

Numerical Modelling and Validation of Railway Vehicle

Bouhlaï Laila, Touati Mohammed, Lamdouar Nouzha

Abstract: The objective of this study was to establish a theoretical model regarding the dynamic behaviour of the railway vehicle in the vertical plan while travelling on a track with irregularities. In this article the vehicle is modelled by seven bi-dimensional rigid elements respectively representing: the body, tow bogies and four wheels. The equations system was numerically integrated by the Newmark's β and Wilson θ method. The model developed was subjected to a validation process based on the comparison between numerically derived acceleration spectra and experimental data.

Keywords: Railway vehicle, theoretical model, dynamic behaviour, track with irregularities, validation.

I. INTRODUCTION

As the speed and intensity of rail traffic increases, a detailed numerical model becomes needed to predict the effect of track irregularities on the behavior of a railway vehicle. The theoretical models, in spite of their complexity, start from a series of simplifying hypotheses consequently the results can deviate from reality. The validation of the theoretical model through experimental measurements remains an essential step. The validation process is based on representative comparisons between experimental results performed on the vehicle and those obtained from numerical simulations.

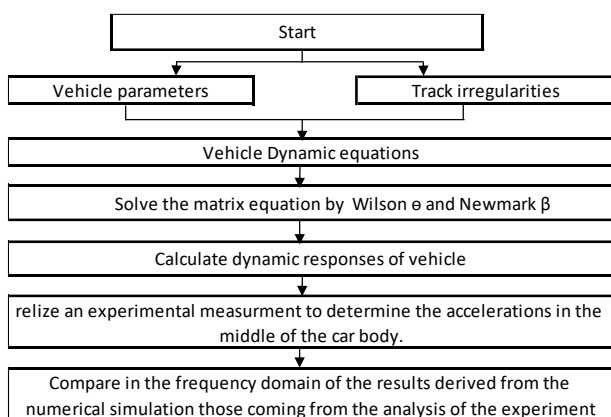


Fig.1. Process of validation

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II. THE THEORETICAL MODEL OF THE VEHICLE

Assuming that the load is distributed symmetrically on both rails, focusing on the vertical acceleration of the vehicle, a 2D model is well adapted to represent it.

The vehicle is modelled through body, two bogies and four wheelsets, connected by spring and damping systems. Hence, the model has 10 DOFs, consisting of vertical and pitching motion of the vehicle body and bogies and; vertical motion of the wheelsets

M_c and J_c represent respectively the mass and rotational inertia of car body. M_{t1} and J_{t1} stand for the mass and rotational inertia of bogie 1 while M_{t2} and J_{t2} belong to bogie 2. l_t and l_c are the half distance of two wheels and two bogies, respectively. k_{t1} , c_{t1} and k_{t2} , c_{t2} are the stiffness and damping of bogies 1 and 2. [1]- [2]-[3]

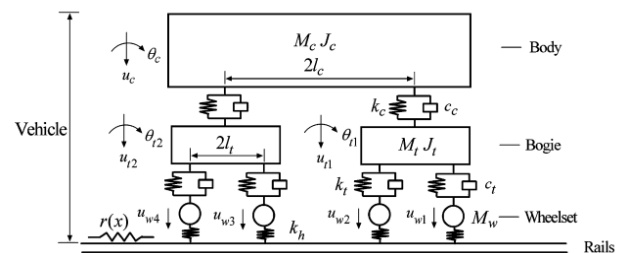


Fig. 2. Model 2D of vehicle [2]

A. The motion equations of vehicle

The set of DOF (degrees of freedom) represented in Figure 1 is governed by the following system of motion equations:

Motion equation of car body

$$M_c \ddot{z}_c + 2k_2 z_c + 2c_2 \dot{z}_c - k_2 z_{t1} - k_2 z_{t2} - c_2 \dot{z}_{t1} - c_2 \dot{z}_{t2} = -M_c g \quad (1)$$

$$J_c \ddot{\beta}_c + 2k_2 l_c^2 \beta_c + 2c_2 l_c^2 \dot{\beta}_c - k_2 l_c z_{t1} + k_2 l_c z_{t2} - c_2 l_c \dot{z}_{t1} + c_2 l_c \dot{z}_{t2} = 0 \quad (2)$$

