Abstract: The research was based on the principle of defining the fuzzy positional function applying the fuzzy filter. Significant properties were obtained and examples were provided to clarify these properties.

Keywords: Fuzzy ideal, Fuzzy local function, Fuzzy filter, Fuzzy positional function

I. INTRODUCTION

The fuzzy sets is one of the important and vital topics as it entered into various applied and pure sciences, and it took place in engineering with its various branches, computers and others. Zadeh is the first to define fuzzy sets in 1965[1]. In 2019 Al-Razzaq AS and AL-Swidi LA They classified the fuzzy sets theory as families [2]. Well, in the same year they Finding and Taxonomy a New Fuzzy Soft points [3] and introduce the definition Soft Generalized Vague Sets [4]. Among the pure sciences took a great deal in general topology. Where our research is based on the fuzzy filter. Lowen is the first to introduce the concept in 1979[5]. After which many scientists and researchers who completed this topic came, including Prada and Saralegui they introduce a characterization of fuzzy filter concept [6] in 1988. Sarker is the first to define the fuzzy ideal as well as the fuzzy local function[7] in 1997. in 2019 AL Mohammed R and AL-Swidi LA. They got New Concepts of Fuzzy Local Function[8]. Where there is a relationship between the filter and the ideal in the usual sets, we also found a relationship between them using the fuzzy sets. The research is based on the study of the fuzzy positional function depending the fuzzy filter.

II. PRELIMINARIES

In this section we will mention the most important concepts used in this paper. Where the triple (1, τ, F) is called a fuzzy filter topological space (simply FTS) for which (1, τ) is fuzzy topology space in Change [10], and F is collection fuzzy filter define in Lowen [5].

Definition2.1.[2].

Let the membership M( X, 1) = {P ; P : X → I} where X any nonempty set, 1= [0, 1]. A fuzzy set A of the space X × I define as follows, A = { (x, P A(x)), ∀ x ∈ X } where the membership P A(x) = { f(x) for x ∈ A 0 for x ∉ A

Example 2.2.

Let X = {1, 2, 3} , A = {1} and B = {2, 3} , when the memberships of A∪B are. P A∪B(x) = { 1 2 for x ∈ A 2 3x − 10 1 for x ∉ A . Hence

A={(1,1),(2,0),(3,0)}, B={(1,0),(2,0.3),(3,0.5)}.

Definition2.3.[4].

A fuzzy point is fuzzy set in ΓX denoted by P f for with support x ∈ X and λ ∈ (0, 1] and the membership is P f(ω) = {E if ω = x 0 if ω ≠ x . NoteP f ∈ A iff E ≤ P f(x) and A ⊆ B if and only if P A(x) ≤ PA-B(x) where A and B are fuzzy sets for which

Definition2.4.[11].

Let A, B be a fuzzy sets, then A is called quasi-coincident with B denoted by AqB iff there exists y ∈ X η f A(y) + g B(y) > 1. The collection of all quasi-coincident denoted by q − N(P f x) . Otherwise, we called A not quasi-coincident with B if f A(y) + g B(y) ≤ 1, ∀x ∈ X and denoted by AqB.

Lemma 2.4.[12].

Let A and B be a fuzzy set in ΓX. Then the following are satisfying.

1. If (A∩B) ≠ ∅ , then ∃ x ∈ X and λ ∈ (0, 1] such that λ ≤ f A(x) and λ ≤ g B(x).
2. If A ≠ ∅ , then ∃ x ∈ X and λ ∈ (0, 1] such that P A(x) ∈ A.
3. If A ≠ ∅ , then ∃ x ∈ X and λ ∈ (0, 1] such that P A(x) ∈ A.
4. If (A∪B) = ∅ , then ∃ x ∈ X and λ ∈ (0, 1] such that P A∪B(x) and P A∪B(x).
5. If A ≠ ∅ , then ∃ x ∈ X and λ ∈ (0, 1] such that P A∪B(x) and P A∪B(x).

Definition 2.5.[9].

