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A Statistical Analysis of Fuzzy Balanced Incomplete Block Designs with Intra-Block Analysis using Trapezoidal Method

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***Abstract:** The Balanced Incomplete Block Designs (BIBD) plays a major role in the design of experiments, particularly in agricultural experiments. In an intra-block analysis of BIBD, the practical situations are not measurable the exact observations due to a shortage of experimental apparatus or facilities and the physical size of the block. In such circumstances, the analyses of fuzzy observations are used. In this research work, a statistical analysis of BIBD with intra-block analysis through $\alpha-$ cut interval method of trapezoidal fuzzy observations is taken for study. A numerical example is provided by the primary data to show the preciseness of the yield of cotton crop seeds.*

***Keywords:** Analysis of Variance, Balanced Incomplete Block Designs, Decision Rule, Trapezoidal Fuzzy Numbers, α -Cut Method.*

I. INTRODUCTION

The analysis of variance (ANOVA) is the most important technique for the test of significance. One-way ANOVA is the single factor that has two or more level of treatments and two-way ANOVA is that two factors are associated with the independent and dependent factors. The design of experiments is said to be the construction of work based on a degree of uncertainty. There are three principles of experimental designs. They are replication, randomization and local control. Replication means the repetition of treatment under investigation. When all the treatments have equal opportunity to be assigned to varying experimental units, it is called randomization. Local control is used for minimizing experimental error and maximizing efficiency. There are three basic complete block designs namely Completely Randomized Design (CRD), Randomized Block Design (RBD) and Latin Square Design (LSD). CRD is the simplest randomized experiment for comparing several treatments that can be applied only to homogeneous experiments. In this procedure, the principle of local control is not accepted. Usually, the experimental material will not be homogeneous. RBD is adopted in local control and the experimental material is grouped into homogeneous subgroups. Since each block contains the entire treatment, one block equals one replicate. LSD is defined as the method to eliminate the variation of two factors known as row and column. RBD is most efficient when compared to CRD and LSD. Incomplete Block Designs (IBD) does not receive all the treatments in its block designs. When the number of treatments to be compared is huge, a huge number of blocks are required to take all the treatments. Therefore this needs further experimental objects and experiments are high in terms of labor, money and time. In such situations, CRD and RBD cannot be applicable to a large number of experimental units. If the treatment combinations are large, the suitable designs are known as BIBD. Analysis of BIBD can be divided into two objects, one is inter-block analysis also called as random effects and the other, intra-block analysis further called as fixed effects. If the data presented in this study are vague, so we need an extended version of the BIBD intra-block analysis to investigate these fuzzy observations is almost inevitable. Fuzzy sets were originally introduced by Lotfi A. Zadeh in 1965 [16]. A fuzzy set represents linguistic variables expressed by membership functions of Triangular Fuzzy Numbers (TFNs) and Trapezoidal Fuzzy Numbers (TZFNs). The TFNs are uniquely appropriated by triplet (a,b,c) with membership functions which can be more sophisticatedly and efficiently expressed as $a \leq b \leq c$. The trapezoids fuzzy numbers are located on the left and right and can be adjusted as desired by choosing the compact resistance values of a variable. TZFNs have lots of uses in the efficient adjustment of imprecise information. This rule is expressed by $a \leq b \leq c \leq d$. Many authors have discussed this method. Some suitable references are W. Connor, Jr.[15] has proposed analytical methods for constructing two unsymmetrical balanced incomplete block designs such that there is an inequality in the number of treatments common to both blocks. S. Chanas [2] has introduced the interval approximation of a fuzzy number by following the two conditions for measuring width and Hamming distance.

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Montenegro *et al.*[4] have analyzed two different fuzzy variables that are presented to solve the one-way ANOVA hypothesis testing to deal with fuzzy random variables. Wu [13] has proposed a traditional ANOVA for triangular fuzzy data using h-level set and decision rule on pessimistic and optimistic degree by solving optimization problems. Wu [14] proposed a central limit theorem to construct the unknown fuzzy parameters for fuzzy confidence intervals that are used to a fuzzy random variable.

Gonzalez-Rodriguez *et al* [3] have proposed that One-way ANOVA data can be used as a fuzzy functional data, which is considered a functional Hilbert space. Sanjib Kumar Behera and Dhyan Singh [1] has investigated impact of continuous use of fertilizer on fractions of manganese in soil and stated that it directly influenced randomized block designs. Nourbakhsh *et al.* [7] has proposed one-way ANOVA method based on fuzzy ANOVA observed data and were fuzzy observations rather than real numbers. Kalpanapriya and Pandian [8] have proposed a single factor ANOVA model with a new statistical decision on fuzzy hypothesis testing. Alireza Jiryaei *et al.* [9] have analyzed traditional ANOVA based on the real-valued random variables with triangular and exponential fuzzy environments to estimate fuzzy parameters. Parthiban and Gajivaradhan [10] have proposed a new technique for testing the hypotheses under one-factor ANOVA model using trapezoidal method through α -cut interval model and followed by the author [11] they proposed a comparative study of two-factor ANOVA method under fuzzy environments using different models with trapezoidal method and also the author [12] continued they discussed with numerical examples using three-factor ANOVA model under trapezoidal fuzzy environments. In this paper, a statistical analysis of BIBD with intra-block analysis for trapezoidal fuzzy numbers through α -cut interval method.

