Fuzzy Positional Function Via Fuzzy Filter

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Abstract: ***The research was based on the principle of defining the fuzzy positional function applying the fuzzy filter. Significant properties were obtained and examples were provided to clarify these properties.***

Keywords: Fuzzy ideal, Fuzzy local function, Fuzzy filter, Fuzzy positional function

I. INTRODUCTION

The fuzzy sets is one of the important and vital topics as it entered into various applied and pure sciences, and it took place in engineering with its various branches, computers and others. Zadeh is the first to define fuzzy sets in 1965[1].In 2019 Al-Razzaq AS and AL-Swidi LA They classified the fuzzy sets theory as families [2]. Well, in the same year they Finding and Taxonomy a New Fuzzy Soft points [3] and introduce the definition Soft Generalized Vague Sets [4].Among the pure sciences took a great deal in general topology. Where our research is based on the fuzzy filter. Lowen is the first to introduce the concept in 1979[5]. After which many scientists and researchers who completed this topic came, including Prada and Saralegui they introduce a characterization of fuzzy filter concept [6]in1988. Sarker is the first to define the fuzzy ideal as well as the fuzzy local function[7] in 1997.in 2019 AL Mohammed R and AL-Swidi LA. They got New Concepts of Fuzzy Local Function[8] .Where there is a relationship between the filter and the ideal in the usual sets, we also found a relationship between them using the fuzzy sets . The research is based on the study of the fuzzy positional function depending the fuzzy filter.

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II. PRELIMINARIES

In this section we will mention the most important concepts used in this paper. Where the triple $(\breve{\mathrm{I}},\tau,\breve{\mathcal{F}})$ is called a fuzzy filter topological space (simply $\mathrm{F}\mathcal{F}\mathrm{T}\mathcal{S}$) for which $(\breve{\mathrm{I}},\tau)$ is fuzzy topology space in Change [10], and $\breve{\mathcal{F}}$ is collection fuzzy filter define in Lowen [5].

Definition2.1.[2].

Let the membership $\mathcal{M}(\mathrm{X},\mathrm{I}) = \{\mathcal{P};\mathcal{P}:\mathrm{X}\to \mathrm{I}\}$ where X any nonempty set, I= [0, 1]. A fuzzy set $\breve{\mathcal{A}}$ of the space X × I define as follows,

$\breve{\mathcal{A}} = \{(x,\mathcal{P}_{\mathcal{A}}(x)), \forall x \in \mathrm{X}\}$

where the membership $\mathcal{P}_{\mathcal{A}}(x) = \begin{cases} f(x) & for\ \ x \in \mathcal{A} \\ 0 & for\ \ x \notin \mathcal{A} \end{cases}$

Example 2.2.

Let $\mathrm{X} = \{1,2,3\}$, $\mathcal{A} = \{1\}$ and $\mathcal{B} = \{2,3\}$, when the memberships of $\breve{\mathcal{A}},\breve{\mathcal{B}}$ are.

$\mathcal{P}_{\mathcal{A}}(x) = \frac{1}{x^3}$, $\mathcal{T}_{\mathcal{B}}(x) = \frac{2x-1}{10}$. Hence

$\breve{\mathcal{A}}$={(1,1), (2, 0), (3,0)}, $\breve{\mathcal{B}}$={(1, 0), (2, 0.3), (3,0.5)}.

Definition2.3.[4].

A fuzzy point is fuzzy set in Γ^{X} denoted by $\mathrm{P}_x^{\mathcal{E}}$ for with support $x \in \mathrm{X}$ and $\lambda \in (0,1]$ and the membership is $\mathrm{P}_x^{\mathcal{E}}(w) = \begin{cases} \mathcal{E} & \text{if } w = x \\ 0 & \text{if } w \neq x \end{cases}$,

Note $\mathrm{P}_x^{\mathcal{E}} \subset \breve{\mathcal{A}}$ iff $\mathcal{E} \le \mathcal{P}_{\mathcal{A}}(x)$ and $\breve{\mathcal{A}} \subseteq \breve{\mathcal{B}}$ if and only if $\mathcal{P}_{\mathcal{A}}(x) \le \mathcal{T}_{\mathcal{B}}(x)$ .where $\breve{\mathcal{A}}$ and $\breve{\mathcal{B}}$ are fuzzy sets for withs

Definition 2.4.[11].

Let $\breve{\mathcal{A}}$, $\breve{\mathcal{B}}$ be a fuzzy sets, then $\breve{\mathcal{A}}$ is called quasi-coincident with $\breve{\mathcal{B}}$ denoted by $\breve{\mathcal{A}}\mathrm{q}\breve{\mathcal{B}}$ iff there exists $y \in \mathrm{X} \ni f_{\mathcal{A}}(y) + g_{\mathcal{B}}(y) > 1$. The collection of all quasi-coincident denoted by $q - \mathcal{N}(P_x^{\lambda})$. Otherwise, we called $\breve{\mathcal{A}}$ not quasi-coincident with $\breve{\mathcal{B}}$ if $f_{\mathcal{A}}(x) + g_{\mathcal{B}}(x) \le 1, \forall x \in \mathrm{X}$ and denoted by $\breve{\mathcal{A}}\wp\breve{\mathcal{B}}$.

Lemma 2.4.[12].

Let $\breve{\mathcal{A}}$ and $\breve{\mathcal{B}}$ be a fuzzy set in Γ^{X}. Then the following are satisfying.

