

Numerical Computation of First Three Frequencies for Circular Plate with Transcendental Thickness

Neetu Singh, Vipin Saxena

Abstract: In the present work, a very important approach Rayleigh-Ritz method has been used to compute the first few frequencies of a circular plate. The boundary conditions of circular plate are considered as a clamped and simply-supported. Different types of thickness variation of circular plate have been considered by researchers and a vast numbers of numerical results are available in the literature but none of the researchers consider the transcendental thickness variation which has been considered in the present work. The type of circular plate is considered as isotropic plate and significant numerical computations have been done for finding first three frequencies by varying the order of approximation and also the taper parameter. In special cases, results have been compared for uniform, linearly varying and transcendental thickness variations of circular plate and computed result are presented in the form of tables and graphs.

Keywords: Rayleigh-Ritz, Natural Frequencies, Transcendent Thickness, Circular Plate, Taper Parameter, Boundary Conditions.

I. NOMENCLATURE

x',y' =Co-ordinate of a point on the plate
 t =Time
 w =Displacement
 ω =Angular frequency
 W =Defined by $W(x,y,t)=W(x,y) \cos \omega t$
 R =Domain occupied of the plate
 E =Young modulus of the material
 ρ =Density of the plate
 ν =Poisson ratio of the plate
 h =Thickness of circular plate
 x,y = Non-dimensional co-ordinates
 $\phi_j(x,y)$ =Basis function
 N =Order of approximation
 C_j =Constants
 a_{ij}, b_{ij} =Matrices ($N \times N$)
 h_0 =Thickness at a origin
 λ =Frequency

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α =Taper parameter

$f(x,y)$ =Function for thickness variation.

II. INTRODUCTION

The study of vibration of plates plays an important role in the design of aircraft, naval structure and also for the engineering design. The various research papers and books are available on the transverse vibrations of different types of plates, thickness variations and boundary conditions. Let us describe some of the important references related to the present work. Conway et.al [1] has obtained transverse vibration resonant frequencies of clamped tapered circular plated with taper parameter and boundary conditions. Leissa [2-8] is admirable source of information on survey vibration of different types of plates. This is one of the oldest references related of the circular plate. Celep [9] has computed and compared numerical results for use of free vibration of the circular plate by the Classical, Reissner and Mindlin theories for free vibration of two circular plates.

In the year 1980, Grossi and Laura [10] have done exhalative study for transverse vibrations of a circular plate with boundary conditions and linearly varying thickness. Gupta and Mishra [11] used Bessel function in mathematical modeling to obtain the numerical results for circular plate. All numerical results have been compared between two theories classical theories and shear theories. Leissa and Narita [12] have evaluated the natural frequencies of simply-supported circular plate with boundary conditions. Geloset.al. [13] have studied vibrations of circular plate with variable thicknesses which have been obtained by varying the thickness variation. Zhixin [14] has studied large deflection mathematics analysis of clamped circular plate with boundary conditions and in constant thickness and also consider the distribution of loads. Irie et.al. [15] have obtained stability and vibration of circular plate with uniformly varying thickness.

In the year 1986, Ficcaclenti and Laura [16] have investigated natural frequencies of circular plate of non-uniform variable thickness with rational conditions. Kim and Dickinson [17] examined the free transverse vibration of annular and circular thin sectional plates with boundary conditions. Mode shapes and frequencies have been obtained for circular and annular elastic plates computing the natural frequencies by Weisened [18]. Singh and Chakraverty [19] have used the Rayleigh-Ritz mechanism for figuring the frequencies and mode shapes with orthogonal polynomials for elliptic and circular plates.