II. PRELIMINARIES

2.1 Definition of BIBD

A set of v treatments in b blocks each with containing $(k < v)$ treatments is said to be a balanced incomplete block design if it satisfies the following parametric relations: (i) $vr = bk = N$, (ii) $\lambda(v-1) = r(k-1)$ and (iii) $b \geq v$. The quantities $v,\ b,\ r,\ k$ and λ are usually called the parameters. Where, v = treatments, b = blocks, k = size of the block, r = replication, λ = all pair of treatments appear together in blocks.

2.2 Definition of Fuzzy Numbers

A fuzzy set is normal, convex $\forall \alpha \in (0,1]$ and their membership functions are defined in $\mathbb{R}$ and piecewise continuous, is known as fuzzy numbers.

2.3 Definition of α -Cut Method

The set $\tilde{A}_{\alpha}$ is made up of members whose membership is greater than α is known as α -cut method.

$$\tilde{A}_{\alpha} = \left\{ x \in X \,/\, \mu_{\tilde{A}}(x) \geq \alpha \right\}$$

2.4 Definition of TZFNs

The trapezoidal membership functions are identifying by four parameters $\tilde{A} = a, b, c, d \in \mathbb{R}$ as follows:

$$\mu_{\tilde{A}}(x) = \begin{cases} 0 & ; x < a \\ \dfrac{x-a}{b-a} & ; a \leq x \leq b \\ 1 & ; b \leq x \leq c \\ \dfrac{d-x}{d-c} & ; c \leq x \leq d \\ 0 & ; x > d \end{cases}$$

we get crisp interval by $\alpha -$cut operation interval $\tilde{A}_{\alpha}$ shall be obtained as follows

$$\text{Let } \frac{x-a}{b-a} = \alpha \ ; \ \frac{d-x}{d-c} = \alpha$$

$$\Rightarrow \ x = (b-a)\alpha + a \ ; \ x = d-(d-c)\alpha$$

If we denote $\alpha-$ cut interval for fuzzy number $\tilde{A}$ as $\tilde{A}_{\alpha}$, the obtained interval is defined as

$$\tilde{A}_{\alpha} = [a+(b-a)\alpha, d-(d-c)\alpha] \ ; \ \alpha \in [0,1]$$

when $b = c,$ the trapezoidal fuzzy number coincides with triangular one. TZFNs defined by lower limit a , upper limit d , a lower support limit b and the upper support limit c . Trapezoid rule is indicated by $a \leq b \leq c \leq d$. The following aspects of the membership functions are defined. They are:

(i) The core of membership function is $\mu_{\tilde{A}}(x) > 0.$ (ii) The support of membership function is $\mu_{\tilde{A}}(x) = 1.$ (iii) The boundaries usually denoted by the interval $0 < \mu_{\tilde{A}}(x) > 1.$ Figure 2.4.1 assists in these details.

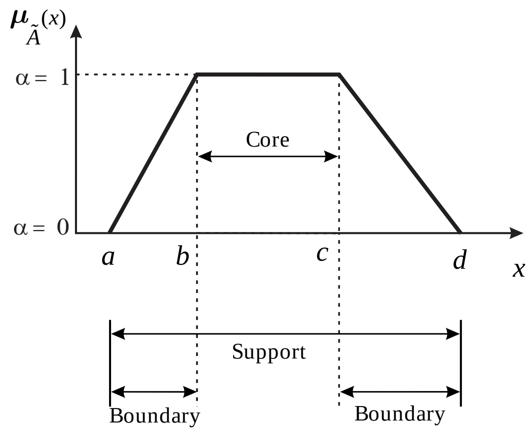

Figure 2.4.1. Trapezoidal Membership Functions

III. A STATISTICAL ANALYSIS OF BIBD WITH INTRA-BLOCK ANALYSIS

The general linear model for BIBD with intra-block analysis is given by

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$$y_{ij} = \mu + \tau_i + \beta_j + \varepsilon_{ij} \quad ; \; i = 1,2,...,v \; ; \; j = 1,2,...,b \qquad (1)$$

Where, y_{ij} is the observation in the j^{th} block receiving the i^{th} treatments. μ is the general mean effect which is fixed; τ_i is the fixed effect due to the i^{th} treatment, β_j is the fixed effect due to the j^{th} block; and ε_{ij} is the random error effect which follows $N(0,\sigma_e^2)$.

The required sums of squares are obtained as follows:

The Total Sum of Squares (TSS) is

$$TSS = \sum_i \sum_j y_{ij}^2 - \frac{G^2}{N} \; ; G = \sum_i \sum_j y_{ij}^2 ; \; N = vr = bk \qquad (2)$$

The Blocks (uadjusted) Sum of Squares (BSS) is

$$BSS = \sum_j \frac{B_j^2}{k} - \frac{G^2}{N} \qquad (3)$$

The Treatments (adjusted) Sum of Squares (Ad.TrtSS) is

$$Ad.TrtSS = \frac{k}{\lambda v} \sum_i \left(\mathbb{Q}_i^2 \right) \qquad (4)$$

Where $\mathbb{Q}_i$ is the adjusted total for the i^{th} treatment, which is computed as $\mathbb{Q}_i = T_i - \frac{1}{k} \sum_j n_{ij} B_j$

Error Sum of Squares (ESS) is obtained by subtraction

$$ESS = TSS - BSS - Ad.TrtSS \qquad (5)$$

Where, TSS, BSS and ESS are based on sum of $N-1, b-1, v-1$ and $N-v-b+1$ degrees of respectively.