1. If $(\breve{\mathcal{A}} \wedge \breve{\mathcal{B}}) \neq \breve{0}$, then $\exists\ x \in X$ and $\lambda \in (0,1]$ such that $\lambda \le f_{\mathcal{A}}(x)$ and $\lambda \le g_{\mathcal{B}}(x)$.
2. If $\breve{\mathcal{A}} \neq \breve{0}$, then $\exists\ x \in X$ and $\lambda \in (0,1]$ such that $\mathrm{P}_x^{\lambda} \in \breve{\mathcal{A}}$
3. If $\breve{\mathcal{A}} \neq \breve{0}$, then $\exists\ x \in X$ and $\lambda \in (0,1]$ such that $\mathrm{P}_{\mathrm{x}}^{\lambda} q\breve{\mathcal{A}}$
4. If $(\breve{\mathcal{A}} \wedge \breve{\mathcal{B}}) \neq \breve{0}$, then $\exists\ x \in X$ and $\lambda \in (0,1]$ such that $\mathrm{P}_{\mathrm{x}}^{\lambda} q\breve{\mathcal{A}}$ and $\mathrm{P}_{\mathrm{x}}^{\lambda} q\breve{\mathcal{B}}$.
5. If $\breve{\mathcal{A}} \neq \breve{0}$, then $\exists\ x \in X$ and $\lambda \in (0,1]$ such that $\mathrm{P}_{\mathrm{x}}^{\lambda} \wp\breve{\mathcal{A}}$.

Definition 2.5 .[9].

Let $\breve{\mathcal{A}}$, $\breve{\mathcal{B}}$ any fuzzy set in Γ^{X}. stander intersection, The stander of union, difference, complement are from.

1. $\breve{\mathcal{A}} \wedge \breve{\mathcal{B}}$ = {(x, min { $f_{\mathcal{A}}(x), \mathrm{g}_{\mathcal{B}}(x)$}), $\forall x \in \mathrm{X}$ }
2. $\breve{\mathcal{A}} \vee \breve{\mathcal{B}}$ = {(x, max { $f_{\mathcal{A}}(x), \mathrm{g}_{\mathcal{B}}(x)$}), $\forall x \in \mathrm{X}$}
3. Let $\breve{\mathcal{A}},\breve{\mathcal{B}}$ be a fuzzy sets the difference defines by:

- $(\breve{\mathcal{A}} - \breve{\mathcal{B}})_{max} = \{(x, \max\{f_{\mathcal{A}}(x) - \mathrm{g}_{\mathcal{B}}(x), 0\}), \forall x \in \mathrm{X}\}$.
- $(\breve{\mathcal{A}} - \breve{\mathcal{B}})_{min} = \{(x, \min\{f_{\mathcal{A}}(x), \mathrm{g}_{(1-\mathcal{B})}(x)\}), \forall x \in \mathrm{X}\}$.

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4. $1-\breve{\mathcal{A}} = \{(x, 1 - f_{\mathcal{A}}(x)),$
$\forall x \in X\}$.

Definition 2.7.[7].

Let $(\breve{1}, \tau, \breve{I})$be a $FI\mathcal{TS}$. The fuzzy local function of $\breve{\mathcal{A}}$ of the first type $\breve{\mathcal{A}}^{*1}(\breve{I}, \tau)$ is defined by:

$\breve{\mathcal{A}}^{*1}(\breve{I}, \tau) = \vee\{P_x^\lambda;\ \forall \breve{Z} \in q - \mathcal{N}(P_x^\lambda), \exists\ y \in X \ni f_Z(y) + g_{\mathcal{A}}(y) - 1\} > h_l(y)$for every$\breve{l} \in \breve{I}\}$.The fuzzy local function $\breve{\mathcal{A}}^{*1}$ or $\breve{\mathcal{A}}^{*1}(\breve{I})$ or $\breve{\mathcal{A}}^{*1}(\breve{I}, \tau)$.

Therefore, if $P_x^\lambda \notin \breve{\mathcal{A}}^{*1}(\breve{I}, \tau)$.Then there exist $\breve{\mathcal{W}} \in q - \mathcal{N}(P_x^\lambda) \ni \forall x \in X, f_W(x) + g_{\mathcal{A}}(x) - 1 \le h_J(x)$ for some $\breve{J} \in \breve{I}$.

III. FUZZY POSITIONAL FUNCTION

In this section we will introduce a new concept called the fuzzy positional function while discussing the most important characteristics associated with this concept and giving examples showing those characteristics.

Definition 3.1.

Let $(\breve{1}, \tau, \breve{\mathcal{F}})$ be a $F\mathcal{FTS}$. the fuzzy positional function of $\breve{\mathcal{A}}$ of the first type $\breve{\mathcal{A}}^{\#1}(\breve{\mathcal{F}}, \tau)$ defined by:

$\breve{\mathcal{A}}^{\#1}(\breve{\mathcal{F}}, \tau) = \vee\{P_x^\lambda;\ \forall \breve{u} \in q - \mathcal{N}(P_x^\lambda), \forall\ y \in X \ni max\{0, f_u(y) + g_{\mathcal{A}}(y) - 1\} \ge h_F(y)$ for some $\breve{F} \in \breve{\mathcal{F}}\}$. The fuzzy positional function of $\breve{\mathcal{A}}$ Denoted by$\widetilde{\mathcal{A}}^{\#1}$or $\breve{\mathcal{A}}^{\#1}(\breve{\mathcal{F}})$ or $\breve{\mathcal{A}}^{\#1}(\breve{\mathcal{F}}, \tau)$.

Therefore, if $P_x^\lambda \notin \breve{\mathcal{A}}^{\#1}(\breve{\mathcal{F}}, \tau)$ then there exist $\breve{n} \in q - \mathcal{N}(P_x^\lambda)$, $x \in X$ $max\{0, f_n(x) + g_{\mathcal{A}}(x) - 1\} < h_f(x)$ for every $\breve{f} \in \breve{\mathcal{F}}$.

Example 3.2.

