van Hiele Level of Geometric Thinking among Secondary School Students

Muhammad Ammar Naufal, Abdul Halim Abdullah, Sharifah Osman, Mohd Salleh Abu, Hisyam Ihsan

Abstract*: Geometric thinking plays an important role in geometric achievement. It is also important in other fields, such as architecture, engineering, film, science, graphics, and arts. However, in Indonesian education curriculum, teaching and learning geometry does not emphasise the geometric thinking skills. Several studies revealed that Indonesian students could not come out from the lowered zone of the International exam, such as Trends in International Mathematics and Science Study (TIMSS), which caused by van Hiele levels of geometric thinking. Therefore, the purpose of this study was to investigate the van Hiele levels of geometric thinking among secondary school students in Makassar, Indonesia. A total of 298 respondents randomly took part in this study. The van Hiele geometric thinking test was used to assess the student's level of geometric thinking. Data were in ordinal form analysed according to the weighted van Hiele geometric thinking test scores presented in the table. The findings showed that most of the students were at the lowest level of geometric thinking. Several 123 and 93 respondents were at Level 0 (Visualisation) and Level 1 (Analysis), respectively. Meanwhile, 70 respondents were lower Level 0 and only a few respondents were in the upper Level 1. The result might be used as a fundamental source to produce a learning strategy in elevating van Hiele levels of geometric thinking.

Keywords*: geometric thinking, van hiele level of geometric thinking, secondary school student.

I. INTRODUCTION

Geometry is a subject that is learned in the mathematics curriculum from elementary to the secondary school level. However, many researchers [1–4] found that learning Geometry was difficult to understand and most of the students failed to master Geometry concepts. This issue has also been studied by van Hiele [5], who found that students' difficulty in understanding the Geometry concept is because of a lack of prerequisite knowledge about Geometry and a low level of geometric thinking. According to him, this is a requirement for students to understand the Geometry concept.

Geometry is one of the mathematic subjects that students around the world cannot easily master. Trends in International Mathematics and Science Study (TIMSS) 2015 showed that students in Malaysia, Turkey, Georgia and Thailand have low achievement in Geometry subject [6]. Including Indonesian students, TIMSS 2011 [7] revealed that Indonesia is only in rank five from below in Geometry subject. Puspendik [8] also reported that the Indonesian students in TIMSS results have declined from year to year. Indonesia was in rank 38th with scores 377 from 42 countries involved.

Recent studies in Indonesia such as Abu and Abidin [9], Burais and Husna [10] and Kurniawati et al. [11] asserted that the main cause of low students Geometry achievement is due to the low level of geometric thinking. They also found that students who learn the concept of geometry with the memorisation approach often fail in recognising Geometry shapes and relationships among them. In addition, Hardianti et al. [2] found that Indonesian students' level of geometric thinking is still at a low level. One of the reason is that learning in the classroom focuses only on transfer of knowledge from the teacher to the students regardless of the van Hiele learning phases or even do not consider van Hiele level of geometric thinking. As a result, students only memorise without understanding the concept given in the learning process. Though, according to van Hiele [5], Geometry learning needs to pay attention to the level of geometric thinking and a learning process in order for students to increase their achievement in Geometry.

Even though Abu and Abidin [9], Hardianti et al. [2] and Kurniawati et al. [11] have proved the positive impact of van Hiele learning phases in elevating level of geometric thinking, most of the teacher failed to improve and implement teaching and learning Geometry based on van Hiele model [12]. Hence, based on the information mentioned earlier, learning difficulties by Indonesian students are associated with visualisation, which is the basic level of van Hiele geometric thinking. Other weaknesses encountered by Indonesian students are associated with inconsistency in the van Hiele level of geometric thinking.