The mean sum of squares is obtained as follows:

$$MSS_B = \frac{BSS}{b-1}, MSS_{Ad.Trt} = \frac{Ad.TrtSS}{v-1} \text{ and } MSS_E = \frac{ESS}{N-v-b+1}$$

Where, MSS_B, $MSS_{Ad.Trt}$ and MSS_E stands for mean sum of square blocks, mean sum of square adjusted treatment, mean sum of square error. For testing the hypotheses, it is are considered.

$$H_0 : b_1 = b_2 = ... = b_b \;\; \text{Vs} \;\; H_1 : b_1 \neq b_2 \neq ... \neq b_b$$

$$H_0 : t_1 = t_2 = ... = t_v \;\; \text{Vs} \;\; H_1 : t_1 \neq t_2 \neq ... \neq t_v$$

The test statistic to be used is $F = \frac{MSS_B}{MSS_E}$ and $F = \frac{MSS_{Ad.Trt}}{MSS_E}$ when the null hypothesis H_0 holds true, it is known that F is distributed as with corresponding degrees of freedom $(b-1),(v-1)$ and $(N-v-b+1)$ that is $F_{\left[(b-1),(v-1),(N-v-b+1)\right]}$.

All these values are referred in the ANOVA table and the inference is drawn.

Table 1: ANOVA Table for BIBD with Intra-Block Analysis

SV	df	SS	MSS	F - Ratio
Blocks (unadjusted)	$(b-1)$	$BSS = \sum_j \frac{B_j^2}{k} - \frac{G^2}{N}$	$MSS_B = \frac{SS_B}{(b-1)}$	$F = \frac{MSS_B}{MSS_E}$
Treatment (adjusted)	$(v-1)$	$Ad.TrtSS = \frac{k}{\lambda v} \sum_i \left(\mathbb{Q}_i^2 \right)$	$MSS_{Ad.Trt} = \frac{SS_{Ad.Trt}}{(v-1)}$	$F = \frac{MSS_{Ad.Trt}}{MSS_E}$
Total Due to Remainder	$(N-v-b+1)$	$ESS = SS_E (by\ subtraction)$	$MSS_E = \frac{SS_E}{(N-v-b+1)}$	-
Total	$(N-1)$	$TSS = \sum_i \sum_j y_{ij}^2 - \frac{G^2}{N}$	-	-

Decision rules of F - Ratio

(i) If $F = \frac{MSS_B}{MSS_E} < F_t$ where F_t is the tabulated value of F with $[(b-1),(N-v-b+1)]$ degrees of freedom at r level of significance, then the null hypothesis H_0 is accepted, if not the alternative hypothesis H_1 is accepted.

(ii) If $F = \frac{MSS_{Ad.Trt}}{MSS_E} < F_t$ where F_t is the tabulated value of F with $[(v-1),(N-v-b+1)]$ degrees of freedom at r level of significance, then it accepts the null hypothesis H_0, if not the alternative hypothesis H_1 is accepted.

Note: Here, the notation for level of significance is to be r as an alternative to α, so as to avoid getting confused with $\alpha -$cut value found in trapezoidal fuzzy numbers.

3.1 A Statistical Analysis of Balanced Incomplete Block Designs with Trapezoidal Method Using Fuzzy Observations

In this section, to test the hypotheses of BIBD with intra-block analysis using $\alpha-$ cut interval method of trapezoidal fuzzy numbers is proposed. Using this condition, convert the crisp BIBD intra-block analysis to fuzzy BIBD intra-analysis. Fetching the fuzzy BIBD, let us consider lower and upper levels of BIBD's. In this planned approach, a crisp BIBD intra-block models are chosen in terms of lower and upper level. To conclude, we have analyzed lower-level and upper-level model using crisp BIBD intra-block procedure. The generalized linear model of fuzzy BIBD is $\tilde{y}_{ij} = \mu + \tau_i + \beta_j + \varepsilon_{ij}$. Where $\tilde{y}_{ij}$ is the observation in the j^{th} block receiving the i^{th} treatments. τ_i is the fixed effect due to the i^{th} treatment, β_j is the fixed effect due to the j^{th} block and ε_{ij} is the random error effect which follows $N(0,\sigma_e^2)$. A fuzzy BIBD linear model can be divided into two models. Namely, the lower-level and upper-level models are respectively. If the lower-level model is $a_{ij} + (b_{ij} - a_{ij})\alpha$; $i = 1, 2, ..., v$; $j = 1, 2, ..., b$, the α -cut interval method can be represented as in the table 2. Similarly, if the Upper-level model is $d_{ij} - (d_{ij} - c_{ij})\alpha$; $i = 1, 2, ..., v$; $j = 1, 2, ..., b$, the α -cut interval method can be represented as in the table 3.