Let $(\breve{1}, \tau, \breve{\mathcal{F}})$ be a $F\mathcal{FTS}$ and $X = \{1, 2, 3\}$, A, E, K and F are subset of X s.t

$A = \{1, 2, 3\}$ with the memberships of $\breve{A}$,

$$T_A(x) = \begin{cases} \frac{10-X}{10} & , x \in A \\ 0, & x \notin A \end{cases}$$

$E = \{1, 2, 3\}$ with the memberships of $\breve{E}$,

$$L_E(x) = \begin{cases} \frac{9}{10} & \text{if } x \text{ is odd}, x \in E \\ \frac{3}{5} & \text{if } x \text{ } is\ even, x \in E \\ 0 & \text{if } \quad , x \notin E \end{cases}$$

$K = \{3\}$ with the memberships of $\breve{K}$,

$$R_K(x) = \begin{cases} \frac{x^2-2}{10} & , x \in K \\ 0, & x \notin K \end{cases}$$

$F = \{1, 2, 3\}$ with the memberships of $\breve{F}$,

$$h_F(x) = \begin{cases} \frac{2x+1}{10} & , x \in F \\ 0, & x \notin F \end{cases}$$

we get,

$\breve{A} = \{(1, 0.9), (2, 0.8), (3, 0.7)\}$,

$\breve{E} = \{(1, 0.9), (2, 0.6), (3, 0.9)\}$,

$\breve{K} = \{(1, 0), (2, 0), (3, 0.7)\}$,

Let $\tau = \{\breve{0}, \breve{1}, \breve{E}, \breve{K}\}$ and

$\breve{\mathcal{F}} = \{\breve{1}, \breve{F}\} \cup \{\breve{\mathcal{G}}; \breve{\mathcal{G}} \ge \breve{F}\}$.Where

$\breve{F} = \{(1, 0.3), (2, 0.5), (3, 0.7)\}$.

Then, $\breve{\mathcal{A}}^{\#1} = \{(1, 0.1), (2, 0.4), (3, 0.1)\}$.

Proposition 3.3.

Let $(\breve{1}, \tau, \breve{\mathcal{F}})$ be a $F\mathcal{FTS}$, let $\breve{\mathcal{F}}$ be a filter fuzzy, $\breve{I}$ fuzzy ideal, $\breve{\mathcal{A}}$any fuzzy set if $\breve{\mathcal{A}} \notin \breve{I}$ then $\breve{\mathcal{A}}^{\#1}(\breve{\mathcal{F}}) \subseteq \breve{\mathcal{A}}^{*1}(\breve{I})$, with the filter $\breve{\mathcal{F}} = \{\breve{\mathcal{A}};\ \breve{\mathcal{A}}^c \in \breve{I}\}$, but the converse may be not true.

Proof.

Let $P_x^\lambda \in \breve{\mathcal{A}}^{\#1}(\breve{\mathcal{F}})$ for every $\forall \breve{u} \in q - \mathcal{N}(P_x^\lambda), \forall\ y \in X$; $max\{0, f_u(y) + g_{\mathcal{A}}(y) - 1\} \ge h_F(y)$ for some $\breve{F} \in \breve{\mathcal{F}}$ }, but $\breve{\mathcal{F}} = \{\breve{\mathcal{A}};\ \breve{\mathcal{A}}^c \in \breve{I}\}$ $\forall \breve{u} \in q - \mathcal{N}(P_x^\lambda), \exists\ y \in X \ni f_u(y) + g_{\mathcal{A}}(y) - 1 > h_\ell(y)$ for every $\breve{\ell} \in \breve{I}\}$. Thus $P_x^\lambda \in \breve{\mathcal{A}}^{*1}(\breve{I})$.

The following example shows that the converse of property is not true.

Example 3.4.

Let $(\breve{1}, \tau, \breve{\mathcal{F}})$ be a $F\mathcal{FTS}$, $X = \{1, 2, 3\}$. A, B, C and I are subset of X such that

$A = \{1, 2, 3\}$ s.t the memberships of $\breve{A}$,

$$f_A(x) = \begin{cases} \frac{10-X}{10} & , x \in A \\ 0, & x \notin A \end{cases}$$

$B = \{1, 3\}$ s.t the memberships of $\breve{B}$,

$$K_B(x) = \begin{cases} \frac{1}{5} & \text{if } x \text{ is odd}, x \in B \\ \frac{7}{10} & \text{if } x \text{ } is\ even, x \in B \\ 0 & \text{if } \quad , x \notin B \end{cases}$$

$C = \{2, 3\}$ s.t the memberships of $\breve{C}$,

$$g_C(x) = \begin{cases} \frac{3}{10} & \text{if } x \text{ is odd}, \quad x \in C \\ \frac{6}{10} & \text{if } x \text{ } is\ even, x \in C \\ 0 & \text{if } \quad , \quad x \notin C \end{cases}$$

$I = \{1, 2, 3\}$ s.t the memberships of $\breve{I}$,

$$h_I(x) = \begin{cases} \frac{3x}{10} & , x \in I \\ 0 & , x \notin I \end{cases}$$

we get,

$\breve{A} = \{(1, 0.9), (2, 0.8), (3, 0.7)\}$,

$\breve{B} = \{(1, 0.7), (2, 0), (3, 0.7)\}$,

$\breve{C} = \{(1, 0), (2, 0.6), (3, 0.3)\}$,

$\breve{B} \wedge \breve{C} = \{(1, 0), (2, 0), (3, 0.7)\}$,

$\breve{B} \vee \breve{C} = \{(1, 0.5), (2, 0.6), (3, 0.3)\}$.

Let $\tau = \{\breve{0}, \breve{1}, \breve{B}, \breve{C}, \breve{B} \wedge \breve{C}, \breve{B} \vee \breve{C}\}$ and

$\breve{I} = \{\breve{0}, \breve{\jmath}\} \cup \{\breve{\ell}; \breve{\ell} \le \breve{\jmath}\}$.Where

$\breve{\jmath} = \{(1, 0.3), (2, 0.6), (3, 0.9)\}$.

$\breve{\mathcal{F}} = \{\breve{1}, \breve{F}\} \cup \{\breve{\mathcal{G}}; \breve{\mathcal{G}} \ge \breve{F}\}$.Where

$\breve{F} = \{(1, 0.7), (2, 0.4), (3, 0.1)\}$.

thus, $\breve{A}^{*1} = \{(1, 1), (2, 0.4), (3, 0.3)\}$.