Motion equation of bogie 1

$$J_c \ddot{\beta}_c + 2k_2 l_c^2 \beta_c + 2c_2 l_c^2 \dot{\beta}_c - k_2 l_c z_{t1} + k_2 l_c z_{t2} - c_2 l_c \dot{z}_{t1} + c_2 l_c \dot{z}_{t2} = 0 \quad (3)$$

$$J_c \ddot{\beta}_c + 2k_2 l_c^2 \beta_c + 2c_2 l_c^2 \dot{\beta}_c - k_2 l_c z_{t1} + k_2 l_c z_{t2} - c_2 l_c \dot{z}_{t1} + c_2 l_c \dot{z}_{t2} = 0 \quad (4)$$

Motion equation of bogie 2

$$J_c \ddot{\beta}_c + 2k_2 l_c^2 \beta_c + 2c_2 l_c^2 \dot{\beta}_c - k_2 l_c z_{t1} + k_2 l_c z_{t2} - c_2 l_c \dot{z}_{t1} + c_2 l_c \dot{z}_{t2} = 0 \quad (5)$$

$$J_c \ddot{\beta}_c + 2k_2 l_c^2 \beta_c + 2c_2 l_c^2 \dot{\beta}_c - k_2 l_c z_{t1} + k_2 l_c z_{t2} - c_2 l_c \dot{z}_{t1} + c_2 l_c \dot{z}_{t2} = 0 \quad (6)$$

Motion equation of wheel

$$J_c \ddot{\beta}_c + 2k_2 l_c^2 \beta_c + 2c_2 l_c^2 \dot{\beta}_c - k_2 l_c z_{t1} + k_2 l_c z_{t2} - c_2 l_c \dot{z}_{t1} + c_2 l_c \dot{z}_{t2} = 0 \quad (7)$$

$$J_c \ddot{\beta}_c + 2k_2 l_c^2 \beta_c + 2c_2 l_c^2 \dot{\beta}_c - k_2 l_c z_{t1} + k_2 l_c z_{t2} - c_2 l_c \dot{z}_{t1} + c_2 l_c \dot{z}_{t2} = 0 \quad (8)$$

$$J_c \ddot{\beta}_c + 2k_2 l_c^2 \beta_c + 2c_2 l_c^2 \dot{\beta}_c - k_2 l_c z_{t1} + k_2 l_c z_{t2} - c_2 l_c \dot{z}_{t1} + c_2 l_c \dot{z}_{t2} = 0 \quad (9)$$

$$J_c \ddot{\beta}_c + 2k_2 l_c^2 \beta_c + 2c_2 l_c^2 \dot{\beta}_c - k_2 l_c z_{t1} + k_2 l_c z_{t2} - c_2 l_c \dot{z}_{t1} + c_2 l_c \dot{z}_{t2} = 0 \quad (10)$$

The system of equations can be written in the matrix form:

$$M\ddot{X} + C\dot{X} + KX = [F(t)] \quad (11)$$

The displacement vector X, the forces vector F(t), the mass matrix M, the stiffness matrix K and the damping matrix C are defined by:

$$X = \begin{bmatrix} Z_c \\ \beta_c \\ Z_{t1} \\ \beta_{t1} \\ Z_{t2} \\ \beta_{t2} \end{bmatrix} \quad M = \begin{bmatrix} M_c & 0 & 0 & 0 & 0 & 0 \\ 0 & J_c & 0 & 0 & 0 & 0 \\ 0 & 0 & M_{t1} & 0 & 0 & 0 \\ 0 & 0 & 0 & \beta_{t1} & 0 & 0 \\ 0 & 0 & 0 & 0 & M_{t2} & 0 \\ 0 & 0 & 0 & 0 & 0 & \beta_{t2} \end{bmatrix}$$

$$K = \begin{bmatrix} 2k_2 & 0 & -k_2 & 0 & -k_2 & 0 \\ 0 & 2k_2 l_c^2 & -k_2 l_c & 0 & k_2 l_c & 0 \\ 0 & 0 & 2k_1 + k_2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2k_1 l t^2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2k_1 + k_2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 2k_1 l t^2 \end{bmatrix}$$

$$C = \begin{bmatrix} 2C_2 & 0 & -C_2 & 0 & -C_2 & 0 \\ 0 & 2C_2 l_c^2 & -C_2 l_c & 0 & C_2 l_c & 0 \\ 0 & 0 & 2C_1 + C_2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2C_1 l t^2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2C_1 + C_2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 2C_1 l t^2 \end{bmatrix}$$