Table 2: The Layout of Lower-Level Model for α -Cut Interval Method of BIBD

Treatments i	Blocks j					
	1	2	…	j	…	b
1	$a_{11}+(b_{11}-a_{11})\alpha$	-	…	$a_{1j}+(b_{1j}-a_{1j})\alpha$	…	$a_{1b}+(b_{1b}-a_{1b})\alpha$
2	$a_{21}-(b_{21}-a_{21})\alpha$	$a_{22}-(b_{22}-a_{22})\alpha$	…	$a_{2j}-(b_{2j}-a_{2j})\alpha$	…	-
⋮	⋮	⋮	…	⋮	⋮	⋮
i	$a_{i1}-(b_{i1}-a_{i1})\alpha$	$a_{i2}-(b_{i2}-a_{i2})\alpha$	…	-	…	$a_{ib}-(b_{ib}-a_{ib})\alpha$
⋮	⋮	⋮	⋮	⋮	⋮	⋮
v	-	$a_{v2}-(b_{v2}-a_{v2})\alpha$	…	$a_{vj}-(b_{vj}-a_{vj})\alpha$	…	$a_{vb}-(b_{vb}-a_{vb})\alpha$

The Total Sum of Squares (TSS) is

$$TSS^L = \sum_i \sum_j \left[a_{ij} + \left(b_{ij} - a_{ij}\right)\alpha \right]^2 - \frac{\left(G^2\right)}{N} \qquad \text{(Using 2)}$$

The Grand Total (GT) is

$$G^L = \sum_i \sum_j \left[a_{ij} + \left(b_{ij} - a_{ij}\right)\alpha \right]$$

The Correction Factor (CF) is

$$CF^L = \frac{\left(G^2\right)}{N}$$

The Blocks (unadjusted) Sum of Squares (BSS) is

$$BSS^L = \frac{1}{k} \sum_j \left[a_{ij} + \left(b_{ij} - a_{ij}\right)\alpha \right]^2 - \frac{\left(G^2\right)}{N} \qquad \text{(Using 3)}$$

The Treatments (adjusted) Sum of Squares (Ad.TrtSS) is

$$Ad.Trt\,SS^L = \frac{k}{\lambda v} \sum_i \left(\mathbb{Q}_i^2\right) \qquad \text{(Using 4)}$$

Where,

$$\mathbb{Q}_i^L = \left[\left(\sum_j \left[a_{ij} + \left(b_{ij} - a_{ij}\right)\alpha \right] \right) - \frac{1}{k} \sum_j n_{ij} \left(\sum_i \left[a_{ij} + \left(b_{ij} - a_{ij}\right)\alpha \right] \right) \right]$$

Error Sum of Squares is obtained by subtraction

$$ESS^L = TSS^L - BSS^L - Ad.Trt\,SS^L \qquad \text{(Using 5)}$$

Test of Hypotheses for Two Sets of Blocks and Treatments

The null hypothesis for lower-level model of blocks:

$$\tilde{H}_0^L : b_1^L = b_2^L = \cdots = b_b^L \;\; Vs \;\; \tilde{H}_1^L : b_1^L \neq b_2^L \neq \cdots \neq b_b^L$$

The null hypothesis for lower- level model of *treatments*:

$$\tilde{H}_0^L : t_1^L = t_2^L = \cdots = t_v^L \;\; Vs \;\; \tilde{H}_1^L : t_1^L \neq t_2^L \neq \cdots \neq t_v^L$$

Decision Rules of Lower-Level Model

(i) If $\tilde{F}_B^L < F_t$ at r level of significance with $\left[(b-1),(N-v-b+1)\right]$ degrees of freedom, then the null hypothesis $\tilde{H}_0^L$ is accepted for certain value of $\alpha \in [0,1]$, if not the alternative hypothesis $\tilde{H}_1^L$ is accepted.

(ii) If $\tilde{F}_{Ad.Trt}^L < F_t$ at r level of significance with $\left[(v-1),(N-v-b+1)\right]$ degrees of freedom, then the null hypothesis $\tilde{H}_0^L$ is accepted for certain value of $\alpha \in [0,1]$, if not the alternative hypothesis $\tilde{H}_1^L$ is accepted.

Table 3: ANOVA Table for Fuzzy BIBD with Intra-Block Analysis of Lower-Level

SV	Df	SS	MSS	$\tilde{F}$ - ratio
Blocks (unadjusted)	$(b-1)$	BSS^L	MSS_B^L	$\tilde{F}=\frac{MSS_B^L}{MSS_E^L}$
Treatments (adjusted)	$(v-1)$	$Ad.Trt\,SS^L$	$MSS_{Ad.Trt}^L$	$\tilde{F}=\frac{MSS_{Ad.Trt}^L}{MSS_E^L}$
Total Due to Remainder	$(N-v-b+1)$	ESS^L	MSS_E^L	-
Total	$(N-1)$	TSS^L	-	-

