Numerical Modelling and Validation of Railway Vehicle

Bouhla Laila, Touati Mohammed, Lamdouar Nouzha

Abstract: The objective of this study was to establish a theoretical model regarding the dynamic behaviour of the railway vehicle in the vertical plan while travelling on a track with irregularities. In this article the vehicle is modelled by seven bi-dimensional rigid elements respectively representing: the body, tow boogies and four wheels. The equations system was numerically integrated by the Newmark’s β and Wilson θ method. The model developed was subjected to a validation process based on the comparison between numerically derived acceleration spectra and experimental data.

Keywords: Railway vehicle, theoretical model, dynamic behaviour, track with irregularities, validation.

I. INTRODUCTION

As the speed and intensity of rail traffic increases, a detailed numerical model becomes needed to predict the effect of track irregularities on the behavior of a railway vehicle. The theoretical models, in spite of their complexity, start from a series of simplifying hypotheses consequently the results can deviate from reality. The validation of the theoretical model through experimental measurements remains an essential step. The validation process is based on representative comparisons between experimental results performed on the vehicle and those obtained from numerical simulations.

II. THE THEORETICAL MODEL OF THE VEHICLE

Assuming that the load is distributed symmetrically on both rails, focusing on the vertical acceleration of the vehicle, a 2D model is well adapted to represent it. The vehicle is modelled through body, two bogies and four wheelsets, connected by spring and damping systems. Hence, the model has 10 DOFs, consisting of vertical and pitching motion of the vehicle body and bogies and; vertical motion of the wheelsets. Mc and Jc represent respectively the mass and rotational inertia of car body. Mt1 and Jt1 stand for the mass and rotational inertia of bogie 1 while Mt2 and Jt2 belong to bogie 2. Lt and Lc are the half distance of two wheels and two bogies, respectively. Kt1, Ct1 and Kt2, Ct2 are the stiffness and damping of bogies 1 and 2. [1]- [2]-[3]

Fig. 2. Model 2D of vehicle [2]

A. The motion equations of vehicle

The set of DOF (degrees of freedom) represented in Figure 1 is governed by the following system of motion equations:

Motion equation of car body

\[ M_c \ddot{z}_c + 2k_c z_c + 2c_c \dot{z}_c - k_2 z_{t1} - k_2 z_{t2} - c_2 \dot{z}_{t1} - c_2 \dot{z}_{t2} = -M_g \]

(1)

Motion equation of bogie 1

\[ J_c \ddot{\beta}_c + 2k_c l_c^2 \dot{\beta}_c + 2c_c l_c \dot{\beta}_c - k_c z_{t1} + k_c l_c \dot{z}_{t1} - c_c l_c \ddot{z}_{t1} + c_c l_c \ddot{z}_{t2} = 0 \]

(2)

Motion equation of bogie 2

\[ J_c \ddot{\beta}_c + 2k_c l_c^2 \dot{\beta}_c + 2c_c l_c \dot{\beta}_c - k_c z_{t1} + k_c l_c \dot{z}_{t1} - c_c l_c \ddot{z}_{t1} + c_c l_c \ddot{z}_{t2} = 0 \]

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Motion equation of bogie 2

\[ J_c \ddot{\beta}_c + 2k_c l_c^2 \dot{\beta}_c + 2c_c l_c \dot{\beta}_c - k_c z_{t1} + k_c l_c \dot{z}_{t1} - c_c l_c \ddot{z}_{t1} + c_c l_c \ddot{z}_{t2} = 0 \]

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(2)

Motion equation of bogie 2

\[ J_c \ddot{\beta}_c + 2k_c l_c^2 \dot{\beta}_c + 2c_c l_c \dot{\beta}_c - k_c z_{t1} + k_c l_c \dot{z}_{t1} - c_c l_c \ddot{z}_{t1} + c_c l_c \ddot{z}_{t2} = 0 \]

(3)
The system of equations can be written in the matrix form:

$$M\dot{\mathbf{X}} + C\ddot{\mathbf{X}} + K\mathbf{X} = \mathbf{F}(t)$$

The displacement vector $\mathbf{X}$, the forces vector $\mathbf{F}(t)$, the mass matrix $M$, the stiffness matrix $K$ and the damping matrix $C$ are defined by:

$$M = \begin{bmatrix}
Z_c & 0 & 0 & 0 & 0 \\
\beta_c & 0 & J_c & 0 & 0 \\
Z_{x1} & 0 & 0 & M_{x1} & 0 \\
\beta_{x1} & 0 & 0 & 0 & 0 \\
Z_{x2} & 0 & 0 & 0 & M_{x2} \\
\beta_{x2} & 0 & 0 & 0 & 0 \\
\end{bmatrix}$$

