



Detection of cracks in Micro structured Cantilever Beam using Wavelet Transforms

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Abstract: *Wavelet Analysis, the improved version of Fourier transform is used to investigate and analyze the variant transient signals in time-frequency domain with higher accuracy and precision. Wavelet theory found its promising application in various fields not limited to Physics, Biology, Geophysics, Engineering and Medicine which becomes a common tool to analyze data. In this work we present new insight using wavelet transform to detect the cracks present in micro structured cantilever beam which found its application in various Micro Electro Mechanical System (MEMS) devices such as Transducers, Sensors, Switches, Actuators and Probes. Even a small change in microstructure will reflect in its dynamic output, so it is desired to locate the presence of cracks or damages over the device structure accurately. The modeling of such microstructure is designed and simulated using COMSOL Multiphysics. The displacement (Static Response) and stress of the beam for simulated damage were analyzed by wavelet transform using MATLAB. The obtained results highlights this method of analysis provides accurate location and effect of the crack over the Micro cantilever structure.*

Keywords: *Wavelet Analysis, Crack Detection, Cantilever Beam, MEMS, COMSOL.*

I. INTRODUCTION

Wavelet transform (WT) is one among the most efficient mathematical method to analyze the signals in both time and frequency domains. In 1930 Haar used basic functions to investigate the small complicated details of random signals. Later in 1980s Jean Morlet introduces the first concept of wavelets and constructed nontrivial wavelets which are continuously differentiable. Later Mayer, Mallet and Daubechies contributed with important developments in WT. In 1990s the wavelet theory caused a paradigm shift and revolutionized digital signal processing similar to Fourier Transform (FT) in 1960s[1]. Researchers used to analyze the signals with FT until the entry of WT on screen for signal

analysis in both the domains. FT played a major role in analyzing the communication signals by differential equations, however wavelet analysis also have very strong bond over this analysis. These both were similar in linear operation in data processing and their basic functions were localized in frequency domain. FT have a single set of basic functions of only sine and cosine functions, but WT have an infinite set of possible basic functions which provides accurate and immediate access to information[2],[3]. FT is inappropriate in analysis of time varying signals due to the unavailability of temporary or local information. Another major drawback of FT is the frequency resolution obtained will remains constant for the whole signal. The difficulties arise in Fourier analysis can be resolved by wavelet theory[4].

WT measures the frequency in time localized bands and has an infinite set of possible basic functions which provides accurate and immediate access to information. Wavelets are well suited for approximating data with sharp discontinuities. Wavelet adapts to prototype function known as mother wavelets. WT found its application in various fields including Science, Engineering and Medicine[5],[6],[8]. Wavelets have become an active area of research in these fields. In Engineering, Digital Signal Processing, Transient Signal Analysis, Image Processing, Communication Systems, Electrical Signal Analysis are some of the applications where WT plays a vital role[7],[9],[10].

In this proposed work, we utilize the wavelets which are very sensitive to local changes in original signal to detect the changes in dynamic property of a micro cantilever beam. Presence of damages or cracks in a beam implies a change in its dynamic property. Here we had compared the properties of a healthy micro cantilever beam which is the reference state with a cantilever beam with crack on its surface.

II. MICRO-CANTILEVER BEAM

Cantilever is a type of beam fixed and constraint in its one end. Cantilever beams find its applications in various fields with respect to the parameters of the beam. The length of the beam can be from few meters to several hundred meters. In contrast the micro cantilever devices will be in the range of few microns to several hundred microns. Micro cantilever beam is one of the most effective and flexible mechanical sensor[18]. It found its major applications in micro electro mechanical systems (MEMS) as transducers, sensors, switches, actuators and probes. This device will transduce the mechanical behavior like deflection and stress of the beam into a measurable signal.

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This type of devices allows fabricating themselves in the integrated chips which makes them cost effective. The basic structure of a cantilever beam fixed at one end is shown in the Fig. 1 where L represents the length of the beam, W represents its width and T is its thickness or height.

This type of devices will be fabricated mostly by the materials like silicon, gold, platinum and polymers with respect to their applications. Mostly silicon will be preferred for electronic and sensing applications due to its material properties at room temperature as its processing technology is

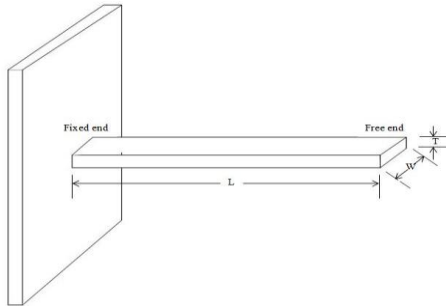


Fig. 1. Structure of cantilever beam

well developed[16]. Since this devices were inevitable today for their exclusive properties. Their advantages such as cost, sensitivity, selectivity and less power consumption make them very popular[17]. Despite of this, it is very important to design a micro cantilever device for specific output and to detect the variance occurs in its structure. Even a small damage or change in the micro structured devices can reflect its output over large extend so it is all important to identify the dissimilarities occurs in the device structure[18]. Here we had discussed a method for identifying a crack location over a beam surface and its impact on its overall performance in terms of deflection and stress.