Table 4: The Layout of Upper -Level Model for α -Cut Interval Method of BIBD

Treatments i	Blocks j					
	1	2	…	j	…	b
1	$d_{11}-(d_{11}-c_{11})\alpha$	-	…	$d_{1j}-(d_{1j}-c_{1j})\alpha$	…	$d_{1b}-(d_{1b}-d_{1b})\alpha$
2	$d_{21}-(d_{21}-c_{21})\alpha$	$d_{22}-(d_{22}-c_{22})\alpha$	…	$d_{2j}-(d_{2j}-c_{2j})\alpha$	…	-
⋮	⋮	⋮	…	⋮	⋮	⋮
i	$d_{i1}-(d_{i1}-c_{i1})\alpha$	$d_{i2}-(d_{i2}-c_{i2})\alpha$	…	-	…	$d_{ib}-(d_{ib}-c_{ib})\alpha$
⋮	⋮	⋮	⋮	⋮	⋮	⋮
v	-	$d_{v2}-(d_{v2}-c_{v2})\alpha$	…	$d_{vj}-(d_{vj}-c_{vj})\alpha$	…	$d_{vb}-(d_{vb}-c_{vb})\alpha$

The Total Sum of Squares is

$$TSS^U=\sum_i\sum_j\left[d_{ij}-\left(d_{ij}-c_{ij}\right)\alpha\right]^2-\frac{\left(G^2\right)}{N} \qquad \text{(Using 2)}$$

The Grand Total is

$$G^U=\sum_i\sum_j\left[d_{ij}-\left(d_{ij}-c_{ij}\right)\alpha\right]$$

The Correction Factor is

$$CF^U=\frac{\left(G^2\right)}{N}$$

The Blocks (unadjusted) Sum of Squares is

$$BSS^U=\frac{1}{k}\sum_j\left[d_{ij}-\left(d_{ij}-c_{ij}\right)\alpha\right]^2-\frac{\left(G^2\right)}{N} \qquad \text{(Using 3)}$$

The Treatments (adjusted) Sum of Squares is

$$Ad.Trt\,SS^U=\frac{k}{\lambda v}\sum_i\mathbb{Q}_i^2 \qquad \text{(Using 4)}$$

where,

$$\mathbb{Q}_i^U=\left[\left(\sum_j\left[d_{ij}-\left(d_{ij}-c_{ij}\right)\alpha\right]\right)-\frac{1}{k}\sum_j n_{ij}\left(\sum_i\left[d_{ij}-\left(d_{ij}-c_{ij}\right)\alpha\right]\right)\right]$$

The Error sum of squares is obtained by subtraction

$$ESS^U=TSS^U-BSS^U-Ad.Trt\,SS^U \qquad \text{(Using 5)}$$

Test of Hypotheses for Two Sets of Blocks and Treatments

The null hypothesis for upper- level model of *blocks*:

$\tilde{H}_0^U: b_1^U=b_2^U=\cdots=b_b^U$ Vs $\tilde{H}_1^U: b_1^U\neq b_2^U\neq\cdots\neq b_b^U$

The null hypothesis for upper- level model of *treatments*:

$\tilde{H}_0^U: t_1^U=t_2^U=\cdots=t_v^U$ Vs $\tilde{H}_1^U: \mu_1^U\neq\mu_2^U\neq\cdots\neq\mu_v^U$

Decision Rules of Upper- Level Model

(i) If $\tilde{F}_B^U<F_t$ at r level of significance with $\left[(b-1),(N-v-b+1)\right]$ degrees of freedom, then the null hypothesis $\tilde{H}_0^U$ is accepted for certain value of $\alpha\in[0,1]$, if not the alternative hypothesis $\tilde{H}_1^U$ is accepted.

(ii) If $\tilde{F}_{Ad.Trt}^U<F_t$ at r level of significance with $\left[(v-1),(N-v-b+1)\right]$ degrees of freedom, then the null hypothesis $\tilde{H}_0^U$ is accepted for certain value of $\alpha\in[0,1]$, if not the alternative hypothesis $\tilde{H}_1^U$ is accepted.

Table 5: ANOVA Table for Upper-Level Fuzzy BIBD with Intra-Block Analysis

SV	df	SS	MSS	$\tilde{F}$ - ratio
Blocks (unadjusted)	$(b-1)$	BSS^U	MSS_B^U	$\tilde{F} = \frac{MSS_B^U}{MSS_E^U}$
Treatments (adjusted)	$(v-1)$	$Ad.Trt^U$	$MSS_{Ad.Trt}^U$	$\tilde{F} = \frac{MSS_{Ad.Trt}^U}{MSS_E^U}$
Total due to remainder	$(N-v-b+1)$	ESS^U	MSS_E^U	-
Total	$(N-1)$	TSS^U	-	-

IV. APPLICATIONS

The yields were collected through primary data in the cotton crop seeds in the Salem District in Tamilnadu. Four varieties of cotton crop seeds [MCU 5, Suvin, TCHB 213, MCU 13] with four different types of fertilizers Nitrogen (N), Phosphorous (P), Potassium (K) and urea $[(NH_2)_2CO]$ are tested in an intra-block analysis of BIBD. These fertilizers are provided in more addition to natural manure. The exact observations in the BIBD with intra-block analysis tested were not observed to the levels of yield. Therefore, fuzzy observations are needed to quantify the blurring nature of this yield. For this purpose, the present data are based on TZFNs in kilograms per hectare.