In the year 1992 vibration of circular plate with stepped on non-homogenous elastic foundation were discussed by Wang [20]. Further Singh and Chakraverty [21] have discussed the first few natural frequencies and mode shapes of elliptic and circular plates with aspect, poisson ratio and different boundary conditions. All frequencies have been calculated by Rayleigh-Ritz method and compared with existing result in special cases. Yang [22] has investigated the vibration of circular plate with boundary condition and varying thickness and computed frequencies and mode shapes. Singh and Saxena [23-25] have used exponential, double linear thickness variation in the Rayleigh-Ritz technique for computation of frequencies, mode shapes and nodal radii with clamped and simply-supported boundary conditions. Roy [26] has computed eigen system by the use of Jacobi method. This method has investigated frequencies of different types of plates. Singh and Hassan [27] have determined numerical results by Rayleigh-Ritz approach with uniform thickness and boundary conditions. Gupta and Goyal [28] have used eigen function method classical plate theory for obtaining the numerical results of linearly tapered circular plate.

Huge quantities of research papers are available on vibration of circular plate with various types of thickness variation and different types of boundary conditions. The type of method may be numerical or approximate or analytical for different types of plates. Researchers take the thickness variation as linearly thickness, quadratic thickness, exponential thickness etc. Some important are references [29-31] and author computed natural frequencies, mode shapes for circular plate. Monterrubio [32] has solved eigen value problem and Rayleigh-Ritz approach for computation of positive and negative penalty parameter for simple condition system. Askari et. al. [33] studied hydro-elastic vibration of circular plate immersed in a liquid filled container with a free surface. Xing et. al. [34] have computed exact solution of circular cylindrical shells with classical boundary and obtained the frequencies based on Donnell-Mushtari shell theory.

In the year 2013, Baghani and Fereidoonzed [35] used the limit analysis theorem, variation iteration method on a circular plate with the simply- supported boundary condition. Wang et.al. [36] have also determined the fundamental frequency and mode shape of circular micro plates and numerical result determined by Kantorovich and shooting methods of a given problem. Szemela [37] has calculated approximation high frequencies of circular plate with clamped and simply-supported boundary conditions. Ilanko and Monterrubio [38] have discussed Rayleigh-Ritz method and applied for accurate results of different types of plates like circular, skew, rectangular, square and elliptic plates. Plaut [39] has studied large axisymmetric deflection of a circular plate by generalized Reissner theory. Abbasi et.al. [40] have also obtained semi-analytical solution for static analysis of functionally graded circular plates. This problem has been solved by two different types of methods, i.e solution method, and governing different equation method with various boundary conditions. Wang et.al [41] has computed uni-field solution of circular, annular and sector plate with general boundary conditions.

On the basis of above review of literature, it is recognized by the authors that transcendental thickness variation has not been studied by the researchers although it satisfies the clamped, simply-supported boundary conditions for circular

plate. The main objective of this work is to compute first three frequencies for circular plate with two types of boundary conditions for the transcendental thickness variation. Computed results by varying taper parameter have been compared with the existing result. Convergence of computed results has been expressed by taking significant order of approximations.

III. METHOD OF SOLUTION

Rayleigh-Ritz method is a proximate method used for computation of frequencies for different types of plates. Let us consider a circular plate which is defined in the domain R and represented in following figure1. For free vibrations of circular plate, the displacement ω at time t and point (x,y) is given below.