Let A, B any fuzzy set in ΓX. stander intersection. The stander of union, difference, complement are from.

1. A ∩ B = {x, min { f A(x), g B(x))}, ∀x ∈ X }
2. A ∪ B = {x, max { f A(x), g B(x))}, ∀x ∈ X }
3. Let A, B be a fuzzy sets the difference defines by:
   • (A − B) max = { (x, max { f A(x) − g B(x), 0)}, ∀x ∈ X }
   • (A − B) min = { (x, min { f A(x), g B(1−B(x)), 0)}, ∀x ∈ X }.
Fuzzy Positional Function Via Fuzzy Filter

4. \(1 - A = \{ (x, 1 - f_A(x)) : \forall x \in X \}\).

**Definition 2.7.** Let \((1, \tau, I)\) be a FTS. The fuzzy local function of \(A\) of the first type \(A^{21}(1, \tau)\) is defined by:
\[
A^{21}(1, \tau) = \{ P_{x} \mid \forall y \in q - N(P_{x}), \exists y \in X \exists f_{2}(y) + g_{A}(y) - 1 > h_{y}(y) \text{ for every } y \in I \}.
\]

The fuzzy local function \(A^{21}(I, \tau)\) or \(A^{21}(1, \tau)\).

Therefore, if \(P_{x} \in A^{21}(I, \tau)\). Then there exist \(W \in q - N(P_{x}) \forall x \in X, f_{10}(x) + g_{A}(y) - 1 \leq h_{y}(x)\) for some \(y \in I\).

III. FUZZY POSITIONAL FUNCTION

In this section we will introduce a new concept called the fuzzy positional function while discussing the most important characteristics associated with this concept and giving examples showing those characteristics.

**Definition 3.1.**

Let \((1, \tau, \bar{F})\) be a FTS. The fuzzy positional function of \(A\) of the first type \(A^{21}(1, \tau)\) is defined by:
\[
A^{21}(1, \tau) = \{ P_{x} \mid \forall \bar{u} \in q - N(P_{x}), \forall y \in X \exists \max\{0, f_{u}(y) + g_{A}(y) - 1 \geq h_{y}(y)\} \text{ for every } \bar{u} \in \bar{F} \}.
\]

The fuzzy positional function of \(A\) Denoted by \(A^{21}(I, \tau)\) or \(A^{21}(1, \tau)\).

Therefore, if \(P_{x} \in A^{21}(I, \tau)\) then there exist \(\bar{n} \in q - N(P_{x}) \forall x \in X \max\{0, f_{u}(x) + g_{A}(x) - 1 \leq h_{y}(x)\} \text{ for every } \bar{u} \in \bar{F} \).

**Example 3.2.**

Let \((1, \tau, \bar{F})\) be a FTS and \(X = \{1, 2, 3\}, A, E, K, F\) and \(I\) are subset of \(X\) such that

\[
\begin{align*}
A & = \{1, 2, 3\} \text{ s.t the membershisp of } A, \\
E & = \{9, 10\} \text{ s.t the membershisp of } E, \\
K & = \{3\} \text{ s.t the membershisp of } K, \\
F & = \{2, 3\} \text{ s.t the membershisp of } F,
\end{align*}
\]

we get,

\[
\begin{align*}
A & = \{(1, 0, 9), (2, 0, 8), (3, 0, 7)\}, \\
E & = \{(1, 0, 9), (2, 0, 6), (3, 0, 9)\}, \\
K & = \{(1, 0), (2, 0), (3, 0)\}, \\
F & = \{(2, 3)\} \text{ s.t the membershisp of } F, \\
h_{F}(x) & = \{2, 0, x \in F, 0, x \in K
\end{align*}
\]

we get,

**Proposition 3.3.**

Let \((1, \tau, \bar{F})\) be a FTS. \(\bar{F}\) be a filter fuzzy, \(\bar{A}\) fuzzy ideal, \(A\) any fuzzy set if \(A \in \bar{A}\) then \(A^{21}(\bar{F}) \subseteq A^{21}(I)\), with the filter \(\bar{F} = \{A : \bar{A} \in 1\}\), but the converse may be not true.