Table 6: Table for Yield of Cotton Crop Seeds

Treatments (Fertilizers)	Blocks (Varieties)			
	MCU 5	Suvin	TCHB 213	MCU 13
N	$[70, 72, 75, 78]$	$[71, 73, 76, 78]$	-	$[68, 70, 73, 76]$
P	-	$[71, 74, 77, 79]$	$[64, 66, 69, 72]$	$[69, 71, 74, 77]$
K	$[70, 72, 75, 78]$	$[71, 74, 77, 79]$	$[64, 67, 70, 73]$	-
$(NH_2)_2CO$	$[73, 74, 77, 79]$	-	$[69, 71, 74, 77]$	$[71, 74, 77, 79]$

Test whether there is any significant difference between varieties and fertilizers of the yield in cotton crop seeds.

Table 7: Table for Lower and Upper Levels using $\alpha-$cut Interval Method with TZFNs

Treatments (Fertilizers)	Blocks (Varieties)			
	MCU 5	Suvin	TCHB 213	MCU 13
N	$[70+2\alpha, 78-3\alpha]$	$[71+2\alpha, 78-2\alpha]$	-	$[68+2\alpha, 76-3\alpha]$
P	-	$[71+3\alpha, 79-2\alpha]$	$[64+2\alpha, 72-3\alpha]$	$[69+2\alpha, 77-3\alpha]$
K	$[70+2\alpha, 78-3\alpha]$	$[71+3\alpha, 79-2\alpha]$	$[64+3\alpha, 73-3\alpha]$	-
$(NH_2)_2CO$	$[73+\alpha, 79-2\alpha]$	-	$[69+2\alpha, 77-3\alpha]$	$[71+3\alpha, 79-2\alpha]$

Table 8: Lower-Level Model for α - Cut Interval Method using TZFNs

Treatments (Fertilizers)	Blocks (Varieties)				Total
	MCU 5	Suvin	TCHB 213	MCU 13	
N	$[70+2\alpha]$	$[71+2\alpha]$	-	$[68+2\alpha]$	$[209+6\alpha]$
P	-	$[71+3\alpha]$	$[64+2\alpha]$	$[69+2\alpha]$	$[204+7\alpha]$
K	$[70+2\alpha]$	$[71+3\alpha]$	$[64+3\alpha]$	-	$[205+8\alpha]$
$(NH_2)_2CO$	$[73+\alpha]$	-	$[69+2\alpha]$	$[71+3\alpha]$	$[213+6\alpha]$
Total	$[213+5\alpha]$	$[213+8\alpha]$	$[197+7\alpha]$	$[208+7\alpha]$	$[831+27\alpha]$

(i) $\tilde{H}_0^L$: There is no significant difference between varieties of yield in cotton crop seeds.

(ii) $\tilde{H}_0^L$: There is no significant difference between fertilizers of yield in cotton crop seeds.

Correction factor is

$$CF^L = \frac{(G^2)}{N} = \frac{729\alpha^2 + 44874\alpha + 690561}{12}$$

Total sum of squares is

$$TSS^L = \sum_i \sum_j \left[a_{ij} + \left(b_{ij} - a_{ij}\right)\alpha\right]^2 - \frac{\left(G^2\right)}{N}$$

$$TSS^L = \frac{51\alpha^2 - 90\alpha + 1011}{12}$$

Blocks (unadjusted) sum of squares is

$$BSS^L = \frac{1}{k}\sum_j \left[a_{ij} + \left(b_{ij} - a_{ij}\right)\alpha\right]^2 - \frac{\left(G^2\right)}{N}$$

$$BSS^L = \frac{19\alpha^2 - 42\alpha + 683}{12}$$

Treatments (adjusted) sum of squares is

$$Ad.Trt\,SS^L = \frac{k}{\lambda v}\sum_i \left(\mathbb{Q}_i^2\right)$$

$$\mathbb{Q}_i^L = \left[\left(\sum_j \left[a_{ij} + \left(b_{ij} - a_{ij}\right)\alpha\right]\right) - \frac{1}{k}\sum_j n_{ij}\left(\sum_i \left[a_{ij} + \left(b_{ij} - a_{ij}\right)\alpha\right]\right)\right]$$

$$\left(\mathbb{Q}_i^2\right)^L = \left\{\left(\frac{2\alpha+7}{3}\right)^2 + \left(\frac{\alpha+6}{3}\right)^2 + \left(\frac{4\alpha-8}{3}\right)^2 + \left(\frac{-\alpha+21}{3}\right)^2\right\}$$

$$Ad.Trt\,SS^L = \frac{11\alpha^2 - 18\alpha + 295}{12}$$

Error sum of square is obtained by subtraction

$$ESS^L = TSS^L - BSS^L - Ad.Trt\,SS^L$$

$$ESS^L = \frac{21\alpha^2 - 30\alpha + 33}{12}$$

All these results are represented in the ANOVA of fuzzy BIBD and mean sum of square is drawn.