A. van Hiele Level of Geometric Thinking

Pierre Marie van Hiele and Dina van Hiele-Geldof in 1956 produced a model in geometric learning, which was levels of geometric thinking and learning phases. Crowley [13] emphasised that teachers should

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produce Geometry learning materials to enable students to mastering the content in each level of geometric thinking and move to the next level.

van Hiele model has been integrated into the geometry learning curriculum in developing countries, including the United States, Russia, and Japan [7]. This model has been referred to designing and developing learning instruction to enhance students' higher-order thinking skills in Geometry [14]. Integration of this model has successfully boosted students Geometry achievements as reported in TIMSS. In TIMSS 2011 [7], United States was in rank 9th with average score 509, while Russia and Japan were rank 6th and 5th with score 539 and 570, respectively. Meanwhile, in TIMSS 2015 [6], the average score of international mathematics achievement of the United States and Japan were 518 and 586, increasing 9 and 17 points from 2011, while Russia decreased a point. However, they were stable at the top position from 2011 to 2015.

van Hiele model explained five students' level of geometric thinking in learning Geometry [15]. First, visualisation (Level 0), that is visual or basic level in which students only recognise Geometry shapes from visual appearance. Not focusing on the properties of the object been seen so that students cannot determine geometric properties. Second, analysis (Level 1), that is descriptive level, in which students enable to look at and analyse the concept and geometric properties by doing observation, measurement, experiment, drawing and creating a model. However, students cannot fully explain the relationship between these traits. Third, informal deduction (Level 2), that is a relationship or theoretical, in which students have seen the relationship between the properties of geometric objects and able to classify the objects hierarchically. Fourth, formal deduction (Level 3), that is level, in which students can receive and construct a proof and a theorem in the axiom system. Fifth, rigour (Level 4), that is axiomatic in which student's mathematical thinking develops formally and be able to analyse the effects of axiomatic manipulation and definition. However, most high school students can only achieve informal deduction level (Level 2) [16], [17]. Supposedly, students in secondary school should be at a formal deduction level (Level 3) [18].

B. van Hiele Phase-Based Learning

van Hiele [19] considered that the development of student's geometric thinking does not depend on the age and maturity of students, yet depending on student's thinking ability and also the learning experiences. Thus, van Hiele proposed five learning phases in order for the student to improve the level of geometric thinking from one stage to the next [13].

First, information, which is an interaction between teacher and students through discussion. Connolly [20] explain that teachers need to propose new information in each of the best-designed questions so that students are able to express the relevance of the initial concepts with the content to be learned; Second, guided orientation, in which students make exploration through guided activities. Clements and Battista [21] and Crowley [13] emphasise that teachers need to instruct the students to study the learned objects. Activity giving instruction to the students is a series of short tasks in getting certain stimuli from students; Third, explication, in which students enable to explain and express their views on their work. Clements and Battista [21] explained that in this phase, the teachers bring a combination of objects (eg geometry ideas, relationships, patterns, etc.) to the level of understanding through discussions among students in using language accuracy; Fourth, free orientation, in which students can solve more complex questions. Crowley [13] suggested that students are given the opportunity to learn how to solve the problems by the students themselves so that the students will see more clearly the relationships among the properties of solid; Fifth, integration, in which students make a conclusion from the learning. Crowley [13] emphasised that by the end of this phase, students have reached a certain level of geometric thinking and are ready to repeat these five phases of learning for the next level of geometric thinking.

However, the learning method used in Indonesia still depends on transferring knowledge from teacher to students so that students only memorise without understanding the geometry concept given. This is in line with the study by Abu and Abidin [9] and Hardianti et al. [2] which showed that traditional geometry teaching and learning method contributed to the low level of geometric thinking among secondary school students. The study by Nadjib [22] in South Sulawesi context showed unsatisfactory results. From 25 respondents, only a student reached informal deduction level (Level 2) and most respondents were at the analysis level (Level 1). It is due to two factors, which were internal and external factors. The internal factor was the students psychology and mental or readiness to learn, whereas the external factor was the environment and instrumental (eg curriculum, learning program, activity, teacher, and facility) [23]. In addition, traditional geometry learning through lecture using school textbooks has not assisted yet students in understanding geometry concepts [24].

Based on the discussion of van Hiele levels of geometric thinking above, this study strives to investigate the van Hiele levels of geometric thinking among secondary school students in Makassar, South Sulawesi Province, Indonesia.

II. METHOD

A total of 298 secondary school students (17-18-year-old) in the Senior High School of 5 Makassar was randomly involved in this study. van Hiele Geometry Test (vHGT) was used to assess the students level of geometric thinking, which was developed by The Cognitive Development and Achievement in Secondary School Geometry project (CDASSG) and applied by Usiskin [15] upon 2900 secondary school students with $r = 0.64$ [25]. vHGT comprised 25 items that if the respondents answered correctly at least 3 of 5 items at any level, the respondents were considered being mastered at that level. In addition, Usiskin [15] established weighted van Hiele geometric thinking test scores to define the van Hiele levels of geometric thinking, as portrayed in Table I.