**Proof.**

Let \(P_{x} \in A^{21}(\bar{F})\) for every \(\bar{u} \in q - N(P_{x}), \forall y \in X \max\{0, f_{u}(y) + g_{A}(y) - 1 \geq h_{y}(y)\} \text{ for every } \bar{u} \in \bar{F}\). But \(\bar{F} = \{A : \bar{A} \in 1\}\) \(\forall \bar{u} \in q - N(P_{x}), \forall y \in X \max\{0, f_{u}(y) + g_{A}(y) - 1 \geq h_{y}(y)\} \text{ for every } \bar{u} \in \bar{F}\). Thus \(P_{x} \in A^{21}(1)\).

The following example shows that the converse of property is not true.

**Example 3.4.**

Let \((1, \tau, \bar{F})\) be a FTS, \(X = \{1, 2, 3\}, A, B, C, I\) are subset of \(X\) such that

\[
\begin{align*}
A & = \{(1, 2, 3)\} \text{ s.t the membershisp of } A, \\
B & = \{(1, 2, 3)\} \text{ s.t the membershisp of } B, \\
C & = \{(1, 2, 3)\} \text{ s.t the membershisp of } C, \\
I & = \{(1, 2, 3)\} \text{ s.t the membershisp of } I,
\end{align*}
\]

we get,

**Theorem 3.5.**

Let \((1, \tau, \bar{F})\) be a FTS. \(\bar{F}\) fuzzy filter and \(\bar{A}, \bar{B}\) any two fuzzy sets. Then the following are satisfying.

1. \(A \subseteq B \Rightarrow A^{#1} \subseteq B^{#1}\)
2. \((A \cup B)^{#1} = A^{#1} \cup B^{#1}\)
3. \((A \land B)^{#1} = A^{#1} \land B^{#1}\)

**Proof.**

1. Let \(P_{x} \in A^{#1}\) for every \(\bar{u} \in q - N(P_{x}), \forall y \in X \max\{0, f_{u}(y) + g_{A}(y) - 1 \geq h_{y}(y)\} \text{ for every } \bar{u} \in \bar{F}\)

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2. \( \mathcal{A} \subseteq (\mathcal{A} \lor \mathcal{B}) \), this implies \( \mathcal{A}^1 \subseteq (\mathcal{A} \lor \mathcal{B})^1 \), by (1).

Also \( \mathcal{B} \subseteq (\mathcal{A} \lor \mathcal{B}) \) this implies \( \mathcal{B}^1 \subseteq (\mathcal{A} \lor \mathcal{B})^1 \), by (1).

Thus \( \mathcal{A}^1 \lor \mathcal{B}^1 \subseteq (\mathcal{A} \lor \mathcal{B})^1 \).

Conversely, let \( P_2^1 \subseteq (\mathcal{A} \lor \mathcal{B})^1 \) if possible that \( P_2^1 \in (\mathcal{A}^1 \lor \mathcal{B}^1) \), which means \( P_2^1 \in \mathcal{A}^1 \) and \( P_2^1 \in \mathcal{B}^1 \), then there exist \( \bar{U} \in q \cdot \mathcal{N}(P_2^1) \) such that, \( \exists x \in X \ni \max \{0, f_{u_1}(x) + g_{A_1}(x) - 1\} \geq h_{Y_1}(x) \) for every \( F_1 \in \mathcal{F} \) and there exists \( \bar{V} \in q \cdot \mathcal{N}(P_2^1) \) such that \( \exists x \in X \ni \max \{0, f_{u_2}(x) + g_{A_2}(x) - 1\} < h_{Y_2}(x) \) for every \( F_2 \in \mathcal{F} \).