Table 9: ANOVA Table for Fuzzy BIBD with Intra-Block Analysis in Lower-Level Model

SV	df	SS	MSS	$\tilde{F}$ - ratio
Varieties unadjusted	3	$BSS^L = \frac{19\alpha^2 - 42\alpha + 683}{12}$	$MSS_B^L = \frac{19\alpha^2 - 42\alpha + 683}{36}$	$\tilde{F}_B^L = \frac{5}{9}\left[\frac{19\alpha^2 - 42\alpha + 683}{7\alpha^2 - 10\alpha + 11}\right]$
Fertilizers adjusted	3	$Ad.Trt\,SS^L = \frac{11\alpha^2 - 18\alpha + 295}{12}$	$MSS_{Ad.Trt}^L = \frac{11\alpha^2 - 18\alpha + 295}{36}$	$\tilde{F}_{Ad.Trt}^L = \frac{5}{9}\left[\frac{11\alpha^2 - 18\alpha + 295}{7\alpha^2 - 10\alpha + 11}\right]$
Total due to remainder	5	$ESS^L = \frac{21\alpha^2 - 30\alpha + 33}{12}$	$MSS_E^L = \frac{21\alpha^2 - 30\alpha + 33}{60}$	-
Total	11	$TSS^L = \frac{51\alpha^2 - 90\alpha + 1011}{12}$	-	-

$\tilde{F}$ -Ratio for Varieties (unadjusted) is

$\tilde{F}_B^L = \frac{MSS_B^L}{MSS_E^L} = \frac{5}{9}\left[\frac{19\alpha^2 - 42\alpha + 683}{7\alpha^2 - 10\alpha + 11}\right]; \alpha \in [0,1]$. Now, the tabulated value of F at 5% level of significance with $\left[(b-1), (N-v-b+1)\right] = (3,5)$ degrees of freedom is 5.41.

$$\Rightarrow F_{t(5\%)}^L = 5.41 \text{ Here, } \tilde{F}_b^L > F_{t(5\%)}^L \ \forall (0 \le \alpha \le 1).$$

(i) The null hypothesis $\tilde{H}_0^L$ is rejected at 5% level of significant $(0 \le \alpha \le 1)$ at lower- level model.

(ii) The difference between varieties is significant. Therefore, the four varieties differ significantly with respect to the cotton crop seeds in yield.

$\tilde{F}$ -Ratio for Treatments (adjusted) is

$$\tilde{F}_{Ad.Trt}^L = \frac{MSS_{Ad.Trt}^L}{MSS_E^L} = \frac{5}{9}\left[\frac{11\alpha^2 - 18\alpha + 295}{7\alpha^2 - 10\alpha + 11}\right]; \alpha \in [0,1].$$

Now, the tabulated value of F at 5% level of significance with $\left[(v-1), (N-v-b+1)\right] = (3,5)$ degrees of freedom is 5.41.

$$\Rightarrow F_{t(5\%)}^L = 5.41 \text{ Here, } \tilde{F}_{Ad.Trt}^L > F_{t(5\%)}^L \ \forall (0 \le \alpha \le 1).$$

(i) The null hypothesis $\tilde{H}_0^L$ is rejected at 5% level of significant $(0 \le \alpha \le 1)$ at lower-level model.

(ii) The difference between fertilizers is significant. Therefore, the four fertilizers differ significantly with respect to the cotton crop seeds in yield

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Table 10: Upper-Level Model for α - cut Interval Method using TZFNs

Treatments (Fertilizers)	Blocks (Varieties)			
	MCU 5	Suvin	TCHB 213	MCU 13
N	$[78-3\alpha]$	$[78-2\alpha]$	-	$[76-3\alpha]$
P	-	$[79-2\alpha]$	$[72-3\alpha]$	$[77-3\alpha]$
K	$[78-3\alpha]$	$[79-2\alpha]$	$[73-3\alpha]$	-
$(NH_2)_2CO$	$[79-2\alpha]$	-	$[77-3\alpha]$	$[79-2\alpha]$

(i) $\tilde{H}_0^U$: There is no significant difference between varieties of yield in cotton crop seeds.

(ii) $\tilde{H}_0^U$: There is no significant difference between fertilizers of yield in cotton crop seeds.

Correction factor is $= \frac{(G^2)^U}{N} = \frac{961\alpha^2 - 57350\alpha + 855625}{12}$

Total sum of squares is

$$TSS^U = \sum_i \sum_j \left[d_{ij} - \left(d_{ij} - c_{ij}\right)\alpha\right]^2 - \frac{(G^2)}{N}$$

$$TSS^U = \frac{35\alpha^2 + 206\alpha + 731}{12}$$

Blocks (unadjusted) sum of squares is

$$BSS^U = \frac{1}{k}\sum_j \left[d_{ij} - \left(d_{ij} - c_{ij}\right)\alpha\right]^2 - \frac{(G^2)}{N}$$

$$BSS^U = \frac{19\alpha^2 + 150\alpha + 491}{12}$$

The Adjusted treatment sum of square is

$$Ad.Trt\,SS^U = \frac{k}{\lambda v}\sum_i \mathbb{Q}_i^2$$

$$\mathbb{Q}_i^U = \left[\left(\sum_j \left[d_{ij} - \left(d_{ij} - c_{ij}\right)\alpha\right]\right) - \frac{1}{k}\sum_j n_{ij}\left(\sum_i \left[d_{ij} - \left(d_{ij} - c_{ij}\right)\alpha\right]\right)\right]$$

$$\left(\mathbb{Q}_i^2\right)^U = \left\{\left(\frac{2\alpha+7}{3}\right)^2 + \left(\frac{\alpha+6}{3}\right)^2 + \left(\frac{\alpha+3}{3}\right)^2 + \left(\frac{4\alpha+16}{3}\right)^2\right\}$$

$$Ad.Trt\,SS^U = \frac{11\alpha^2 + 87\alpha + 175}{12}$$

Error sum of square is obtained by subtraction

$$ESS^U = TSS^U - BSS^U - Ad.Trt\,SS^U$$

$$ESS^U = \frac{5\alpha^2 - 31\alpha + 65}{12}$$

All these results are represented in the ANOVA of fuzzy BIBD and mean square is drawn.

