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Skolem Mean Labeling of Six Star Graphs $K_{1,\eta_1} \cup K_{1,\eta_2} \cup K_{1,\eta_3} \cup K_{1,\eta_4} \cup K_{1,\tau_1} \cup K_{1,\tau_2}$ where $0 \le \left|\sum_{i=1}^{2} \tau_i - \sum_{i=1}^{4} \eta_i\right| \le 1$

J. Vinolin, D. S. T. Ramesh, S. Athisayanathan

Abstract*: Skolem mean labeling of Six star graphs $G = K_{1,\eta_1} \cup K_{1,\eta_2} \cup K_{1,\eta_3} \cup K_{1,\eta_4} \cup K_{1,\tau_1} \cup K_{1,\tau_2}$ under the condition $0 \le \left|\sum_{i=1}^{2} \tau_i - \sum_{i=1}^{4} \eta_i\right| \le 1$ is proved in this paper with clear explanation and examples. The aim of this paper is to discuss the 4, 2 partition of the Six star graph and to find the existence of the skolem mean labeling for the Six star graph under the given conditions with clear step by step proof.

Keywords*: mean graphs, skolem mean labeling, skolem mean graphs, n – star graphs.

I. INTRODUCTION

The term skolem mean labeling was first introduced by V.Balaji et. al.[2] in the year 2007. In that paper [2] he gave the definition of skolem mean labeling for the first time and also some basic properties for a graph to be a skolem mean graph. Some of the most important properties are (i) If G is a graph with n vertices and m edges then G is said to be a skolem mean graph only if n ≥ m + 1, (ii) The graphs which satisfies the condition n ≥ m + 1 are paths and star graphs (iii) Every path is skolem mean (iv)G = $K_{1,n}$ where n ≥ 4 is not a skolem mean graph.

II. PRELIMINARIES

Definition 1: A graph labeling is the assignment of labels to edges and also vertices of a graph or simply either edges or vertices of a graph only by integers.

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We define the skolem mean labeling of a graph G with vertex set V and edge set E of order n and size m as follows:

Definition 2: The vertex labeling f : V⟶{1, 2, …, n} and the induced edge labeling f^*: E⟶{2,3…, n} is a skolem mean labeling if both f and f^* are one – one functions such that f^*(e = uv) = [f(u)+f(v)] / 2 if the sum of the vertex label u and v is even and [f(u)+f(v)+1] / 2 if the sum of the vertex label u and v is odd.

III. MAIN RESULT

Theorem 1: Six star graph $G = K_{1,\eta_1} \cup K_{1,\eta_2} \cup K_{1,\eta_3} \cup K_{1,\eta_4} \cup K_{1,\tau_1} \cup K_{1,\tau_2}$ where $\eta_1 \le \eta_2 \le \eta_3 \le \eta_4$ and $\tau_1 \le \tau_2$ is skolem mean graph if $0 \le \left|\sum_{i=1}^{2} \tau_i - \sum_{i=1}^{4} \eta_i\right| \le 1$.

Proof: Take

$N_k = \sum_{i=1}^{k} \eta_k ; 1 \le k \le 4;\ T_k = \sum_{i=1}^{k} \tau_k ; 1 \le k \le 2.$

That is, $N_1 = \eta_1; N_2 = \eta_1 + \eta_2; N_3 = \eta_1 + \eta_2 + \eta_3;$ $N_4 = \eta_1 + \eta_2 + \eta_3 + \eta_4$ and $T_1 = \tau_1; T_2 = \tau_1 + \tau_2$.

Consider the graph

$G = K_{1,\eta_1} \cup K_{1,\eta_2} \cup K_{1,\eta_3} \cup K_{1,\eta_4} \cup K_{1,\tau_1} \cup K_{1,\tau_2}$ having $V = V_1 \cup V_2 \cup V_3 \cup V_4 \cup V_5 \cup V_6$ as its set of vertices of G where $V_k = \{v_{k,i} : 0 \le i \le \eta_k\}$ for $1 \le k \le 4$,

$V_5 = \{v_{5,i} : 0 \le i \le \tau_1\}, V_6 = \{v_{6,i} : 0 \le i \le \tau_2\}$. Let

$E = \cup_{i=1}^{4}\{ v_{i,0}v_{i,j} : 1 \le j \le \eta_i\}$

$\cup_{i=1}^{2}\{ v_{4+i,0}v_{4+i,j} : 1 \le j \le \tau_i\}$ be the set of edges of G.

The condition

$0 \le \left|\sum_{i=1}^{2} \tau_i - \sum_{i=1}^{4} \eta_i\right| \le 1$

$\Rightarrow \eta_1 + \eta_2 + \eta_3 + \eta_4 \le \tau_1 + \tau_2 \le \eta_1 + \eta_2 + \eta_3 + \eta_4 + 1$

$\Rightarrow N_4 \le T_2 \le N_4 + 1$

That is, there are two cases viz. $T_2 = N_4$ and $T_2 = N_4 + 1$.