Table- I: Weighted van Hiele Geometric Thinking Test Scores

Question Number	van Hiele Level of Geometric Thinking	Score
1 – 5	Level 0: Visualisation	1
6 – 10	Level 1: Analysis	2
11 – 15	Level 2: Informal Deduction	4
16 – 20	Level 3: Formal Deduction	8
21 – 25	Level 4: Rigour	16

If the respondents, for instance, obtained the scores at level 0, 1, and 4, then their score was 19 (1 + 2 + 16). However, data was in ordinal form and the Indonesian version of vHGT was obtained by Abidin [26].

III. RESULT AND DISCUSSION

The result in this study showed that most of the respondents (41.6%) successfully mastered at the visualisation level (L0) and only 31.4% of respondents attained Level 1 of analysis. Interestingly, 23.6% of the respondents were below L0 (*L0). Meanwhile, almost all the respondents failed to reach out the informal deduction level (L2) and Level 3 of formal deduction, as reflected in Table II.

Table- II: Descriptive of van Hiele Level of Geometric Thinking of Respondents

Level	Number	Percentage
*L0	70	23.6
L0 – Visualisation	123	41.6
L1 – Analysis	93	31.4
L2 – Informal Deduction	9	3.0
L3 – Formal Deduction	1	0.3
L4 – Rigour	0	0.0
Total	296	100

It has been found that majority of the respondents had achieved van Hiele's visualisation level of geometric thinking. Only 3.0% was at Informal Deduction level. This finding is parallel with Abdullah and Zakaria [16], Abu and Abidin [9], Andini et al. [27], and Wahab et al. [28] who found that most of the students achieved visualisation level of geometric thinking prior to being conducted the intervention. In fact, secondary school students were supposed to reach out level 3, which is formal deduction, before entering the university or college, according to NCTM [29].

Based on the result, the traditional teaching and learning approach has failed to increase the student's van Hiele level of geometric thinking. Hence, teaching and learning geometry in the classroom, need to be improved in assisting students to overcome their difficulties in terms of van Hiele level of geometric thinking. van Hiele [19] proposed five phases of learning to be applied in teaching and learning geometry, which is Information, Guided Orientation, Explication, Free Orientation, and Integration. According to Meng [30], in order to achieve each van Hiele level of geometric thinking, students should go through all five learning phases to advance from the basic level to the next stages. Besides, there is another factor that can overcome the difficulty of students in learning geometry, which is metacognition. The studies by Abdullah et al. [31] and Naufal et al. [32] found that student's performance in mathematical problem solving was unsatisfactory because of the lack of metacognitive skills. This was consistent with Alzahrani's [33] study founded mathematics activities without metacognitive teaching hinder students to perform better. And also, there is a lack of empirical studies which focus on the effect of metacognition towards geometric thinking. Hence, students need also to be prepared with high metacognitive skills in order to increase their level of geometric thinking [34].

IV. CONCLUSION

These results as a fundamental source suggest to the teachers and researchers to design and develop learning instruction properly based on van Hiele model, which is level of geometric thinking and van Hiele learning phases helped improve students' van Hiele level of geometric thinking. This is in line with Abu and Abidin [9] which showed the improvement of the students after learning intervention based on van Hiele learning phases in increasing level of geometric thinking. It is also in accordance with Muhassanah et al. [35] who suggested creating a teaching and learning model adjusted to each level of geometric thinking of students. Apart from that, teaching and learning geometry should also infuse to the metacognition component in order to enhance geometric thinking of students. According to Abdullah et al. [36], Finnel [34], and Rofii et al. [17], the metacognitive skills enable students to assist their thinking skills in understanding, planning, monitoring, implementing strategies, reflecting, and evaluating the answer to the geometrical problems. Therefore, in future investigations, it might be possible to utilize a different approach in which pupils with a high level of metacognition integrated with the van Hiele model in learning and instruction ought to more effective in raising their level of geometric thinking.

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