Clear, \( (\bar{U} \lor \bar{V}) \in q \cdot \mathcal{N}(P_2^1) \) put up \( (\bar{U} \lor \bar{V}) = \bar{W} \in q \cdot \mathcal{N}(P_2^1) \), But \( \max \{0, f_{u_1}(x) + g_{A_1}(x) - 1\} \leq \max \{0, f_{u_1}(x) + g_{A_1}(x) - 1\} < h_{F_1}(x) \), also maxim \( \{0, f_{u_2}(x) + g_{A_2}(x) - 1\} \leq \max \{0, f_{u_2}(x) + g_{A_2}(x) - 1\} < h_{F_2}(x) \), then \( \max \{0, f_{u_2}(x) + g_{A_2}(x) - 1\} \leq \max \{h_{F_1}(x), h_{F_2}(x)\} \) for every \( F_2 \in \mathcal{F} \).

Then \( \bar{W} \in q \cdot \mathcal{N}(P_2^1) \) s.t for some \( x \in X \ni \max \{0, f_{u_2}(x) + g_{A_2}(x) - 1\} \leq \max \{0, f_{u_2}(x) + g_{A_2}(x) - 1\} < h_{F_1}(x) \), so \( \max \{0, f_{u_2}(x) + g_{A_2}(x) - 1\} \leq \max \{h_{F_1}(x), h_{F_2}(x)\} \) for every \( F_2 \in \mathcal{F} \).

Again, let \( P_2^1 \subseteq \mathcal{A}^1 \) and \( P_2^1 \in \mathcal{B}^1 \), \( \forall U \in q \cdot \mathcal{N}(P_2^1) \forall y \in X \ni \max \{0, f_{u_1}(x) + g_{A_1}(x) - 1\} \geq h_{F_1}(x) \) for some \( F_1 \in \mathcal{F} \). So, \( \forall \bar{V} \in q \cdot \mathcal{N}(P_2^1) \forall y \in X \ni \max \{0, f_{u_2}(x) + g_{A_2}(x) - 1\} \geq h_{F_2}(x) \) for some \( F_2 \in \mathcal{F} \).

If possible \( P_2^1 \in \mathcal{A}^1 \land \mathcal{B}^1 \) then \( \exists \bar{W} \in q \cdot \mathcal{N}(P_2^1) \exists y \in X \ni \max \{0, f_{u_1}(x) + g_{A_1}(x) - 1\} \geq h_{F_1}(x) \) for every \( F_2 \in \mathcal{F} \).

If \( \min \{g_{A_1}(x), g_{A_2}(x)\} = g_{A_1}(x) \) this implies that, \( \max \{0, f_{u_2}(y) + g_{A_2}(y) - 1\} < h_1(x) \), for every \( F_1 \in \mathcal{F} \) this contradiction. Also when \( \min \{g_{A_1}(x), g_{A_2}(x)\} = g_{A_2}(y) \) and \( \max \{0, f_{u_2}(y) + g_{A_2}(y) - 1\} = h_1(y) \) for every \( F_2 \in \mathcal{F} \) this contradiction. Hence \( P_2^1 \in (\mathcal{A} \lor \mathcal{B})^2 \). Thus \( \mathcal{A}^1 \lor \mathcal{B}^1 \subseteq (\mathcal{A} \lor \mathcal{B})^1 \).

Remark 3.6

The converse of property 1 is not satisfying as in the example.

Let \( (I, \tau, \mathcal{F}) \) be a FFFTS and \( X = \{1, 2, 3\} \). Let \( A = B = F = C, D = \{2\} \). The membership of \( A, B, C, F \) and \( D \) are.
such that $\bar{u} \land \bar{a} = \bar{0}$, hence $\forall y \in X \ s.t$

$\max(0, f_u(y) + g_a(y) - 1) = 0 \ orall y \in X$, thus $P^k_3 \in \mathcal{A}^1$. 

3. If possible $\mathcal{A}^1 \neq \phi$, there exists $P^k_3 \in \mathcal{O}^{2#2}$ such that $\forall y \in X \ s.t.$

$\max(0, f_u(y) + g_a(y) - 1) \geq \bar{h}_f(y)$ for some $\bar{f} \in \mathcal{F}$, but $g_a(y) = 0$ then $\max(0, f_u(y) - 1) = 0$ this contradiction, thus $\mathcal{A}^1 \neq \phi$.