$$w(x, y, t) = W(x, y) \cos \omega t \tag{1}$$

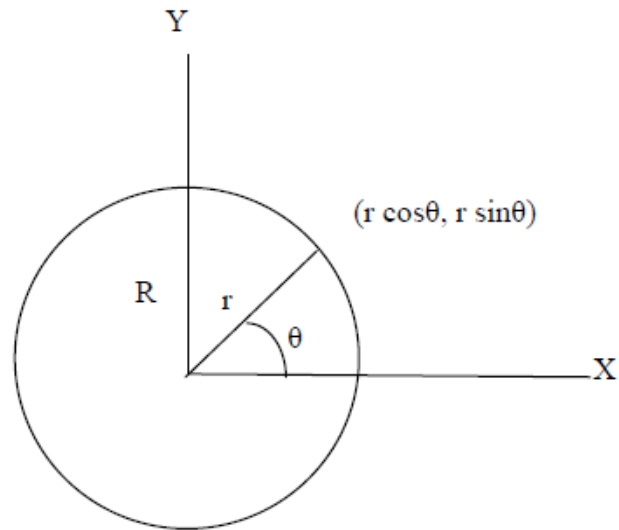


Fig. 1. Representation Circular plate

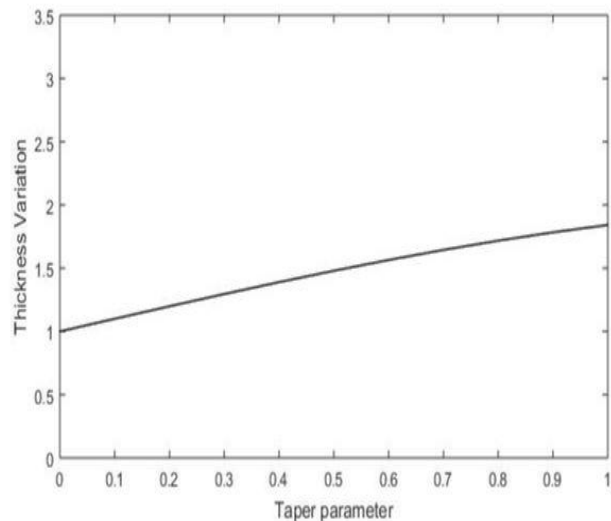


Fig. 2. Circular plate with variable thickness

Where $W(x,y)$ is the maximum displacement at point (x,y) and ω is frequency at the time t of the given plate.

Maximum strain energy and kinetic energy are presented by following equations

$$U_{\max} = \frac{1}{2} \iint_R \frac{Eh^3}{12(1-\nu^2)} \left[\left(\frac{d^2W}{dr^2} + \frac{1}{r} \frac{dW}{dr} \right)^2 + 2(1-\nu) \frac{d^2W}{dr^2} \left(\frac{1}{r} \frac{dW}{dr} \right) \right] r dr \quad (2)$$

$$T_{\max} = \frac{1}{2} \omega^2 \iint_R \gamma h W^2 r dr \quad (3)$$

After solving equation (2) and (3), one can obtain Rayleigh quotient which is given by

$$\omega^2 = \frac{E \iint_R \left[\left(\frac{d^2W}{dr^2} + \frac{1}{r} \frac{dW}{dr} \right)^2 + 2(1-\nu) \frac{d^2W}{dr^2} \left(\frac{1}{r} \frac{dW}{dr} \right) \right] r dr}{12(1-\nu^2) \iint_R \gamma h W^2 r dr} \quad (4)$$

Where r is the radial distance, γ is the distance of the material of the plate.

The main aim of the Rayleigh-Ritz approach is to optimize the Rayleigh quotient. The two types of boundary condition which are considered in the present work are described

Clamped Boundary Condition

$$W = 0$$

$$\frac{dW}{dr} = 0 \quad (5)$$

Simply-Supported Boundary Condition

$$W = 0$$

$$\frac{d^2W}{dr^2} + \left(\frac{\nu}{r} \right) \frac{dW}{dr} = 0 \quad (6)$$

Minimization of the equation (4) leads to determination of natural frequencies i.e λ . Let us consider an approximate solution with displacement ω at time t is given below.

$$W(x, y) = \sum_{j=1}^N C_j \phi_j(x, y) \quad (7)$$

Where C_j 's are the constants $\phi_j(x, y)$ are taken as basis function satisfying the boundary condition and substituting in (4) for minimization of ω^2 over the constants C_j 's, one can lead to

$$\sum_{j=1}^N (a_{ij} - \lambda^2 b_{ij}) C_j = 0 \quad (8)$$

Equation (8) can be rewritten as

$$\sum_{j=1}^N (a_{ij} - \lambda^2 b_{ij}) C_j = 0 \quad j = 1, 2, \dots, N \quad (9)$$

