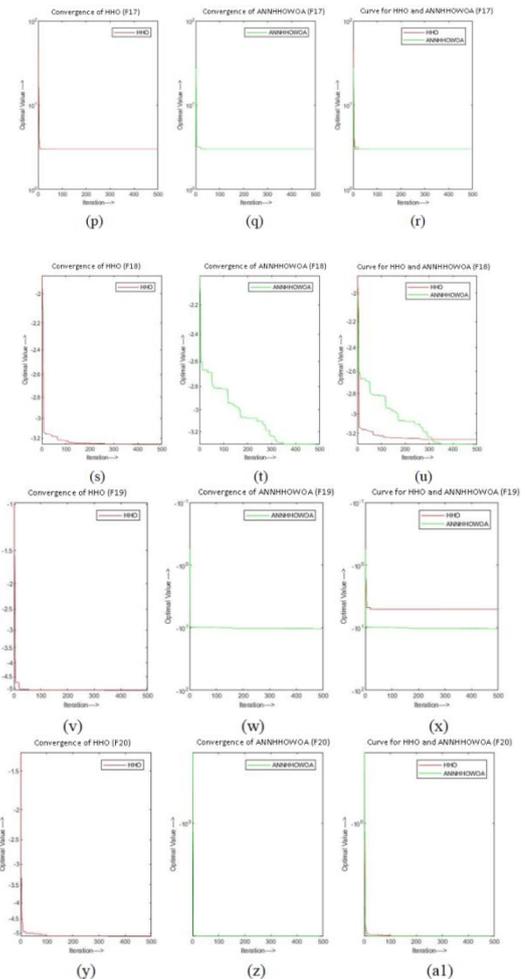
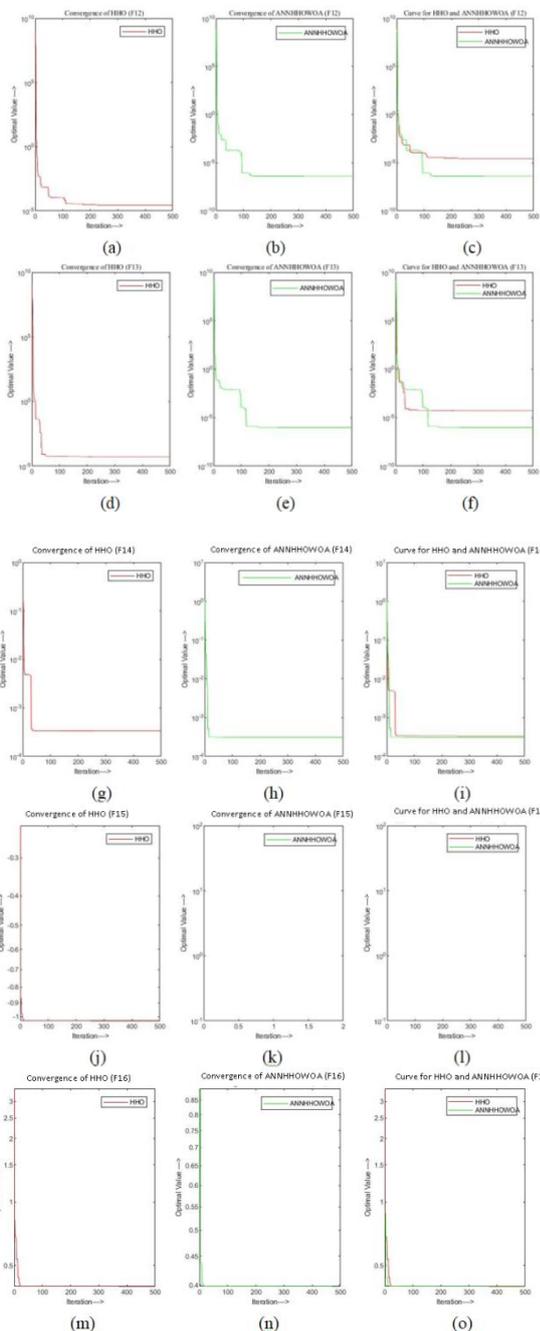
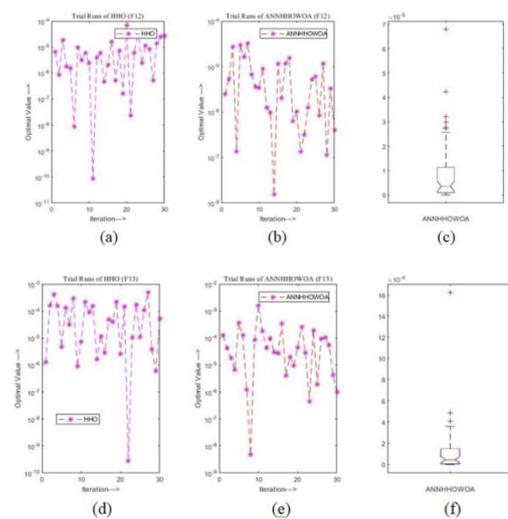