$$K = \begin{bmatrix}
2k_2 & 0 & -k_2 & 0 & -k_2 & 0 \\
0 & 2k_2 & 0 & -k_2 & 0 & k_2 \\
0 & 0 & 2k_1 + k_2 & 0 & 0 & 0 \\
0 & 0 & 0 & 2k_1 + k_2 & 0 & 0 \\
0 & 0 & 0 & 0 & 2k_1 + k_2 & 0 \\
0 & 0 & 0 & 0 & 0 & 2k_1 + 2k_2 \\
\end{bmatrix}$$

$$C = \begin{bmatrix}
2C_2 & 0 & -C_2 & 0 & -C_2 & 0 \\
0 & 2C_2 & 0 & -C_2 & 0 & C_2 \\
0 & 0 & 2C_1 + C_2 & 0 & 0 & 0 \\
0 & 0 & 0 & 2C_1 + C_2 & 0 & 0 \\
0 & 0 & 0 & 0 & 2C_1 + C_2 & 0 \\
0 & 0 & 0 & 0 & 0 & 2C_1 + 2C_2 \\
\end{bmatrix}$$

### B. Track irregularities

The vibrations of the vehicle were mainly excited by defects that may appear on the wheels and/or the rails during their manufacture or use. It was assumed in this study that the wheel was perfect and that the excitement source represented random irregularities on the railway line. These track irregularities were registered in Morocco by a measuring car called EM120. [4]

**Fig. 3. Moroccan geometry measuring car EM120**

### III. NUMERICAL SOLUTION OF EQUATION

- **Newmark β method**

  In this method, the following assumptions for motion and velocity vectors were used:

  $$\ddot{u}_{t+\Delta t} = \ddot{u}_t + [(1-\delta)\ddot{u}_t + \delta \ddot{u}_{t+\Delta t}] \Delta t$$

  $$u_{t+\Delta t} = u_t + \ddot{u}_t \Delta t + \left[\frac{1-\alpha}{2}\ddot{u}_t + \alpha \ddot{u}_{t+\Delta t}\right] \Delta t^2$$

  or $\Delta t = t_{n+1} - t_n$.

  $\alpha$ and $\delta$ were parameters that could be determined in order to obtain stable and accurate integration. $\delta = 1/2, \alpha = 1/4$ brings unconditional stability. To solve the displacement, velocities and accelerations at time $t + \Delta t$, the equilibrium equations were evaluated at the same time $t + \Delta t$.

  $$M\ddot{u}_{t+\Delta t} + C\dddot{u}_{t+\Delta t} + K\dddot{u}_{t+\Delta t} = r_{t+\Delta t}$$

- **Wilson 0 method**

  In Wilson’s method $\theta$, we assumed a linear variation of the acceleration of a time $t$ at a time $t + \theta \Delta t$, where $\theta \geq 1$. If the unconditional stability of the method is required, $\theta$ must be greater than or equal to 1.37; thus, we usually use $\theta = 1.40$.

  $$\ddot{u}_{t+\Delta t} = \ddot{u}_t + \frac{\tau}{\Delta t} (\ddot{u}_{t+\theta \Delta t} - \ddot{u}_t)$$

  The algorithm for solving both methods has been programmed with the MATLAB software.

**Fig. 4. Vertical acceleration of car body**

**Fig. 5. Vertical acceleration of bogie**
IV. EXPERIMENTAL DETERMINATIONS

To validate theoretical models of railway vehicles, an experimental measurement was realized to determine accelerations in the middle of the car body. Experimental measurements were made on the Casablanca-Marrakech railway line between PK236 and PK237. The speed of the vehicle measured in this area was constant and equal to 94km/h. The system used for the experimental measurements included components of the acceleration measurement system, data acquisition and processing system and a GPS receiver for monitoring and recording speed of the vehicle.

V. VALIDATION OF THE VEHICLE THEORETICAL MODEL

The purpose of this chapter was to validate the theoretical model developed in this study. The validation process require the comparison in the frequency domain of the results derived from numerical simulation programs of the vehicle dynamic behaviour and those obtained from the experiment analysis.
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Fig. 12. Comparison between (a) numerical results of car body acceleration calculated by Wilson \(\theta\) (b) Experimental data

Fig. 13. Comparison between (a) numerical results of car body acceleration calculated by Newmark \(\beta\) (b) Experimental data.

VI. CONCLUSION

In this article a theoretical model of the railway vehicle has been established taking into account the mechanical and dynamic characteristics of the vehicle. The numerical model is solved using the Newmark \(\beta\) and Wilson \(\theta\) methods.

The current model is validated by comparing the results obtained from numerical simulation with those from experimental measurements.

The model developed evaluated the dynamic behaviour of the vehicle in a vertical plane during the traffic on an irregular track. The established model allows to:

- Calculate deformation, velocities and accelerations in the center of the body and the center of two bogies.
- Study the influence of several parameters, such as the train speed, the track irregularities, on the vehicle dynamic behaviour.
- Evaluate the dynamic behaviour of railway vehicles in terms of quality ride, vibrating comfort and fatigue stress on the track.

REFERENCES


AUTHORS PROFILE

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