Where

$$a_{ij} = \iint_R f^3 \left(\phi_i \phi_j + \nu (\phi_i \phi_j + \phi_i \phi_j) + \frac{1}{\rho} \phi_i \phi_j \right) d\rho \quad (10)$$

$$b_{ij} = \iint_R f \phi_i \phi_j \rho d\rho \quad (11)$$

$$\lambda^2 = a^4 \omega^2 \rho \quad (12)$$

$$\lambda^2 = \frac{12a^2 \rho (1-\nu^2) \omega^2}{Eh_0} \quad (13)$$

f = thickness variation

Now consider the following non-dimensional variables and parameter $\rho = r/a$, $H = h/h_0$ and h_0 is the thickness of the circular plate at origin and now substituted equation (10) and (11) in equation (9) and solve above equation for λ^2 with constant C_j 's and $j=1,2,3,\dots,N$. The basis function is described below.

$$\phi_i = u^{s+i-1} \quad i = 1, 2, \dots, N \quad (14)$$

$$\phi_i = (1-\rho^2)^{s+i-1} \quad (15)$$

Where $S=1$ leads to frequencies related to simply supported and $S=2$ is for the clamped circular plate. Transcendental thickness variation is defined by

$$f(\rho) = 1 + \sin \alpha \rho \quad (16)$$

Where f is considered as one parameter and ρ controls radius of the circular plate and S is taken as taper parameter. Solution of a_{ij} and b_{ij} through generalized Jacobi method contains (17) the following type of integrals.

$$\iint_R \sin^m x \cos^n x dx = \frac{\frac{m+1}{2} \frac{n+1}{2}}{2 \frac{m+n+2}{2}} \quad (17)$$

IV. NUMERICAL RESULTS AND DISCUSSION

For computation of results, different parameters have been considered and discussed below in brief:

- 1) Taper parameter which is controlling the entire thickness variation of circular plate is considered from 0 to 1 at the interval difference of 0.1.
- 2) The value of poisson ratio is fixed for isotropic plate and taken as 0.3.
- 3) Approximation order N is taken from 1 to 10 for fast convergence of results.
- 4) The parameters which are controlling the different types of boundary conditions of plate, taken as 2 for clamped circular plate and 1 for simply-supported circular plates.
- 5) All the involved integral in computation are in closed form and computed by equation [17] by the use of Rayleigh-Ritz method, an eigen value problem has been solved by generalized Jacobi method. The first three frequencies are represented in table [1] for the circular plate on different boundary conditions of plate. Similar results are defined in the figure [3] and [4] for simply-supported and clamped plates respectively.

In all cases, it is observed that due to increase of taper parameter, the frequencies are increasing. This is because of stiffness of circular plate is increasing by the increasing of taper parameter. When taper parameter is zero then it leads to results related to the uniform thickness variation of circular plates.

All the computed results have been compared up-to the level of five significant digits. These calculated results ensure the convergence of results by varying order of approximate i.e. N and represented below in table [2]. From the table, it is found that results are matching upto five significant digits. In the case of uniform thickness variation the computed results are analyzed with available results and found that results are matching with the existing results.

Table I. First Three Frequencies for clamped and simply-supported circular plate with transcendental thickness variation

Tape-r Para-mete r	Simple-Support Boundary Condition (S=1)			Clamped Boundary Condition (S=2)		
	λ_1	λ_2	λ_3	λ_1	λ_2	λ_3
α						
0	4.935 1	29.72	74.15 6	10.21 5	39.77 1	89.10 4
0.1	5.633 7	33.92 4	84.64 3	11.67 7	45.41 4	101.7 2
0.2	6.263 5	37.70 5	94.06 1	13.02 7	50.51 7	113.0 8
0.3	6.838 5	41.14	102.6	14.26 8	55.18	123.4 2
0.4	7.367 2	44.27 7	110.4	15.43 6	59.46 2	132.8 7
0.5	7.854 7	47.14 7	117.5 1	16.53 2	63.40 3	141.5 3
0.6	8.304 8	49.77 3	124.0 1	17.56 2	67.03	149.4 5
0.7	8.720 2	52.17 3	129.9 3	18.53 1	70.36 5	156.7
0.8	9.103	54.36 3	135.3 2	19.44 2	73.42 5	163.3 1
0.9	9.455	56.35 7	140.2 2	20.29 8	76.22 8	169.3 3
1	9.777 7	58.17	144.6 5	21.10 2	78.79	174.7 9