Figure 12: Graphs (a)-(a1) represents convergence graphs of HHO, ANHHOWOA and comparative graph plotted by taking number of iterations along x-axis and an optimal value generated along y-axis for fixed benchmark functions. It can be clearly depicted that results generated by ANHHOWOA are better than HHO.

F. Trial run Graphs using fixed benchmark functions



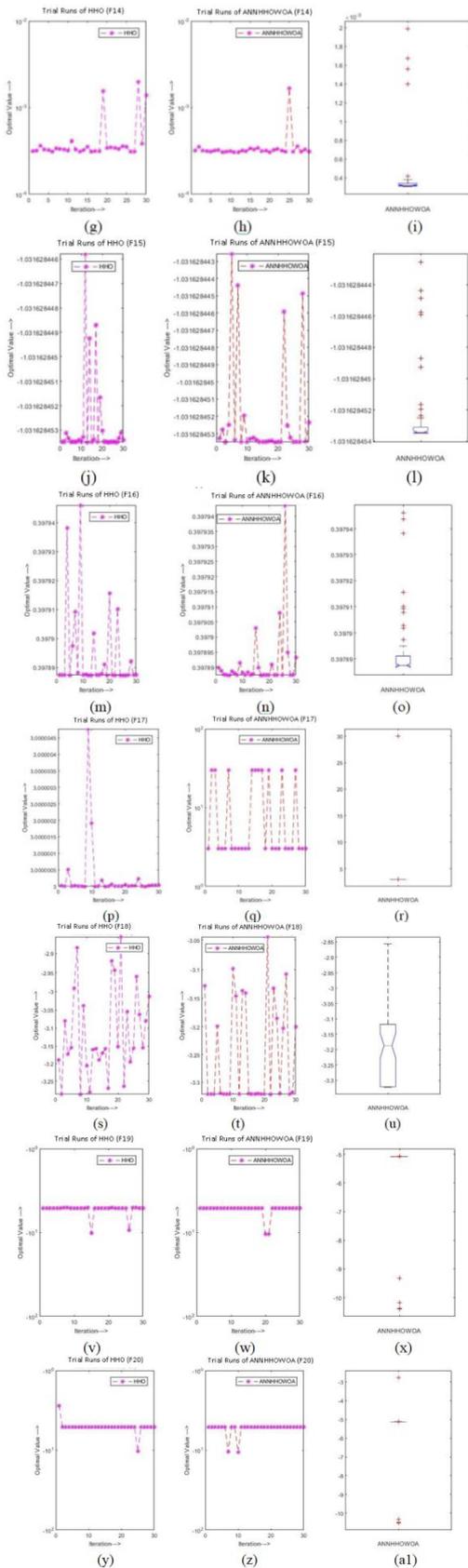


Figure 13: Graphs (a)-(a1) represents trial runs done using HHO, ANNHOWOA and comparative graphs plotted by taking number of iterations along x-axis and an optimal value generated along y-axis for fixed benchmark functions. It can be clearly depicted that results generated by ANNHOWOA are better than HHO.