But $\breve{A}^{\#1} = \{(1, 0.5), (2, 0.4), (3, 0.3)\}$.

Theorem 3.5.

Let $(\breve{1}, \tau, \breve{\mathcal{F}})$be a $F\mathcal{FTS}$. Let $\breve{\mathcal{F}}$ fuzzy filter and $\breve{\mathcal{A}}, \breve{\mathcal{B}}$ any two fuzzy sets. Then the following are satisfying.

1. $\breve{\mathcal{A}} \subseteq \breve{\mathcal{B}} \Longrightarrow \breve{\mathcal{A}}^{\#1} \subseteq \breve{\mathcal{B}}^{\#1}$.
2. $(\breve{\mathcal{A}} \vee \breve{\mathcal{B}})^{\#1} = \breve{\mathcal{A}}^{\#1} \vee \breve{\mathcal{B}}^{\#1}$.
3. $(\breve{\mathcal{A}} \wedge \breve{\mathcal{B}})^{\#1} = \breve{\mathcal{A}}^{\#1} \wedge \breve{\mathcal{B}}^{\#1}$.

Proof.

1. Let $P_x^\lambda \in \breve{\mathcal{A}}^{\#1}$ for every $\breve{\mathcal{U}} \in q^- \mathcal{N}(P_x^\lambda)\ \forall\ y \in$

X $\ni$ $max\{0, f_{\mathcal{U}}(y) + g_{A}(y) - 1\} \geq h_F(y)$ for some $\breve{F} \in \breve{\mathcal{F}}$.

since $\breve{\mathcal{A}} \leq \breve{\mathcal{B}}$,so $g_{\mathcal{A}}(y) \leq g_{\mathcal{B}}(y)$ then $f_{\mathcal{U}}(y) + g_{\mathcal{A}}(y) - 1 \leq f_{\mathcal{U}}(y) + g_{\mathcal{B}}(y) - 1$ this leads to $max\{0, f_{\mathcal{U}}(y) + g_{\mathcal{B}}(y) - 1\} \geq h_F(y)$ for some $\breve{F} \in \breve{\mathcal{F}}$, thus $P_x^\lambda \in \breve{\mathcal{B}}^{\#1}$.

2. $\breve{\mathcal{A}} \subseteq (\breve{\mathcal{A}} \vee \breve{\mathcal{B}})$, this implies $\breve{\mathcal{A}}^{\#1} \subseteq (\breve{\mathcal{A}} \vee \breve{\mathcal{B}})^{\#1}$, by (1) .

Also $\breve{\mathcal{B}} \subseteq (\breve{\mathcal{A}} \vee \breve{\mathcal{B}})$ this implies $\breve{\mathcal{B}}^{\#1} \subseteq (\breve{\mathcal{A}} \vee \breve{\mathcal{B}})^{\#1}$, by (1). Thus $\breve{\mathcal{A}}^{\#1} \vee \breve{\mathcal{B}}^{\#1} \subseteq (\breve{\mathcal{A}} \vee \breve{\mathcal{B}})^{\#1}$

Conversely , let $P_x^\lambda \in (\breve{\mathcal{A}} \vee \breve{\mathcal{B}})^{\#1}$ if possible that $P_x^\lambda \notin (\breve{\mathcal{A}}^{\#1} \vee \breve{\mathcal{B}}^{\#1})$, which means $P_x^\lambda \notin \breve{\mathcal{A}}^{\#1}$ and $P_x^\lambda \notin \breve{\mathcal{B}}^{\#1}$, then there exist $\breve{\mathcal{U}} \in q\text{-}\mathcal{N}(P_x^\lambda)$ such that, $\exists\, x \in X \ni$ $max\{0, f_{1\mathcal{U}}(x) + g_{1\mathcal{A}}(x) - 1\} < h_{1F_1}(x)$for every $\breve{F}_1 \in \breve{\mathcal{F}}$ and there exists $\breve{\mathcal{V}} \in q\text{-}\mathcal{N}(P_x^\lambda)$ such that $\exists\, x \in X \ni$ $max\{0, f_{2\mathcal{V}}(x) + g_{2\mathcal{B}}(x) - 1\} < h_{2F_2}(x)$ for every $\breve{F_2} \in \breve{\mathcal{F}}$

Clear, $(\breve{\mathcal{U}} \wedge \breve{\mathcal{V}}) \in$ q_$\mathcal{N}(P_x^\lambda)$ put up $(\widetilde{\mathcal{U} \wedge \mathcal{V}}) = \breve{\mathcal{W}} \in$ q_$\mathcal{N}(P_x^\lambda)$, But $max\{0, f_{3\mathcal{W}}(x) + g_{1\mathcal{A}}(x) - 1\} \leq max\{0, f_{1\mathcal{U}}(x) + g_{1\mathcal{A}}(x) - 1\} \leq h_{1F_1}(x)$, also max $\{0, f_{3\mathcal{W}}(x) + g_{2\mathcal{B}}(x) - 1\} \leq max\{0, f_{2\mathcal{V}}(x) + g_{2\mathcal{B}}(x) - 1\} \leq h_{2F_2}(x)$, thus max$\{$ $max\{0, f_{3\mathcal{W}}(x) + g_{1\mathcal{A}}(x) - 1\}, max\{0, f_{3\mathcal{W}}(x) + g_{2\mathcal{B}}(x) - 1\}\} \leq$ max $\{h_{1F_1}(x), h_{2F_2}(x)\}$, then $max\{0, f_{3\mathcal{W}}(x) +$ max $\{g_{1\mathcal{A}}(x), g_2(x)\} - 1\} \leq max\ \{h_{1F_1}(x), h_{2F_2}(x)\}$,

then there exists $\breve{\mathcal{W}} \in$ q_$\mathcal{N}(P_x^\lambda)$ s.t for some $x \in X$, $max\{0, f_{3\mathcal{W}}(x) + g_{3(\mathcal{A}\ \vee \mathcal{B})}(x) - 1\} \leq h_{3(F_1 \vee F_2)}(x)$ for every $(\breve{F}_1 \vee \breve{F}_2) \in \breve{\mathcal{F}}$, this means $P_x^\lambda \notin \breve{\mathcal{A}}^{\#1} \vee \breve{\mathcal{B}}^{\#1}$ which contradiction, thus $(\breve{\mathcal{A}} \vee \breve{\mathcal{B}})^{\#1} \subseteq \breve{\mathcal{A}}^{\#1} \vee \breve{\mathcal{B}}^{\#1}$.