B. Track irregularities

The vibrations of the vehicle were mainly excited by defects that may appear on the wheels and / or the rails during their manufacture or use. It was assumed in this study that the wheel was perfect and that the excitement source represented random irregularities on the railway line. These track irregularities were registered in Morocco by a measuring car called EM120. [4]



Fig. 3. Moroccan geometry measuring car EM120

III. NUMERICAL SOLUTION OF EQUATION

• Newmark β method

In this method, the following assumptions for motion and velocity vectors were used:

$$\dot{u}_{t+\Delta t} = \dot{u}_t + [(1 - \delta)\dot{u}_t + \delta\ddot{u}_{t+\Delta t}]\Delta t \quad (11)$$

$$u_{t+\Delta t} = u_t + \dot{u}_t\Delta t + \left[\left(\frac{1}{2} - \alpha\right)\ddot{u}_t + \alpha\ddot{u}_{t+\Delta t}\right]\Delta t^2 \quad (12)$$

or $\Delta t = t(n + 1) - t(n)$, α and δ were parameters that could be determined in order to obtain stable and accurate integration. $\delta = 1/2$, $\alpha = 1/4$ brings unconditional stability. To solve the displacement, velocities and accelerations at time $t + \Delta t$ the equilibrium equations were evaluated at the same time $t + \Delta t$.

$$M\ddot{u}_{t+\Delta t} + C\dot{u}_{t+\Delta t} + Ku_{t+\Delta t} = r_{t+\Delta t} \quad (13)$$

• Wilson θ method

In Wilson's method θ , we assumed a linear variation of the acceleration of a time t at a time $t + \theta\Delta t$, where $\theta > 1$. If the unconditional stability of the method is required, θ must be greater than or equal to equal to 1.37; thus, we usually use $\theta = 1.40$

$$\ddot{u}_{t+\tau} = \ddot{u}_t + \frac{\tau}{\theta\Delta t}(\ddot{u}_{t+\theta\Delta t} - \ddot{u}_t) \quad (14)$$

The algorithm for solving both methods has been programmed with the MATLAB software.