Table 1: Results of time taken by unimodal, multimodal and fixed benchmark function using ANNHOWOA algorithm for calculating parameters like mean, standard deviation, best fitness, worst fitness, median and p-value.

Function	Parameters					
	Mean	SD	Best Fitness	Worst Fitness	Median	p-value
F1	9.5607E-106	5.2355E-105	6.9382E-127	2.8676E-104	5E-114	0.325481349
F2	8.28056E-55	2.81517E-54	2.021E-64	1.22348E-53	7.62E-59	0.11799492
F3	4.07758E-84	2.22006E-83	8.6638E-114	1.21621E-82	8.08E-98	0.322736016
F4	1.00259E-53	4.90939E-53	4.54417E-64	2.68656E-52	3.71E-58	0.272508933
F5	0.014333269	0.027701829	9.41688E-09	0.135941459	0.004474	0.008284051
F6	0.000169533	0.00028556	3.5803E-08	0.001223513	3.1E-05	0.00290648
F7	0.000163266	0.000166649	7.37553E-06	0.000752946	0.000106	9.17383E-06
F8	-12568.43097	1.515775489	-12569.48662	-12563.80623	-12569.1	2.0548E-115
F9	0	0	0	0	0	0
F10	8.88178E-16	0	8.88178E-16	8.88178E-16	8.88E-16	0
F11	0	0	0	0	0	0
F12	7.01051E-06	9.0511E-06	1.55639E-08	3.19892E-05	3.36E-06	0.000206425
F13	0.000131243	0.000298231	4.75375E-09	0.001621269	4.33E-05	0.022503362
F14	0.000365863	0.000246587	0.000307794	0.001669727	0.000318	5.82249E-09
F15	-1.031628452	3.09925E-09	-1.031628453	-1.031628443	-1.03163	6.4234E-249
F16	0.397891518	1.09146E-05	0.397887358	0.397943632	0.397888	4.5978E-134
F17	12.00001759	12.9455338	3	30.00010874	3	2.04682E-05
F18	-3.25060214	0.093259963	-3.321929616	-3.044405339	-3.32028	1.6787E-46
F19	-5.440966039	1.345099226	-10.39316593	-5.086594869	-5.08759	1.00684E-19
F20	-5.487297225	1.366087966	-10.51869957	-5.127936045	-5.12836	1.22006E-19

Table 2: Results of time taken by hybrid ANNHOWOA algorithm to find optimal solution.

Function	Time (in Sec.)		
	Best Time	Mean Time	Worst Time
F1	0.046875	0.084895833	0.3125
F2	0.046875	0.071354167	0.1875
F3	0.21875	0.244270833	0.34375
F4	0.046875	0.061458333	0.078125
F5	0.078125	0.0921875	0.109375
F6	0.046875	0.066145833	0.09375
F7	0.125	0.159375	0.234375
F8	0.078125	0.103645833	0.265625
F9	0.0625	0.08125	0.140625
F10	0.0625	0.084375	0.140625
F11	0.078125	0.102604167	0.328125
F12	0.3125	0.370833333	0.734375
F13	0.328125	0.367708333	0.4375
F14	0.046875	0.089583333	0.296875
F15	0.046875	0.0875	0.28125
F16	0.046875	0.080208333	0.21875
F17	0.046875	0.061979167	0.140625
F18	0.0625	0.0921875	0.328125
F19	0.09375	0.1328125	0.46875
F20	0.09375	0.1375	0.421875

The calculated results show that the ANN based hybrid approach to find optimal solution is better than the individual meta-heuristic algorithms.

