Unbalanced Transportation Problem with Multiple Fuzzy Goals

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Abstract: In most of the transportation related decision making situations, the decision maker face difficulty in case of unbalanced supply-demand units, in particular when total number of supply units is less than total number of required units. Due to shortage of availability of products, it is not possible for the decision maker to fully specify the requirement of various requirement points, with the existing available amount. In these situations, multiple objectives are considered to handle the situation. The objectives, of a multi-objective model are in general conflicting in nature. The traditional transportation approaches are not able to address the problem but goal programming approach is a well suited approach to such transportation models. In this present study, fuzzy goal programming approach is presented for an unbalanced transportation model with multiple number of objectives, in which all supply and demand quantities are specified imprecisely. We have investigated the results by implementing this method to an unbalanced transportation problem with multiple numbers of equally important fuzzy goals. To illustrate the approach, a numerical example is considered and the result is discussed.

Key words: Multi-objective problem, Goal programming approach, Multi-criteria decision making (MCDM)

1. INTRODUCTION

Optimization problems with multiple number of objectives are a specific part of multi criteria decision making (MCDM) problem which are concerned with the problems involving multiple numbers of objectives which are in general conflicting in nature and Goal programming (GP) is a well suited approach to find solution of these types of problems. This approach is analyzed in details by Lee (1972), Ignizio (1976, 82a, 83), and Romero (1986) also discussed the same approach later on.

To solve multi criterion models, Goal Programming approach is being used. In these cases, the aspiration level is kept fixed during goal formulation and the deviations from the particular aspiration level is not under control of the decision maker. Fuzzy goal programming (FGP) is used as another alternative approach to avoid these difficulties. We have investigated the results by implementing this method to an unbalanced transportation problem with multiple numbers of equally important fuzzy goals. Equal weights are assigned to each goal. Almost in all multi criteria decision making models, all the goals as well as the objectives are imprecise in nature and represented in terms of fuzzy quantities.

Bellman and Zadeh (1970) considered decision making problems with fuzzy relations. Narasimhan (1980) and Hannan (1981a, 81b) first implemented fuzzy set theory concept to solve the transportation model by goal programming approach. Tiwari et al. (1986) had investigated about the use of primitive priority structure in goal programming problems with fuzzy parameters and had an additive model to find solution of these problems. Here, goal programming approach with fuzzy parameters are used to find out the solution of an unbalanced transportation model. Basic notations of fuzzy sets are used and the solution of the model is obtained by liner programming approach. Because of equality type goals max-min operator is used for aggregation of the fuzzy goals.

In most of the transportation related issues, decision maker encounters with conflicting nature of different objectives of transportation model. The traditional transportation methods are not able to handle the situations but goal programming approach is applicable to such transportation models. Application of goal programming method to transportation models with multiple objectives are presented by Lee and Moore (1973). Kwak (1979) presented goal programming approach for improved transportation problem. Singh and Kishore (1991) implemented weighted goal programming approach to find solution of an unbalanced transportation model with only one objective function, with consideration of budgetary constraints. A model for transportation of coals and the related-products from different sources to different consumption sites (Bit et al. (1993c) was considered. The model has been developed to meet different requirements of energy with minimum transportation cost.

Sayed A. Zaki et.al (2012) discussed a solution procedure for obtaining solution of transportation models with multiple number of objectives. Another approach under group decision making was presented by Cherryl et.al (2014), to obtain a satisfied solution for MOTP. Maity et.al (2014) investigated the improved solution of MOTP by using utility function method. To find compromise solution, Mohammad Asim Nomani et.al (2017) presented an approach based on goal programming technique. Dutta and Jana (2017) formulated the average of the reductions of type-2 trapezoidal fuzzy parameters and applied it to a solid transportation model with multiple objectives by goal programming approach.

In most of the transport related decision making situations, the decision maker face difficulty in case of unbalanced supply-demand units particularly when total number of supply units is less than total number of required unit. Due to shortage of availability of products, it is not possible for the decision maker to fully satisfy the requirements of various requirement points with the existing amount of availability.
In these situations, multiple objectives are considered and those affect the solution.

1.2 Mathematical Formulation of the Model

Consider m-number of origins \( O_i \) \((i = 1, 2, ..., m)\) and n-number of destinations \( D_j \) \((j = 1, 2, ..., n)\). At origin \( O_i \), let \( X_{ij} \) be the amount of available unit of products which will be transported to destination \( D_j \). A penalty \( C_{ij} \) is the transportation cost per unit of the products from \( I \)-th origin to \( J \)-th destination corresponding to \( k \)-th criterion. The penalty is either cost of transportation, shipping time, quantity of transported amount etc. In many of the real life transportation related problems, are presented as unbalanced problem where the objective or goals of the problem are not specified properly. The goals are imprecise in nature and are represented by fuzzy quantities. Here, for an unbalanced transportation model with multiple numbers of objectives, fuzzy goal programming model is presented in which all supply and demands quantities are specified imprecisely.