3. $(\breve{\mathcal{A}} \wedge \breve{\mathcal{B}}) \subseteq \breve{\mathcal{A}}$, This implies $(\breve{\mathcal{A}} \wedge \breve{\mathcal{B}})^{\#1} \subseteq \breve{\mathcal{A}}^{\#1}$, $(\breve{\mathcal{A}} \wedge \breve{\mathcal{B}}) \subseteq \breve{\mathcal{B}}$, This implies $(\breve{\mathcal{A}} \wedge \breve{\mathcal{B}})^{\#1} \subseteq \breve{\mathcal{B}}^{\#1}$, thus $(\breve{\mathcal{A}} \wedge \breve{\mathcal{B}})^{\#1} \subseteq \breve{\mathcal{A}}^{\#1} \wedge \breve{\mathcal{B}}^{\#1}$.

Again, Let $P_y^\lambda \in (\breve{\mathcal{A}}^{\#1} \wedge \breve{\mathcal{B}}^{\#1})$, this implies $P_y^\lambda \in \breve{\mathcal{A}}^{\#1}$ and $P_y^\lambda \in \breve{\mathcal{B}}^{\#1}$.$\forall\, \breve{\mathcal{U}} \in q - \mathcal{N}(P_y^\lambda) \forall\, y \in X$

s.t $\max\{0, f_{1\mathcal{U}}(y) + g_{1\mathcal{A}}(y) - 1\} \geq h_{2F_1}(y)$ for some $\breve{F}_1 \in \breve{\mathcal{F}}$ .So, $\forall\, \breve{\mathcal{V}} \in q - \mathcal{N}(P_y^\lambda)$; $\forall y \in X \ni$ $\max\{0, f_{2\mathcal{V}}(y) + g_{2\mathcal{B}}(y) - 1\} \geq h_{2F_2}(y)$ for some $\breve{F_2} \in \breve{\mathcal{F}}$.

If possible $P_y^\lambda \notin (\breve{\mathcal{A}} \wedge B^z)^{\#1}$. then $\exists \breve{\mathcal{W}} \in$ $q^-\mathcal{N}(P_x^\lambda) \ni y \in X; \{0, f_{3\mathcal{W}}(y) + min\ \{g_{1\mathcal{A}}(y), g_{2\mathcal{B}}(y)\} - 1\} < h_j(y)$ for every $\breve{j} \in \breve{\mathcal{F}}$.

if $min\ \{g_{1\mathcal{A}}(y), g_{2\mathcal{B}}(y)\} = g_{1A}(y)$ this implies that, $\max\{0,\ f_{3\mathcal{W}}(y) +\ g_{1\mathcal{A}}(y) - 1\} < h_j(y)$ for some $\breve{j} \in \breve{\mathcal{F}}$ this contradiction. Also when $min\ \{g_{1\mathcal{A}}(y), g_{2\mathcal{B}}(y)\} = g_{2\mathcal{B}}(y)$, thus $\max\{0, f_{3\mathcal{W}}(y) +\ g_{2\mathcal{B}}(y) - 1\} < h_j(y)$ for every $\breve{j} \in \breve{\mathcal{F}}$ this contradiction. Hence $P_y^\lambda \in (\breve{\mathcal{A}} \wedge \breve{\mathcal{B}})^{\#2}$. Thus $\breve{\mathcal{A}}^{\#1} \wedge \breve{\mathcal{B}}^{\#1} \subseteq (\breve{\mathcal{A}} \wedge \breve{\mathcal{B}})^{\#1}$.

Remark 3.6.

The converse of property 1 is not satisfying as in the example**.**

Let $(\breve{1}, \tau, \breve{\mathcal{F}})$ be a $F\mathcal{FTS}$ and $X = \{1, 2, 3\} = A = B = F = C$, D={2}.The membership of $\breve{\mathcal{A}}, \breve{\mathcal{B}}, \breve{\mathcal{C}}$, $\breve{F}$and $\breve{\mathcal{D}}$are.

$f_A(x) = \frac{x+4}{10}\ \forall x \in A$, $g_B(x) = \frac{x+6}{10}\ \forall x \in B$,

$K_C(x) = \frac{10-X}{10} \forall x \in C$, $h_D(x) = \frac{x}{10}\ \ \forall x \in D$,

$h_F(x) = \frac{x+3}{10}\ \ \forall x \in F_1$.

$\breve{\mathcal{A}}$={(1, 0.5), (2, 0.6), (3,0.7)},,

$\breve{\mathcal{B}}$={(1,0.7), (2, 0.8), (3,0.9)}

$\breve{\mathcal{C}}$={(1, 0.9), (2, 0.8), (3,7)},

$\breve{\mathcal{D}}$ ={(1, 0), (2, 0.2), (3,0)},

Let $\tau = \{\breve{0}, \breve{1}, \breve{C}, \breve{\mathcal{D}}\}$ and

$\breve{\mathcal{F}} = \{\breve{1},\ \breve{F}\} \cup \{\breve{k}; \breve{k} \geq\ \breve{F_1}\}$. Where

$\breve{F}$= {(1, 0.4), (2, 0.5), (3, 0.6)}.

Then, $\breve{\mathcal{A}}^{\#1}$ ={(1, 0.1), (2, 0.2), (3,0.3)}. And,

$\breve{B}^{\#1}$ ={(1, 1), (2, 0.2), (3,1}.

Hence $\breve{\mathcal{B}}^{\#1} \nsubseteq \breve{\mathcal{A}}^{\#1}$

Theorem 3.7.