Case A: $T_2 = N_4$.

G has $N_4 + T_2 + 6 = 2N_4 + 6$ vertices and $N_4 + T_2 = 2N_4$ edges. The vertex labeling

$f : V \to \{1, 2, 3, \ldots, N_4 + T_2 + 6 = 2N_4 + 6\}$ given as:

$f(v_{1,0}) = 1;\ f(v_{2,0}) = 2;\ f(v_{3,0}) = 3;\ f(v_{4,0}) = 4;$

$f(v_{5,0}) = N_4 + T_2 + 5 = 2N_4 + 5$

$f(v_{6,0}) = N_4 + T_2 + 6 = 2N_4 + 6$

$f(v_{1,i}) = 2i + 4$	$1 \le i \le \eta_1$
$f(v_{2,i}) = 2N_1 + 2i + 4$	$1 \le i \le \eta_2$
$f(v_{3,i}) = 2N_2 + 2i + 4$	$1 \le i \le \eta_3$
$f(v_{4,i}) = 2N_3 + 2i + 4$	$1 \le i \le \eta_4$
$f(v_{5,i}) = 2i + 3$	$1 \le i \le \tau_1$
$f(v_{6,i}) = 2T_1 + 2i + 3$	$1 \le i \le \tau_2$

Their induced labels for edges are as follows:

The induced label of $v_{1,0}v_{1,i}$ is 3 + *i* where $1 \le i \le \eta_1$ (edge labels are 4, 5,…., η_1 + 3 = N_1 + 3), $v_{2,0}v_{2,i}$ is N_1 + 3 + *i* for $1 \le i \le \eta_2$ (edge labels are N_1 + 4, N_1 + 5, …, $N_1 + \eta_2$ + 3 = N_2 + 3), $v_{3,0}v_{3,i}$ is N_2 + 4 + *i* for $1 \le i \le \eta_3$ (edge labels are N_2 + 5, N_2 + 6, …, $N_2 + \eta_3$ + 4 = N_3 + 4), $v_{4,0}v_{4,i}$ is N_3 + 4+ *i* for $1 \le i \le \eta_4$ (edge labels are N_3 + 5, N_3 + 6, …, $N_3 + \eta_4$ + 4 = N_4 + 4), $v_{5,0}v_{5,i}$ is N_4 + 4 + *i* for $1 \le i \le \tau_1$ (edge labels are N_4 + 5, N_4 + 6, …, $N_4 + \tau_1$ + 4), $v_{6,0}v_{6,i}$ is $N_4 + \tau_1$ + 5 + *i* for $1 \le i \le \tau_2$ (edge labels are $N_4 + \tau_1$ + 6, $N_4 + \tau_1$ + 7, …, $N_4 + \tau_1 + \tau_2$ + 5 = $N_4 + T_2$ + 5 = $2N_4$ + 5) .

The labels of edges induced by the labels of vertices of graph G are distinct.

This shows the existence of skolem mean labeling for G.

Case B: $T_2 = N_4 + 1$.

G has $N_4 + T_2 + 6 = 2N_4 + 7$ vertices and $N_4 + T_2 = 2N_4$ +1 edges. The vertex labeling

$f: V \to \{1, 2, 3, \ldots, N_4 + T_2 + 6 = 2N_4 + 7\}$ given as:

$f(v_{1,0}) = 1;\ f(v_{2,0}) = 2;\ f(v_{3,0}) = 3;\ f(v_{4,0}) = 4;$

$$f(v_{5,0}) = N_4 + T_2 + 5 = 2N_4 + 6$$
$$f(v_{6,0}) = N_4 + T_2 + 6 = 2N_4 + 7$$
$$f(v_{1,i}) = 2i + 4 \qquad 1 \le i \le \eta_1$$
$$f(v_{2,i}) = 2N_1 + 2i + 4 \qquad 1 \le i \le \eta_2$$
$$f(v_{3,i}) = 2N_2 + 2i + 4 \qquad 1 \le i \le \eta_3$$
$$f(v_{4,i}) = 2N_3 + 2i + 4 \qquad 1 \le i \le \eta_4$$
$$f(v_{5,i}) = 2i + 3 \qquad 1 \le i \le \tau_1$$
$$f(v_{6,i}) = 2T_1 + 2i + 3 \qquad 1 \le i \le \tau_2$$

Their induced labels for edges are as follows:

The induced label of $v_{1,0}v_{1,i}$ is 3 + *i* where $1 \le i \le \eta_1$ (edge labels are 4, 5,…., η_1 + 3 = N_1 + 3), $v_{2,0}v_{2,i}$ is N_1 + 3 + *i* for $1 \le i \le \eta_2$ (edge labels are N_1 + 4, N_1 + 5, …, $N_1 + \eta_2$ + 3 = N_2 + 3), $v_{3,0}v_{3,i}$ is N_2 + 4 + *i* for $1 \le i \le \eta_3$ (edge labels are N_2 + 5, N_2 + 6, …, $N_2 + \eta_3$ + 4 = N_3 + 4), $v_{4,0}v_{4,i}$ is N_3 + 4 + *i* for $1 \le i \le \eta_4$ (edge labels are N_3 + 5, N_3 + 6, …, $N_3 + \eta_4$ + 4 = N_4 + 4), $v_{5,0}v_{5,i}$ is N_4 + 4 + *i* for $1 \le i \le \tau_1$ (edge labels are N_4 + 5, N_4 + 6, …, $N_4 + \tau_1$ + 4), $v_{6,0}v_{6,i}$ is $N_4 + \tau_1$ + 5 + *i* for $1 \le i \le \tau_2$ (edge labels are $N_4 + \tau_1$ + 6, $N_4 + \tau_1$ + 7, …, $N_4 + \tau_1 + \tau_2$ + 5 = $N_4 + T_2$ + 5 = $2N_4$ + 6) .

The labels of edges induced by the labels of vertices of graph G are distinct.

This shows the existence of skolem mean labeling for G.

We illustrate the above two cases with the following Six star graphs

Fig.1. is an illustartion of Case A where $T_2 = N_4$.

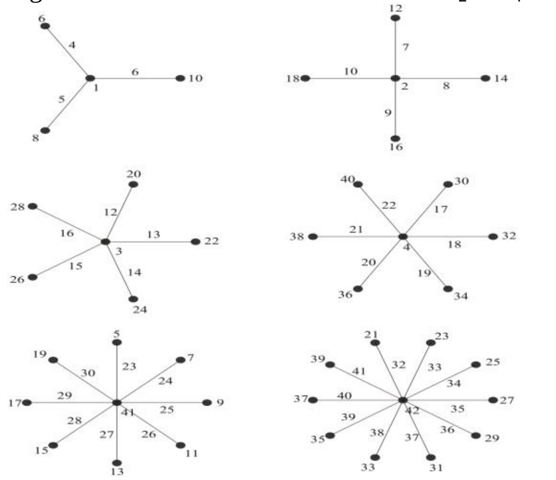

Fig. 1. G = $K_{1,3} \cup K_{1,4} \cup K_{1,5} \cup K_{1,6} \cup K_{1,8} \cup K_{1,10}$

In this graph , $\eta_1 + \eta_2 + \eta_3 + \eta_4$ = 3 + 4 + 5 + 6 = 18 and $\tau_1 + \tau_2$ = 8 + 10 = 18 = N_4.

Fig.2. is an illustartion of Case A where $T_2 = N_4 + 1$.

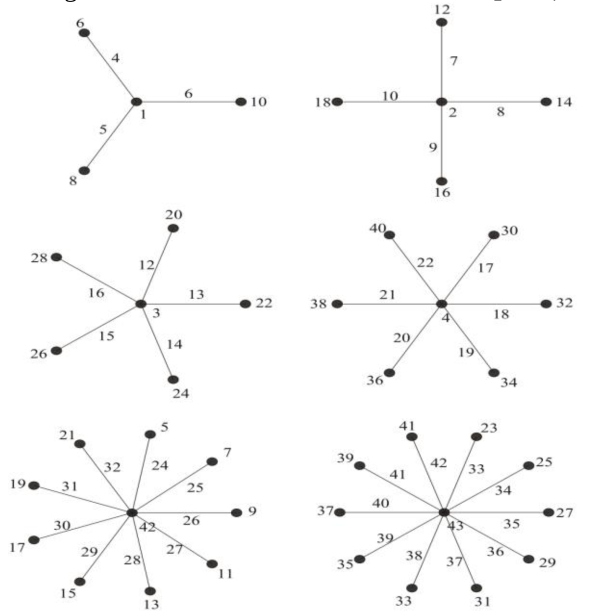

Fig. 1. G = $K_{1,3} \cup K_{1,4} \cup K_{1,5} \cup K_{1,6} \cup K_{1,9} \cup K_{1,10}$

In this graph , $\eta_1 + \eta_2 + \eta_3 + \eta_4$ = 3 + 4 + 5 + 6 = 18 and $\tau_1 + \tau_2$ = 9 + 10 = 19 = N_4+1.

IV. CONCLUSION

Skolem mean labeling of a six star graph $G = K_{1,\eta_1} \cup K_{1,\eta_2} \cup K_{1,\eta_3} \cup K_{1,\eta_4} \cup K_{1,\tau_1} \cup K_{1,\tau_2}$ with a partition of six into 4, 2 is discussed in this paper . We mainly discussed the two cases $T_2 = N_4$ and $T_2 = N_4 + 1$ and gave the labeling which satisfies the condition of skolem mean labeling which exists for graph G. We are in further research to find upto how many cases the graph G will allow skolem mean labeling.

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