VI. CONCLUSION

In this research paper, a new ANNHOWOA optimization algorithm is presented. The proposed method is conducted on twenty three mathematical benchmark functions to analyze standard deviation, mean, best fitness, worst fitness, median and p-value parameters. ANNHOWOA is more competitive with respect to the other meta-heuristic methods. In addition to this, ANNHOWOA resulted better on engineering problems. This algorithm is tested on twenty engineering problems. The proposed algorithm shows the high level of competency over conventional techniques.

REFERENCE

- Roy,S., Das, A.K., P. K. Bose, and Banerjee,R., "ANN metamodel assisted Particle Swarm Optimization of the performance-emission trade-off characteristics of a single cylinder CRDI engine under CNG dual-fuel operation," *J. Nat. Gas Sci. Eng.*, vol. 21, no. x, pp. 1156–1162, 2014.
- Roy,S., Ghosh,A., Das, A.K., and Banerjee,R., "A comparative study of GEP and an ANN strategy to model engine performance and emission characteristics of a CRDI assisted single cylinder diesel engine under CNG dual-fuel operation," *J. Nat. Gas Sci. Eng.*, vol. 21, pp. 814–828, 2014.
- Roy,S. and Banerjee,R., "Multi-objective optimization of the performance-emission trade-off characteristics of a CRDI coupled CNG diesel dual-fuel operation: A GEP meta-model assisted MOGA endeavour," *Fuel*, vol. 211, no. October 2017, pp. 891–897, 2018.
- Deb,M., Majumder,P., Majumder,A., Roy,S., and Banerjee,R., "Application of artificial intelligence (AI) in characterization of the performance–emission profile of a single cylinder CI engine operating with hydrogen in dual fuel mode: An ANN approach with fuzzy-logic based topology optimization," *Int. J. Hydrogen Energy*, vol. 41, no. 32, pp. 14330–14350, 2016.
- Javed,S., Murthy, Y.V.V.S., Baig, R.U., and Rao, D.P., "Development of ANN model for prediction of performance and emission characteristics of hydrogen dual fueled diesel engine with Jatropa Methyl Ester biodiesel blends," *J. Nat. Gas Sci. Eng.*, vol. 26, pp. 549–557, 2015.
- Deb,M., Majumder,P., Majumder,A., Roy,S., and Banerjee,R., "Application of artificial intelligence (AI) in characterization of the performance–emission profile of a single cylinder CI engine operating with hydrogen in dual fuel mode: An ANN approach with fuzzy-logic based topology optimization," *Int. J. Hydrogen Energy*, vol. 41, no. 32, pp. 14330–14350, 2016.
- Kshirsagar, C.M. and Anand,R., "Artificial neural network applied forecast on a parametric study of Calophyllum inophyllum methyl ester-diesel engine out responses," *Appl. Energy*, vol. 189, pp. 555–567, 2017.
- Channapattana, S.V., Pawar, A.A. and Kamble, P.G. "Optimisation of operating parameters of DI-CI engine fueled with second generation Bio-fuel and development of ANN based prediction model," *Appl. Energy*, vol. 187, pp. 84–95, 2017.
- [9] Roy,S., Banerjee,R., Das, A.K., and Bose, P.K., "Development of an ANN based system identification tool to estimate the performance-emission characteristics of a CRDI assisted CNG dual fuel diesel engine," *J. Nat. Gas Sci. Eng.*, vol. 21, no. x, pp. 147–158, 2014.
- Roy,S., Das, A.K., Bhadouria, V.S., Mallik, S.R., Banerjee,R., and Bose, P.K., "Adaptive-neuro fuzzy inference system (ANFIS) based prediction of performance and emission parameters of a CRDI assisted diesel engine under CNG dual-fuel operation," *J. Nat. Gas Sci. Eng.*, vol. 27, pp. 274–283, 2015.
- Roy,S. and Banerjee,R., "Multi-objective optimization of the performance-emission trade-off characteristics of a CRDI coupled CNG diesel dual-fuel operation: A GEP meta-model assisted MOGA endeavour," *Fuel*, vol. 211, no. October 2017, pp. 891–897, 2018.
- Roy,S., Das, A.K., Bhadouria, V.S., Mallik, S.R., Banerjee,R., and Bose, P.K., "Adaptive-neuro fuzzy inference system (ANFIS) based prediction of performance and emission parameters of a CRDI assisted diesel engine under CNG dual-fuel operation," *J. Nat. Gas Sci. Eng.*, vol. 27, pp. 274–283, 2015.
- R. Abbassi, A. Abbassi, A. Asghar, S. Mirjalili, An efficient salp swarm inspired algorithm for parameters identification of photovoltaic cell models, *Energy Convers. Manag.* 179 (2018) (2019) 362–372.
- H. Faris, et al., An intelligent system for spam detection and identification of the most relevant features based on evolutionary Random Weight Networks, *Inf. Fusion*, vol. 48 (2019) 67–83.
- J.F. McCarthy, Block-conjugate-gradient method, *Phys. Rev. D* 40 (6) (1989) 2149–2152.
- G. Wu, Across neighborhood search for numerical optimization, *Inf. Sci. (Ny)*. 329 (61563016) (2016) 597–618.
- G. Wu, W. Pedrycz, P.N. Suganthan, R. Mallipeddi, A variable reduction strategy for evolutionary algorithms handling equality constraints, *Appl.Soft Comput. J.* 37 (2015) 774–786.
- A.A. Heidari, R. Ali Abbaspour, A. Rezaee Jordehi, An efficient chaotic water cycle algorithm for optimization tasks, *Neural Comput. Appl.* (1) (2017) 57–85.
- M. Mafarja et al, Evolutionary population dynamics and grasshopper optimization approaches for feature selection problems, *Knowl. -Based Syst.* 145 (2018) 25–45.
- A.A. Heidari, S. Mirjalili, H. Faris, I. Aljarah, M. Mafarja, H. Chen, Harris hawks optimization: Algorithm and applications, *Futur. Gener. Comput. Syst.* (2019).
- M. Dorigo, M. Birattari, T. Stutzle, Ant colony optimization, *IEEE Comput. Intell. Mag.* 1 (4) (2006) 28–39.
- S. Mirjalili, A. Lewis, Adaptive gbest-guided gravitational search algorithm, *Neural Comput. Appl.* 25 (7–8) (2014) 1569–1584.
- S. Mirjalili, The ant lion optimizer, *Adv. Eng. Softw.* 83 (2015) 80–98.