The problem can be formulated as below:

Decision Variables:
For this model, decision variables are defined as

\[
X_{ij}, i = 1, 2, ..., m; j = 1, 2, ..., n
\]
where \( X_{ij} \geq 0 \) for all \( i \) and \( j \)

Supply constraints:

\[
\sum_{j=1}^{n} x_{ij} \leq a_i, i = 1, 2, ..., m
\]
where \( a_i > 0 \) for all \( i \)

Fuzzy demand goals:

\[
\sum_{i=1}^{m} XU \geq b_j, j = 1, 2, ..., n
\]
where

\[
Z_k = \sum_{i=1}^{m} \sum_{j=1}^{n} C_{ij} x_{ij}, k = 1, 2, ..., K
\]

And

II. MATHEMATICAL MODEL

An unbalanced transportation problem with multiple numbers of fuzzy goals is formulated as:

Find \( X_{ij}, i = 1, 2, ..., m; j = 1, 2, ..., n \):

\[
\sum_{i=1}^{n} x_{ij} \leq a_i, i = 1, 2, ..., m
\]

\[
\sum_{i=1}^{m} XU \geq b_j, j = 1, 2, ..., n
\]

\[
Z_k = \sum_{i=1}^{m} \sum_{j=1}^{n} C_{ij} x_{ij}, k = 1, 2, ..., K
\]

\[
x_{ij} \geq 0, a_i > 0, b_j > 0, c_{ij} \geq 0 \text{ for all } i, j \text{ and } k
\]

(1.1)

In case of unbalanced transportation problem, the total supply and demand units are not equal. The total requirements of the demand points either exceed the total availability (i.e., \( \sum_{i=1}^{m} b_j > \sum_{i=1}^{m} a_i \)) or less than that (i.e., \( \sum_{i=1}^{m} b_j < \sum_{i=1}^{m} a_i \))

In this unbalanced transportation problem, the condition \( \sum_{i=1}^{m} b_j > \sum_{i=1}^{m} a_i \) is considered.

1.3 Solution methods for the present model

To obtain solution for the proposed model we use the methodology which is almost same to the Zimmermann’s (1978) approach in formulation of the fuzzy multi-objective linear programming model. Here each of the objective functions are associated with fuzzy goal which has been stipulated by the decision maker whereas in fuzzy based multi-objective linear programming problem, it is assumed that each objective function should be optimized to the fullest extent. In this proposed model, the fuzzy goal programming model is converted to a linear programming problem with help of linear membership functions and max-min operator. The obtained solution for this linear programming model is considered as the efficient solution for the original problem.

The step wise procedure to obtain solution of the proposed model are given as follows:

Step 1: For both fuzzy demand goals and multiple fuzzy goals frame the membership functions.

\[
\mu_i(x) = \begin{cases} 
1 & \text{if } \sum_{i=1}^{m} x_{ij} \geq b_i \\
\left( \sum_{i=1}^{m} x_{ij} - b_i \right) / \left( b_j - b_i \right) & \text{if } \sum_{i=1}^{m} x_{ij} < b_i \\
0 & \text{if } \sum_{i=1}^{m} x_{ij} < b_j
\end{cases}
\]

(1.2)

tolerance limit of the \( j \)-th demand goal.

defined as follows:

The achievement functions can be developed by using max-min operator as follows: Max \( \lambda \)

\[
\lambda = \min_{j,k} \left[ \frac{\left( \sum_{i=1}^{m} x_{ij} - b_j \right)}{b_j - b_i} \right]
\]

(1.4)

Where

\[
0 \leq \sum_{i=1}^{m} x_{ij} \leq B^k
\]

\( B^k \) \((k = 1, 2, ..., K)\) is the upper level tolerance limit of the \( k \)-th fuzzy goal.

Step 2: Consider the max-min operator.

Step 3: Develop the corresponding linear programming problem.
The corresponding linear programming model of the unbalanced transportation problem with multiple numbers of fuzzy goals (1.1) is stated as follows:

Maximize \( \lambda \) subject to
\[
\sum_{j=1}^{n} x_{ij} \leq a_i, i = 1, 2, \ldots, m
\]
\[
\leq \mu_{ij}(x), j = 1, 2, \ldots, n
\]
\[
\lambda \leq \mu_{k+1}(x), k = 1, 2, \ldots, k
\]
\[
x_{ij} \geq 0 \quad \text{for all } i \text{ and } j
\]
\[
\lambda \geq 0
\]

This is a LPP which can be solved by using any of the appropriate linear programming algorithms.