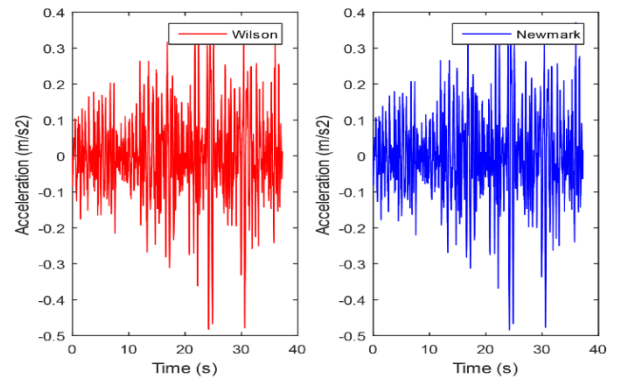


Fig.4. Vertical acceleration of car body

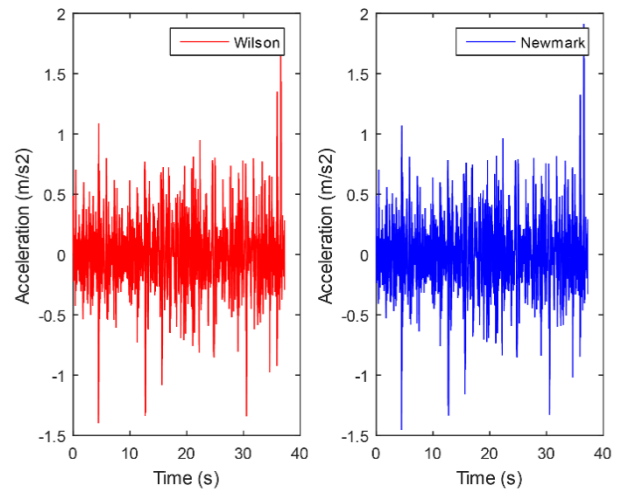


Fig.5. Vertical acceleration of bogie

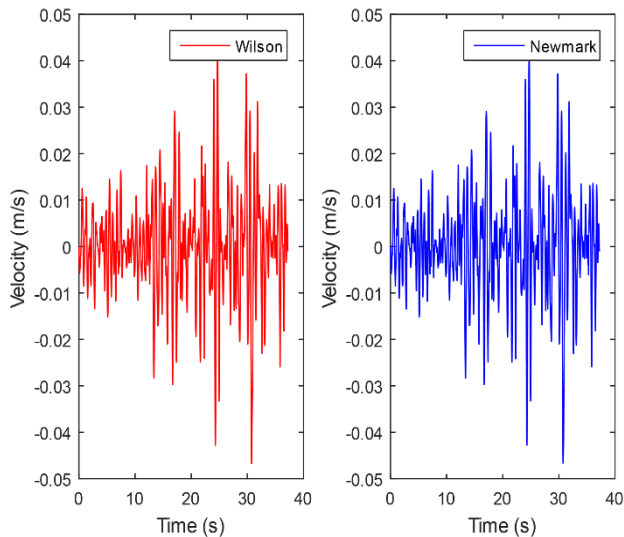


Fig.6. Vertical velocity of car body

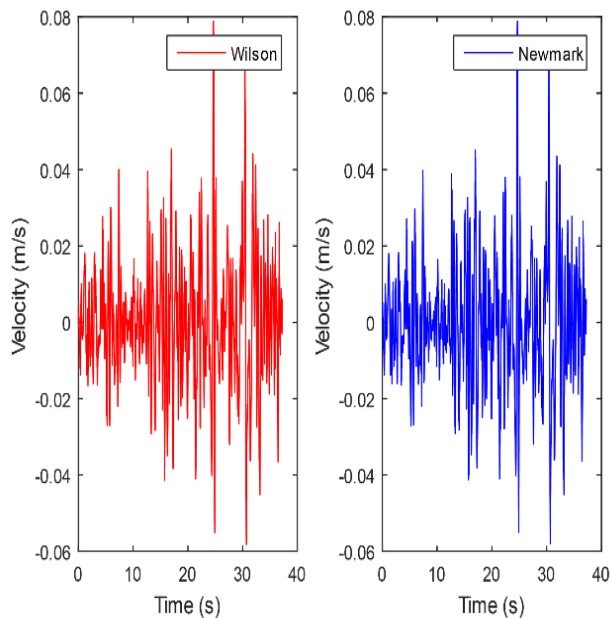


Fig.7. Vertical velocity of bogie

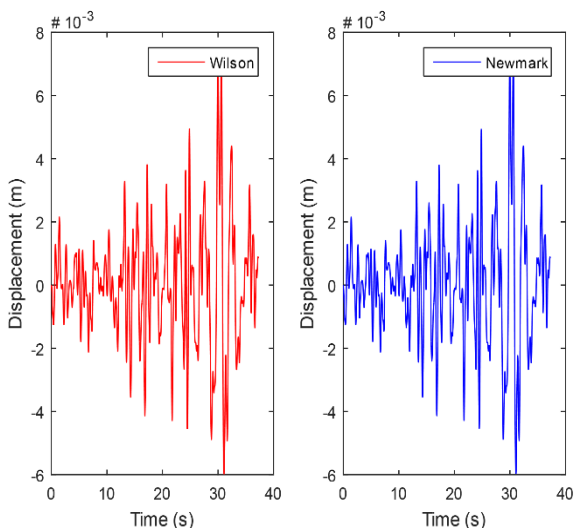


Fig.8. Vertical displacement of car body

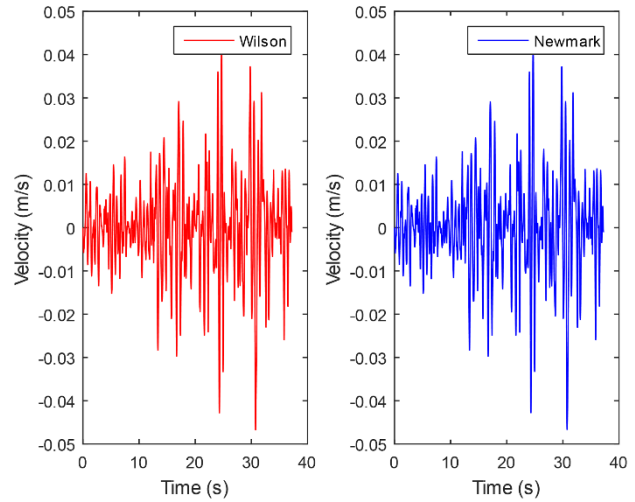


Fig.9. Vertical displacement of bogie

It appears that vertical acceleration, velocities and movements at the center of the body and the bogie, calculated by the Wilson θ and Newmark β method, show similar results

IV. EXPERIMENTAL DETERMINATIONS

To validate theoretical models of railway vehicles, an experimental measurement was realized to determine accelerations in the middle of the car body. Experimental measurements were made on the Casablanca-Marrakech railway line between PK236 and PK237

The speed of the vehicle measured in this area was constant and equal to 94km/h. The system used for the experimental measurements included components of the acceleration measurement system, data acquisition and processing system and a GPS receiver for monitoring and recording speed of the vehicle.