Let $(\breve{1}, \tau, \breve{\mathcal{F}})$be a $F\mathcal{FTS}$. Let $\breve{\mathcal{F}}$ fuzzy filter and $\breve{\mathcal{A}}$ any fuzzy set. Then the following are satisfying.

1. $(\breve{\mathcal{A}}^{\#1})^{\#1} \subseteq \breve{\mathcal{A}}^{\#1}$.
2. $\breve{\mathcal{A}}^{\#1} = cl(\breve{\mathcal{A}}^{\#1}) \subseteq cl(\breve{\mathcal{A}})$.
3. $\breve{0}^{\#1} = \breve{0}\ \ \notin \breve{\mathcal{F}}$.
4. $\breve{\mathcal{A}} \notin \breve{\mathcal{F}}\ \Longrightarrow \breve{\mathcal{A}}^{\#1} = \phi \notin \breve{\mathcal{F}}$.
5. $\breve{F} \notin \breve{\mathcal{F}} \Longrightarrow ((\breve{\mathcal{A}} - \breve{F})_{(max\ )\text{or}(\ min)})^{\#1} \subseteq \breve{\mathcal{A}}^{\#1} =$ $(\mathcal{A} \vee F)\#1$.

Proof.

1. Let $P_x^\lambda \in (\breve{\mathcal{A}}^{\#1})^{\#1}$, this leads $\forall\, \breve{\mathcal{U}} \in$ q_$\mathcal{N}(P_x^\lambda)\ \forall\ y \in X$ such that $max\{0, f_{\mathcal{U}}(y) + g_{\breve{\mathcal{A}}^{\#1}}(y) - 1\} > h_j(y)$ for some $\breve{J} \in \breve{\mathcal{F}}$ .This mean $f_{\mathcal{U}}(y) > h_j(y)$ and $g_{\breve{\mathcal{A}}^{\#1}}(y) > h_j(y)$ for every $\breve{J} \in \breve{\mathcal{F}}$ . .

If possible$P_x^\lambda \notin \breve{\mathcal{A}}^{\#1}$, this implies $\exists \breve{\mathcal{V}} \in q -$ $\mathcal{N}(P_x^\lambda)$, $\exists\ x \in X$ then, $\max\ \{0, k_{\mathcal{V}}(x) + H_{\mathcal{A}}(x) - 1\} <$ $h_J(x)$ for some $\breve{J} \in \breve{\mathcal{F}}$ .This means either $k_{\mathcal{V}}(x) \leq h_J(x)$ for some $\breve{J} \in \breve{\mathcal{F}}$ this is a contradiction, or $H_{\mathcal{A}}(x) \leq h_J(x)$ for some $J^z \in \breve{\mathcal{F}}$ this implies $\breve{\mathcal{A}}^{\#1} = \breve{0}$ that is also contradiction, thus $P_x^\lambda \in \breve{\mathcal{A}}^{\#1}$.

2. Clearly, $\breve{\mathcal{A}} \subseteq cl(\breve{\mathcal{A}})$ this implies $\breve{\mathcal{A}}^{\#1} \leq cl(\breve{\mathcal{A}}^{\#1})$

Let $P_x^\lambda \in cl(\breve{\mathcal{A}}^{\#1})$ for each $\breve{\mathcal{U}} \in$ q-$\mathcal{N}(P_x^\lambda)\ \forall\, y \in X \ni$ $f_{\mathcal{U}}(y) + g_{\breve{\mathcal{A}}^{\#1}}(y) >\ 1$, this means $\breve{\mathcal{U}} \wedge \breve{\mathcal{A}}^{\#1} \neq \breve{0}$, thus $g_{\breve{\mathcal{A}}^{\#2}}(y) \neq 0$,

by lemma 2.4 part (2) there exists fuzzy point $P_{y_0}^{\mathcal{E}} \in \breve{\mathcal{A}}^{\#2}$ which means that $0 < \mathcal{E} \leq 1$ and $\forall\ y_0 \in X \ni$,

$max\{0, f_{\mathcal{V}}(y_0) + g_{\mathcal{A}}(y_0) - 1\} \geq h_j(y_0)$ for some$\breve{J} \in \breve{\mathcal{F}}$ and $\forall\, \breve{\mathcal{V}} \in$ q-$\mathcal{N}(P_{y_0}^{\mathcal{E}})$,

but $f_{\mathcal{U}}(y) + \mathcal{E}$ >1, this means $\breve{\mathcal{U}}$is also q-$\mathcal{N}(P_y^{\mathcal{E}})$, so we get that

$max\{0, f_{\mathcal{U}}(y_0) + g_{\mathcal{A}}(y_0) - 1\} > h_J(y_0)$for some$\breve{J} \in$ $\breve{\mathcal{F}}$ but $\breve{\mathcal{U}}$is also q-$\mathcal{N}(P_x^\lambda)$, thus$P_x^\lambda \in \breve{\mathcal{A}}^{\#1}$.

Again $P_x^\lambda \notin cl(\breve{\mathcal{A}})$ there exist $\breve{\mathcal{U}} \in q\text{-}\mathcal{N}(P_x^\lambda)\ \forall y \in X$

such that $\breve{\mathcal{U}} \wedge \breve{\mathcal{A}} = \breve{0}$, hence $\forall\, y \in X$ s.t $max\{0, f_{\mathcal{U}}(y) + g_{\mathcal{A}}(y) - 1\} = 0 \notin \breve{\mathcal{F}}$, thus $P_x^{\lambda} \notin \breve{\mathcal{A}}^{\#1}$.

3. If possible $\breve{0}^{\#1} \neq \breve{0}$, there exists $P_x^{\lambda} \in 0^{z\#2}$ such that $\forall \breve{\mathcal{U}} \in$ q_$\mathcal{N}(P_x^{\lambda})\ \forall\, y \in X \ni max\{0, f_{\mathcal{U}}(y) + g_{\mathcal{A}}(y) - 1\} \geq h_F(y)$ for some $\breve{F} \in \breve{\mathcal{F}}$, but $g_0(y) = 0$ then $max\{0, f_{\mathcal{U}}(y) - 1\} = 0$ this contradiction, thus $\breve{0}^{\#1} = \breve{0} \notin \breve{\mathcal{F}}$.