Table II. Convergence of the Results for Circular Plate ($\alpha=0, S=1, 2$)

Reference	λ_1	λ_2	λ_3
Present Result	10.215	39.771	89.104
[10]	10.217	34.94	-
[12]	13.9	48.48	102.77
[13]	10.21	-	-
[19]	10.215	34.877	-
[20]	10.26	-	-
[23]	10.216	-	-
[24]	10.216	39.771	89.104
[25]	10.216	39.771	89.104
[29]	10.216	34.877	-

Table III. Comparisons of the Results for Uniform Circular Plate

N	Simply-Supported Circular Plate (S=1)			Clamped Circular Plate (S=2)		
	λ_1	λ_2	λ_3	λ_1	λ_2	λ_3
1	5.5856	-	-	10.327	-	-
2	4.9405	46.706	-	10.217	43.058	-
3	4.9351	30.503	163.23	10.215	39.921	109.59
4	4.9351	29.736	81.896	10.215	39.773	91.157
5	4.9351	29.72	74.683	10.215	39.771	89.203
6	4.9351	29.72	74.171	10.215	39.771	89.106
7	4.9351	29.72	74.156	10.215	39.771	89.104
8	4.9351	29.72	74.156	10.215	39.771	89.104
9	4.9351	29.72	74.156	10.215	39.771	89.104
10	4.9351	29.72	74.156	10.215	39.771	89.104

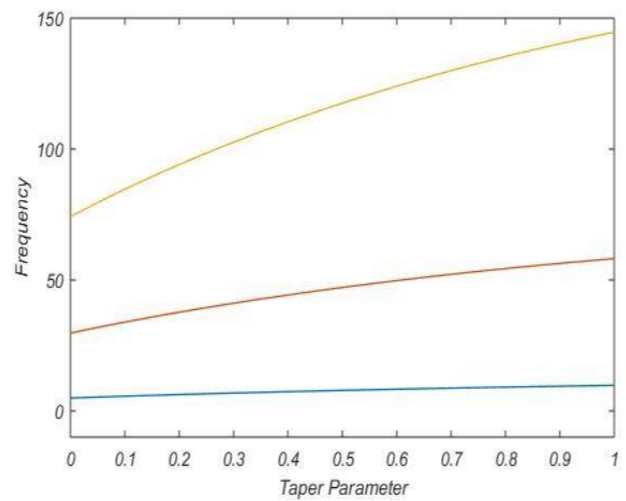


Fig.3. First three frequencies for simply-supported circular plate(S=1)

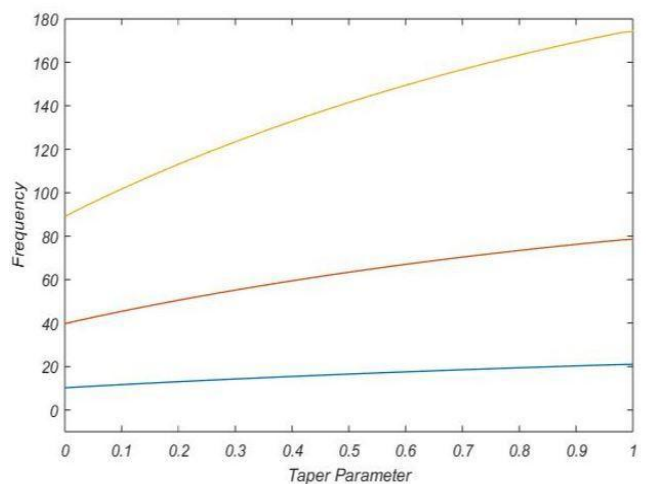


Fig. 4. First three frequencies for clamped circular plate (S=2)

V. CONCLUSION

The current problem investigated the natural frequencies through well known Rayleigh-Ritz technique with two boundary conditions and one taper parameter on the circular plate. When the taper parameter α is increasing from zero to one at the interval (0.1), then frequencies are also increasing of circular plate. The natural frequency has been computed for circular plate upto the five significant digits. Comparison with available result in special cases confirms that accuracy is comparable with best known result.

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