**Numerical Example**

An industry has three storage units from where the products are supplied to four distinct requirement points. Considering the preference of decision maker, the cost of transportation and the deterioration amount of transported products should be around 110 dollars & 140 kilograms, respectively. A total of 8, 10, and 17 quintals are the available amounts at the three storage points and the demands at the requirement points are 11, 5, 14 and 10 quintals. The shipping cost \( c_{ij} \) and the deterioration rate \( d_{ij} \) per quintals of the goods from \( i \)-th storage points to the \( j \)-th requirement point is given in matrix \( C_1 \) and \( C_2 \) respectively. Let us assume that he wants to fulfill at least half of the actual requirements of every requirement points. The DM is also wants to know, how much of products in quintals to be shipped from the \( i \)-th store to the \( j \)-th destination by satisfying all the requirements.

The matrix for cost of transportation is given as follows:

\[
C_1 = \begin{bmatrix}
1 & 2 & 7 & 7 \\
1 & 9 & 3 & 4 \\
8 & 9 & 4 & 6 \\
\end{bmatrix}
\]

And the deterioration matrix is given as:

\[
C_2 = \begin{bmatrix}
4 & 4 & 3 & 4 \\
5 & 8 & 9 & 10 \\
6 & 2 & 5 & 1 \\
\end{bmatrix}
\]

Considering the 130 dollars as maximum tolerance limit for the cost of transportation and 160 kilograms as the maximum amount of deterioration of goods the problem can be formulated mathematically as below:

Evaluate \( x_{ij}, i = 1, 2, 3; j = 1, 2, 3, 4 \) so as to satisfy
\[
\sum_{j=1}^{4} x_{ij} \leq 8, \quad \sum_{j=1}^{4} x_{2j} \leq 10, \quad \sum_{j=1}^{4} x_{3j} \leq 17
\]
\[
\sum_{i=1}^{3} x_{i1} \geq 11, \quad \sum_{i=1}^{3} x_{i2} \geq 5, \quad \sum_{i=1}^{3} x_{i3} \geq 17, \quad \sum_{i=1}^{3} x_{i4} \geq 10
\]
For the problem (1.6), the corresponding linear programming model is developed as follows:

Maximize $\lambda$ subject to

\[
\begin{align*}
\sum_{j=1}^{4} x_{ij} & \leq 8, \quad \sum_{j=1}^{4} x_{ij} \leq 10, \quad \sum_{j=1}^{4} x_{ij} \leq 17 \\
\sum_{i=1}^{3} x_{ij} - 5.5\lambda & \geq 5.5 \\
\sum_{i=1}^{3} x_{ij} - 2.5\lambda & \geq 2.5 \\
\sum_{i=1}^{3} x_{ij} - 7\lambda & \geq 7 \\
\sum_{i=1}^{3} x_{ij} - 5\lambda & \geq 5 \\
Z_1 + 20\lambda & \leq 130 \\
Z_2 + 20\lambda & \leq 160 \\
x_{ij} & \geq 0 \text{ for all } i \text{ and } j \\
\lambda & \geq 0
\end{align*}
\]

The problem (5.13) is solved by using Simplex method and the solutions are obtained as follows:

\[
\begin{align*}
x_{11} &= 3.5642, \quad x_{21} = 5.9682, \quad x_{31} = 0, \\
x_{12} &= 4.3330, \quad x_{22} = 0, \quad x_{32} = 0, \\
x_{13} &= 0.1026, \quad x_{23} = 3.6957, \quad x_{33} = 8.3340, \\
x_{14} &= 0, \quad x_{24} = 0, \quad x_{34} = 8.6660, \quad \text{and } \\
\lambda &= 0.7332
\end{align*}
\]

From the above results, following points are observed:

- Amount of products supplied from source $0_1 = 0.7332$ quintals
- Amount of products supplied from source $0_2 = 9.6639$ quintals
- Amount of products supplied from source $0_3 = 17.00$ quintals
- Demand fulfilled at destination $D_1 = 9.5324$ quintals
- Demand fulfilled at destination $D_2 = 4.3330$ quintals
- Demand fulfilled at destination $D_3 = 12.1323$ quintals
- Demand fulfilled at destination $D_4 = 8.6620$ quintal
- Total amount of demand fulfilled = 34.6637 quintals
- Total amount of supply unit = 34.6637 quintals
- Total cost of transportation = 115.3359 dollars
- Total amount of deterioration = 145.3357 kilograms

### III. CONCLUSIONS

In fuzzy goal programming method, one can fix up the relevant aspiration levels of the fuzzy goals. In fuzzy goal programming approach, the deviations from the actual goals are within the control of the decisions maker. In the above example, the fuzzy demand goals are 5.5, 2.5, 7 and 5 units (which are equal to 50% of the fuzzy demand at each demand point) and are fulfilled as 9.53295, 4.3320, 12.1323 and 8.6660 units respectively, which are within the reasonable limits. The transportation cost 115.34 dollars and the total deterioration of goods 145.34 kilograms are also within the reasonable limits. Thus fuzzy goal programming method is a suitable approach to solve an unbalanced transportation problem with multiple fuzzy goals.

### REFERENCE