4. If possible $\breve{\mathcal{A}}^{\#1} \neq \phi$, there exists $P_x^{\lambda} \in \breve{\mathcal{A}}^{\#1}$,for every $\breve{\mathcal{U}} \in q - \mathcal{N}(P_x^{\lambda})$; $\forall\, y \in X \ni max\{0, f_{\mathcal{U}}(y) + g_{\mathcal{A}}(y) - 1\} \geq h_F(y)$ for some $\breve{F} \in \breve{\mathcal{F}}$, but $max\{0, f_{\mathcal{U}}(y) + g_{\mathcal{A}}(y) - 1\} \leq g_{\mathcal{A}}(y)$, thus $g_{\mathcal{A}}(y)) \leq h_F(y)\ \ \forall\, y \in X$ .therefore $\breve{\mathcal{A}} \notin \breve{\mathcal{F}}$ contradiction. Thus,$\breve{\mathcal{A}}^{\#1} = \phi \notin \breve{\mathcal{F}}$.

5. Let $P_x^{\lambda} \in (\breve{\mathcal{A}} - \breve{F})^{\#1}$, $\forall\ \widetilde{\mathcal{U}} \in$ q_$\mathcal{N}(P_x^{\lambda})\ \forall\, y \in X \ni max\{0, f_{\mathcal{U}}(y) + g_{(\breve{\mathcal{A}}-\breve{F})}(y) - 1\} \geq h_j(y)$ for some $\breve{J} \in \breve{\mathcal{F}}$,since $\breve{\mathcal{A}} - \breve{F} \leq \breve{\mathcal{A}}$, this means $max\{0, f_{\mathcal{U}}(y) + g_{\mathcal{A}}(y) - 1\} > h_j(y)$ for some $\breve{J} \in \breve{\mathcal{F}}$, thus $P_x^{\lambda} \in \breve{\mathcal{A}}^{\#1}$.

Again, $(\breve{\mathcal{A}} \vee \breve{F})^{\#1} = \breve{\mathcal{A}}^{\#1} \vee \breve{F}^{\#1} = \breve{\mathcal{A}}^{\#1} \vee \phi = \breve{\mathcal{A}}^{\#2}$

Theorem 3.8.

Let $(\breve{1}, \tau, \breve{\mathcal{F}})$be a F$\mathcal{FTS}$. Let $\breve{\mathcal{F}}_1$, $\breve{\mathcal{F}}_2$ two fuzzy filters and $\breve{\mathcal{A}}$,$\breve{\mathcal{B}}$ be a fuzzy sets. Then the following are satisfying.

1. $\breve{\mathcal{F}}_1 \subseteq \breve{\mathcal{F}}_2 \implies \breve{\mathcal{A}}^{\#1}(\breve{\mathcal{F}}_2) \subseteq \breve{\mathcal{A}}^{\#1}(\breve{\mathcal{F}}_1)$.
2. $\breve{\mathcal{A}}^{\#1}(\breve{\mathcal{F}}_1 \wedge \breve{\mathcal{F}}_2) = \breve{\mathcal{A}}^{\#1}(\breve{\mathcal{F}}_1) \vee \breve{\mathcal{A}}^{\#1}(\breve{\mathcal{F}}_2)$.
3. $\breve{\mathcal{A}}^{\#1}(\breve{\mathcal{F}}_1 \vee \breve{\mathcal{F}}_2) \subseteq \breve{\mathcal{A}}^{\#1}(\breve{\mathcal{F}}_1) \vee \breve{\mathcal{A}}^{\#1}(\breve{\mathcal{F}}_2)$.

Proof.

1. Let $P_x^{\lambda} \in \breve{\mathcal{A}}^{\#1}(\breve{\mathcal{F}}_2)$, for every $\breve{\mathcal{U}} \in q - \mathcal{N}(P_x^{\lambda})\ \forall\, y \in X \ni max\{0, f_{\mathcal{U}}(y) + g_{\mathcal{A}}(y) - 1\} \geq h_F(y)$ for some $\breve{F} \in \widetilde{\mathcal{F}_2}$, since $\widetilde{\mathcal{F}_1} \subseteq \widetilde{\mathcal{F}_2}$, this implies $max\{0, f_{\mathcal{U}}(y) + g_{\mathcal{A}}(y) - 1\} \geq h_F(y)$ for some $\breve{F} \in \widetilde{\mathcal{F}_1}$, thus $P_x^{\lambda} \in \breve{\mathcal{A}}^{\#1}(\widetilde{\mathcal{F}_1})$.

2. $\breve{\mathcal{F}}_1 \wedge \breve{\mathcal{F}}_2 \leq \breve{\mathcal{F}}_1$, this leads $\breve{\mathcal{A}}^{\#1}(\breve{\mathcal{F}}_1) \leq \breve{\mathcal{A}}^{\#1}(\breve{\mathcal{F}}_1 \wedge \breve{\mathcal{F}}_2)$, also

$\breve{\mathcal{F}}_1 \wedge \breve{\mathcal{F}}_2 \leq \mathcal{F}_2{}^{z}$, this leads$\breve{\mathcal{A}}^{\#1}(\breve{\mathcal{F}}_2) \leq \breve{\mathcal{A}}^{\#1}(\breve{\mathcal{F}}_1 \wedge \breve{\mathcal{F}}_2)$, then

$\breve{\mathcal{A}}^{\#1}(\breve{\mathcal{F}}_1) \vee \breve{\mathcal{A}}^{\#1}(\breve{\mathcal{F}}_2) \leq \breve{\mathcal{A}}^{\#1}(\breve{\mathcal{F}}_1 \wedge \breve{\mathcal{F}}_2)$.