Figure 10 -11: Experimental equipment in vehicle

V. VALIDATION OF THE VEHICLE THEORETICAL MODEL

The purpose of this chapter was to validate the theoretical model developed in this study.

The validation process require the comparison in the frequency domain of the results derived from numerical simulation programs of the vehicle dynamic behaviour and those obtained from the experiment analysis.

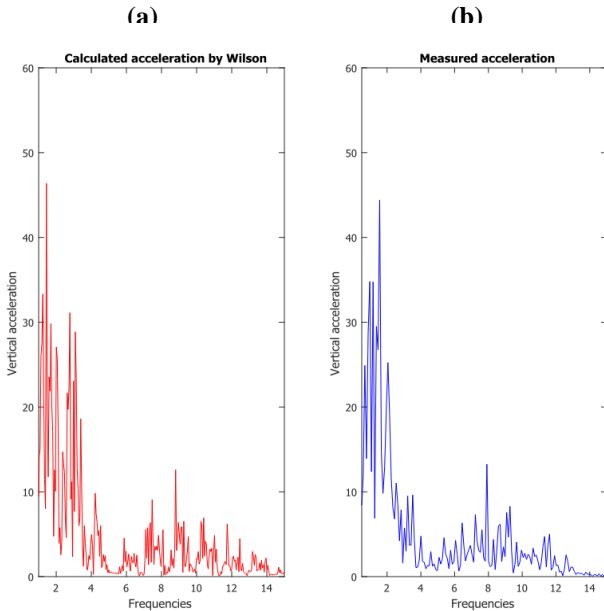


Fig. 12. Comparison between (a) numerical results of car body acceleration calculated by Wilson θ (b) Experimental data

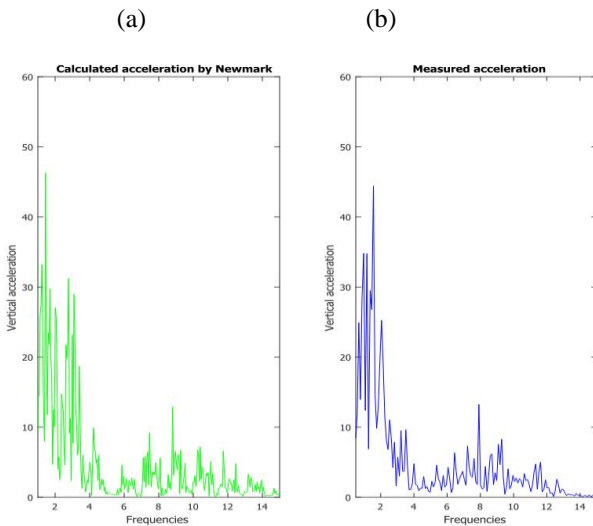


Fig.13. Comparison between (a) numerical results of car body acceleration calculated by Newmark β (b) Experimental data.

The comparison the two diagrams should that:

- The vertical accelerations in the center of the body issued from experimental recordings and those obtained numerical simulation were in harmony.
- The accelerations measured and calculated have the similar value range on both spectra.
- The spectra of the accelerations presented peaks similar around the same frequency in the experimental data and results of the model respectively.

These comparisons concluded that the theoretical model of the vehicle is validated.

VI. CONCLUSION

In this article a theoretical model of the railway vehicle has been established taking into account the mechanical and dynamic characteristics of the vehicle. The numerical model

is solved using the Newmark β and Wilson θ methods.

The current model is validated by comparing the results obtained from numerical simulation with those from experimental measurements.

The model developed evaluated the dynamic behaviour of the vehicle in a vertical plane during the traffic on an irregular track. The established model allows to:

- Calculate deformation, velocities and accelerations in the center of the body and the center of two bogies.
- Study the influence of several parameters, such as the train speed, the track irregularities, on the vehicle dynamic behaviour.
- Evaluate the dynamic behaviour of railway vehicles in terms of quality ride, vibrating comfort and fatigue stress on the track.

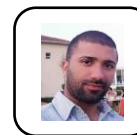
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