24. C. Anitescu, E. Atroshchenko, N. Alajlan, T. Rabczuk, Artificial neural network methods for the solution of second order boundary value Problems, 59 (1) 2019 345–359.
25. A.I. Cohen, M. Yoshimura, A branch-and-bound algorithm for unit commitment, IEEE Trans. Power Appar. Syst 102 (2) (1983) 444–451.
26. [26] A. Yang X-s, New metaheuristic bat-inspired algorithm, in: Nature Inspired Cooperative Strategies for Optimization (NICSO 2010), Springer,2010, pp. 65–74.
27. R.Y.M. Nakamura, L.A.M. Pereira, K.A. Costa, D. Rodrigues, J.P. Papa, X.S. Yang, BBA: A binary bat algorithm for feature selection, Brazilian Symp.Comput. Graph. Image Process. (2012) 291–297.
28. D. Simon, Biogeography-based optimization, IEEE Trans. Evol. Comput. 12 (6) (2008) 702–713.
29. E. Rashedi, H. Nezamabadi-Pour, S. Saryazdi, BGSA: Binary gravitational search algorithm, Nat. Comput. 9 (3) (2010) 727–745.
30. Kamboj, V.K., Nandi, A., Bhadoria, A., Sehgal, S., "An intensify Harris Hawks optimizer for numerical and engineering optimization problems", Applied Soft Computing Journal, pp. 1-35, 2019.