Conversely, let $P_x^{\lambda} \in \breve{\mathcal{A}}^{\#1}(\breve{\mathcal{F}}_1 \wedge \breve{\mathcal{F}}_2)$ for every $\breve{\mathcal{U}} \in q^{-}\mathcal{N}(P_x^{\lambda}), y \in X \ni max\{0, f_{\mathcal{U}}(y) + g_{\mathcal{A}}(y) - 1\} > min\{l_j(y), k_J(y)\}$ for every$\breve{j} \in \breve{\mathcal{F}}_1$,$\breve{J} \in \breve{\mathcal{F}}_2$,

but $min\{l_j(y), k_J(y)\} = \begin{cases} l_j(y) \text{ if } l_j(y) \leq k_J(y) \\ k_J(y) \text{ if } l_j(y) \geq k_J(y) \end{cases}$, hence

$\max\{0, f_{\mathcal{U}}(y) + g_{\mathcal{A}}(y) - 1\} \geq l_j(y)$

or $max\{0, f_{\mathcal{U}}(y), 1g_{\mathcal{A}}(y) - 1\} \geq k_J(y)$,

this implies $P_x^{\lambda} \in \breve{\mathcal{A}}^{\#1}(\breve{\mathcal{F}}_1) \vee \breve{\mathcal{A}}^{\#1}(\breve{\mathcal{F}}_2)$.

3. $\breve{\mathcal{F}}_1 \subseteq (\breve{\mathcal{F}}_1 \vee \breve{\mathcal{F}}_2)$ by part(2)

$\breve{\mathcal{A}}^{\#1}(\breve{\mathcal{F}}_1 \vee \breve{\mathcal{F}}_2) \subseteq \breve{\mathcal{A}}^{\#1}(\breve{\mathcal{F}}_1)$,

also $\breve{\mathcal{F}}_2 \subseteq (\breve{\mathcal{F}}_1 \vee \breve{\mathcal{F}}_2)$ by part (2)

$\breve{\mathcal{A}}^{\#1}(\breve{\mathcal{F}}_1 \vee \breve{\mathcal{F}}_2) \subseteq \breve{\mathcal{A}}^{\#1}(\breve{\mathcal{F}}_2)$, thus

$\breve{\mathcal{A}}^{\#1}(\breve{\mathcal{F}}_1 \vee \breve{\mathcal{F}}_2) \subseteq \breve{\mathcal{A}}^{\#1}(\breve{\mathcal{F}}_1) \vee \breve{\mathcal{A}}^{\#1}(\breve{\mathcal{F}}_2)$.

Remark 3.9.

The converse of properties 1and 3 are not true as in the example**.**

Let $(\breve{1}, \tau, \mathcal{F})$ be a F$\mathcal{FTS}$, and $X = \{1, 2, 3\} = A = B = F_1 = F_2$,C={2}.The membership of $\breve{A}, \breve{B}, \widetilde{F_1}\ \widetilde{F_2}$ and $\breve{C}$,are.

$f_A(x) = \frac{x+6}{10}\quad \forall x \in A$, $g_B(x) = \frac{x+5}{10}\quad \forall x \in B$,

$h_{F_1}(x) = \frac{x}{10}\quad \forall x \in F_1$,

$h_{F_2}(x) = \frac{x+3}{10}\quad \forall x \in F_2$. $K_C(x) = \frac{X^3+1}{10}\ \forall x \in C$.

$\breve{A}$={(1,0.7), (2, 0.8), (3,0.9)},

$\breve{B}$={(1, 0.6), (2, 0.7), (3,0.8)},

$\breve{C}$={(1, 0), (2, 0.9), (3,0)},

$\breve{B} \wedge \breve{C}$= {(1, 0), (2, 0.7), (3,0)},

$\breve{B} \vee \breve{C}$={(1, 0.6), (2, 0.9), (3,0.8)}.

Let $\tau = \{\breve{0}, \breve{1}, \breve{B}, \breve{C}, \breve{B} \wedge \breve{C}, \breve{B} \vee \breve{C}\}$ and

$\breve{\mathcal{F}}_1 = \{\breve{1}, \widetilde{F_1}\} \cup \{\breve{g}; \breve{g} \geq \widetilde{F_1}\}$. Where

$\widetilde{F_1}$= {(1, 0.1), (2, 0.2), (3, 0.3)}.

$\breve{\mathcal{F}}_2 = \{\breve{1}, \widetilde{F_2}\} \cup \{\breve{k}; \breve{k} \geq \widetilde{F_2}\}$. Where

$\widetilde{F_2}$= {(1, 0.4), (2, 0.5), (3, 0.6)}.

Then, $\breve{\mathcal{A}}^{\#1}(\breve{\mathcal{F}}_1)$= {(1, 1), (2, 0.1), (3,1)}. And,

$\breve{\mathcal{A}}^{\#1}(\breve{\mathcal{F}}_2)$= {(1, 0.4), (2, 0.1), (3,0.2)}.

$\breve{\mathcal{A}}^{\#1}(\breve{\mathcal{F}}_1) \nsubseteq \breve{\mathcal{A}}^{\#1}(\breve{\mathcal{F}}_2)$

Also, $\breve{\mathcal{A}}^{\#1}(\breve{\mathcal{F}}_1 \vee \breve{\mathcal{F}}_2)$= {(1, 0.4), (2, 0.1), (3,0.2)}. And,

$\breve{\mathcal{A}}^{\#1}(\breve{\mathcal{F}}_1) \vee \breve{\mathcal{A}}^{\#1}(\breve{\mathcal{F}}_2)$= {(1, 1), (2, 0.1), (3,1)}.

$\breve{\mathcal{A}}^{\#1}(\breve{\mathcal{F}}_1) \vee \breve{\mathcal{A}}^{\#1}(\breve{\mathcal{F}}_2) \nsubseteq \breve{\mathcal{A}}^{\#1}(\breve{\mathcal{F}}_1 \vee \breve{\mathcal{F}}_2)$.

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