4. If possible $\mathcal{A}^1 \neq \phi$, there exists $P^k_3 \in \mathcal{A}^1$, for every $\bar{u} \in q - \mathcal{N}(P^k_3)$:

$\max(0, f_u(y) + g_a(y) - 1) \geq \bar{h}_f(y)$ for some $\bar{f} \in \mathcal{F}$, hence $\max(0, f_u(y) + g_a(y) - 1) \geq \bar{h}_f(y)$ for some $\bar{f} \in \mathcal{F}$, thus $P^k_3 \in \mathcal{A}^1$.

5. Let $P^k_3 \in (\bar{\mathcal{A}} - \bar{\mathcal{F}})^k$, for every $\bar{u} \in q - \mathcal{N}(P^k_3)$:

$\forall y \in X \ s.t.$

$\max(0, f_u(y) + g_a(y) - 1) \geq \bar{h}_f(y)$ for some $\bar{f} \in \mathcal{F}$, thus $P^k_3 \in \mathcal{A}^1$. 

$\bar{A}^1 \cap \mathcal{F} = \mathcal{F}$. 

$\bar{A}^1 \cap \mathcal{F} = \mathcal{F}$. Thus $\bar{A}^1 \neq \phi$.

Theorem 3.8.

Let $(\bar{1}, \tau, \mathcal{F})$ be a FFFTS. Let $\mathcal{F}_1$, $\mathcal{F}_2$ two fuzzy filters and $\mathcal{A}$ be a fuzzy sets. Then the following are satisfying.

$1. \mathcal{F}_1 \cap \mathcal{F}_2 = \mathcal{A}^1(\mathcal{F}_1) \subseteq \mathcal{A}^1(\mathcal{F}_2)$.

$2. \mathcal{A}^1(\mathcal{F}_1 \cap \mathcal{F}_2) = \mathcal{A}^1(\mathcal{F}_2) \vee \mathcal{A}^1(\mathcal{F}_1)$.

$3. \mathcal{A}^1(\mathcal{F}_1 \cap \mathcal{F}_2) \subseteq \mathcal{A}^1(\mathcal{F}_1 \cup \mathcal{F}_2)$.

Proof.

1. Let $P^k_3 \in \mathcal{A}^1(\mathcal{F}_2)$, for every $\bar{u} \in q - \mathcal{N}(P^k_3)$:

$\forall y \in X \ s.t.$

$\max(0, f_u(y) + g_a(y) - 1) \geq \bar{h}_f(y)$ for some $\bar{f} \in \mathcal{F}_2$, since $\mathcal{F}_1 \subseteq \mathcal{F}_2$, this implies $\max(0, f_u(y) + g_a(y) - 1) \geq \bar{h}_f(y)$ for some $\bar{f} \in \mathcal{F}_1$, thus $P^k_3 \in \mathcal{A}^1(\mathcal{F}_1)$.

2. $\mathcal{F}_1 \cap \mathcal{F}_2 \leq \mathcal{F}_1$, this leads $\mathcal{A}^1(\mathcal{F}_1) \leq \mathcal{A}^1(\mathcal{F}_1 \cap \mathcal{F}_2)$ also

$\mathcal{F}_1 \cap \mathcal{F}_2 \leq \mathcal{F}_2$, this leads $\mathcal{A}^1(\mathcal{F}_2) \leq \mathcal{A}^1(\mathcal{F}_1 \cap \mathcal{F}_2)$, then $\mathcal{A}^1(\mathcal{F}_1) \cap \mathcal{A}^1(\mathcal{F}_2) \leq \mathcal{A}^1(\mathcal{F}_1 \cap \mathcal{F}_2)$.

3. $\mathcal{F}_1 \cup \mathcal{F}_2$ by part (2)

4. $\mathcal{A}^1(\mathcal{F}_1 \cup \mathcal{F}_2) \subseteq \mathcal{A}^1(\mathcal{F}_1) \cap \mathcal{A}^1(\mathcal{F}_2)$.

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