Table 11: ANOVA Table for Fuzzy BIBD with Intra-Block Analysis in Upper-Level Model

SV	df	SS	MSS	F - ratio
Varieties (unadjusted)	3	$BSS^U = \frac{19\alpha^2 + 150\alpha + 491}{12}$	$MSS_B^U = \frac{19\alpha^2 - 150\alpha + 491}{36}$	$\tilde{F}_B^U = \frac{5}{3}\left[\frac{19\alpha^2 - 150\alpha + 491}{21\alpha^2 - 30\alpha + 33}\right]$
Fertilizers (adjusted)	3	$Ad.Trt\,SS^U = \frac{11\alpha^2 + 87\alpha + 175}{12}$	$MSS_{Ad.Trt}^U = \frac{11\alpha^2 + 87\alpha + 175}{36}$	$\tilde{F}_{Ad.Trt}^U = \frac{5}{3}\left[\frac{11\alpha^2 + 87\alpha + 175}{5\alpha^2 + 269\alpha + 65}\right]$
Total due to remainder	5	$ESS^U = \frac{5\alpha^2 - 31\alpha + 65}{12}$	$MSS_E^U = \frac{5\alpha^2 - 31\alpha + 65}{60}$	-
Total	11	$TSS^U = \frac{35\alpha^2 + 206\alpha + 731}{12}$	-	-

$\tilde{F}$ -Ratio for Treatments (adjusted) is

$F_{Ad.Trt}^U = \frac{MSS_B^U}{MSS_E^U} = \frac{5}{3}\left[\frac{11\alpha^2 + 87\alpha + 175}{5\alpha^2 + 269\alpha + 65}\right]; \alpha \in [0,1]$. Now, the tabulated value of F at 5% level of significance with $\left[(v-1), \left(N-v-b+1\right)\right] = (3,5)$ degrees of freedom is 5.41.

$\Rightarrow F_{t(5\%)}^L = 5.41$. Here,

$F_{Ad.Trt}^U < F_{t(5\%)}^U$ at $(0 \le \alpha \le 0.2)$ and

$\tilde{F}_{Ad.Trt}^U < F_{t(5\%)}^U$ at $(0.3 \le \alpha \le 1)$.

(i) The null hypothesis $\tilde{H}_0^U$ is accepted at 5% level of significance $\forall \alpha$, $(0 \le \alpha \le 0.2)$ at Upper- level model.

(ii) The difference between fertilizers is not significant. Therefore, the four fertilizers do not differ significantly with respect to the cotton crop seeds in yield.

(iii) The null hypothesis $\tilde{H}_0^U$ is rejected at 5% level of significance $\forall \alpha$, $(0 \le \alpha \le 1)$ at upper-level model.

(iv) The difference between varieties is significant. Therefore, the four varieties do not differ significantly with respect to the cotton crop seeds in yield.

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$\tilde{F}$ **-Ratio for Varieties (unadjusted) is**

$F_B^U = \frac{MSS^U_{Ad.Trt}}{MSS^U_E} = \frac{5}{3}\left[\frac{19\alpha^2 - 150\alpha + 491}{5\alpha^2 - 31\alpha + 65}\right]$ Now, the tabulated value of F at 5% level of significance with $\left[(b-1),(N-v-b+1)\right] = (3,5)$ degrees of freedom is 5.41.

$\Rightarrow F^U_{t(5\%)} = 5.41$. Here, $\tilde{F}^U_b < F^U_{t(5\%)}$ $\forall\alpha, (0 \le \alpha \le 1)$.

(i) The null hypothesis $\tilde{H}^U_0$ is rejected at 5% level of significance $\forall\alpha, (0 \le \alpha \le 1)$ at Upper-level model.

(ii) The difference between varieties is significant. Therefore, the four varieties do not differ significantly with respect to the cotton crop seeds in yield.

V. CONCLUSION

In this paper, BIBD with intra-block analysis based on fuzzy observations is expressed from a mathematical framework for precisely dealing with uncertain phenomena emerged by non-precise numbers. Vague data, rather than crisp observations, may face all sorts of real-life situations. We use a stretch version of BIBD with intra-block analysis to analyze such fuzzy numbers. This can be lead to lack of clarity of the decision and may be important from an application point of view. The statistical analysis of BIBD with intra-block analysis, may lead to either acceptance or rejection of the results of the hypothesis. Fuzzy statistical analysis is done through α - cut interval method of trapezoidal fuzzy observations for different values with probability. We conclude that the proposed fuzzy BIBD with intra-block analysis is a rational transform to the classical analysis of BIBD with intra-block results are shown when the observed data are fuzzy. In forthcoming studies, the proposed method of this research paper can be extended to PBIBD, Lattice designs, and some special designs.

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