III. WAVELET TRANSFORM ANALYSIS

Wavelets are used to analyze the complex signals in time domain. Complex data's such as music, images, patterns and signals are decomposed into elementary forms in different position and scales which can be reconstruct with higher precision. A wavelet is a wave-like oscillation with amplitude that begins at zero, increases, and then decreases back to zero. Equivalent mathematical conditions for wavelets are:

$$(i) \int_{-\infty}^{\infty} |\Psi(t)|^2 dt < \infty;$$

$$(ii) \int_{-\infty}^{\infty} |\Psi(t)| dt = 0;$$

$$(iii) \int_{-\infty}^{\infty} \frac{|\Psi(\omega)|^2}{|\omega|} d\omega < \infty;$$

Where $\Psi(\omega)$ is the Fourier transform of $\Psi(t)$, the above equation is called admissibility condition. The wavelet functions are constructed by translations and dilations of a single function called "mother wavelet" $\Psi(t)$. The mother wavelet can be defines as,

$$\Psi_{a,b}(t) = \frac{1}{\sqrt{|a|}} \Psi\left(\frac{t-b}{a}\right), \quad \text{---- (1)}$$

Where a, b are the scaling parameter and translation parameter respectively, when $|a| < 1$, then the wavelet is the

compressed version of the mother wavelet since the translation parameter determines the time location of the wavelet. However $|a| > 1$ when, then $\Psi_{a,b}(t)$ has larger time-width than $\Psi(t)$.

Here the wavelet method of analysis is applied to the space domain of the device structure rather than time series. Cracks in the device structure are detected by sensing the perturbation caused in the cracked location by wavelet transform [11]-[15].

The continuous wavelet transform (CWT) is a convolution of the data with a scaled and shifted version of the mother wavelet function (Ψ). The mathematical formula for determining CWT is given as equation (2),

$$CWT_{(c,b,\Psi)} = w \int_{-\infty}^{\infty} x(t) \Psi^*\left(\frac{t-b}{c}\right) dt \quad \text{---- (2)}$$

Where $x(t)$ is the signal in time domain, w is a weighting function, Ψ is the wavelet function, c related to scale and b related to the position. $CWT(c,b,\Psi)$ represents the resulting coefficients as a function of c, b and wavelet function Ψ . When using CWT, the desired frequency must be adjusted, since there will be different resolution of time and frequency. The relationship between the scale s , and frequency f is given as equation (3),

$$f = \frac{f_{\Psi} f_s}{s} \quad \text{---- (3)}$$

Where f_{Ψ} , is the central frequency of wavelet function in hertz, f_s is the sampling frequency.

Next, the discrete wavelet transform (DWT) is commonly used to analyze the data which are broken into discrete blocks. DWT requires less memory and less computation time when compared to CWT. DWT employs two set of functions, called scaling function and wavelet function which are associated with low pass and high pass filters respectively. Decomposition of signals is obtained by successive high pass and low pass filtering of time domain followed by sampling[19]. Here DWT analyze the signal by using filters of specific frequency along the axis of the beam.

Selection of wavelet functions were usually done by trial and error method since there is no optimal way to select the best wavelet function for a specific applications like crack or damage detection[20]. Symmetric wavelets are found to be more effective in singularity analysis.

Haar wavelet which is the simplest type of wavelet, serves as a prototype for all other wavelet transforms. Haar wavelet is the only symmetric compactly supported orthogonal wavelet. The Haar wavelet is a sequence of rescaled square typed functions which together form a wavelet family. Its low computing requirements made them more efficient in two dimensional image processing and pattern recognition.

Daubechies wavelet represents a collection of wavelets that improve on the frequency domain characteristics of Haar wavelet. The Daubechies wavelets are chosen to have the highest number A of vanishing moments. A vanishing moment limits the wavelets ability to represent polynomial behavior or information in a signal. Daubechies db1–db10 orthogonal wavelets are commonly used. Daubechies db2, with one vanishing moment, easily encodes polynomials of one coefficient, or constant signal components.

Daubechies D4 encodes polynomials with two coefficients. This ability to encode signals is subject to the phenomenon of scale leakage, and the lack of shift-invariance, which rise from the discrete shifting operation during application of the transform. Daubechies wavelets found to perform well in such applications due to its proven effectiveness in this field. Here both orthogonal and bi-orthogonal wavelet functions were used to detect the location and characteristics of the cantilever beam. Information about the cracks was obtained by the discrete wavelet transform (DWT) coefficients.

IV. MODELING OF MICRO CANTILEVER BEAM

The proposed micro cantilever beam is designed using COMSOL Multiphysics 5.4 simulation software. COMSOL Multiphysics is general-purpose simulation software for modeling designs, devices, and processes in all fields of engineering, manufacturing, and scientific research. It provides an interactive environment for modeling, simulating scientific and engineering problems. It can be used to simulate any combination of add on modules for electromagnetics, structural mechanics, acoustics, fluid flow, heat transfer and chemical engineering. It allows conventional physics-based user interfaces and coupled systems of partial differential equations (PDEs). It encompasses all of the steps in the modeling workflow such as defining geometries, material properties, and the physics that describe specific phenomena to solving and post processing models for producing desired results. Here for our study we have chosen poly crystalline silicon as the device material with following material parameters, Poisson ratio - 0.22, Density - 2320 kg/m3, Young’s Modulus- 160 GPa. Device dimensions length (L), width (W) and Thickness (T) were fixed as 100µm, 10 µm and 1 µm respectively. The force applied (P) over the beam is 2.52x10-6 N.

A. Theoretical calculation:

Deflection of micro cantilever beam with uniform load over entire span is given as equation (4),

$$\delta = \frac{PL^3}{24EI} \left[6 - 4\frac{x}{L} + \frac{x^2}{L^2} \right] \quad \text{----- (4)}$$

For maximum deflection at x=L,

$$\delta_{max} = \frac{PL^3}{8EI} \quad \text{----- (5)}$$

$$I = \frac{1}{12} WT^3 \quad \text{----- (6)}$$

Where, P is force applied, L is length of the beam, W is width of the beam, T is the beam thickness, E is the young’s modulus of the material and I is moment of inertia. From the above equations, we found the maximum deflection of the proposed healthy beam $\delta_{max} = 2.35 \mu\text{m}$.

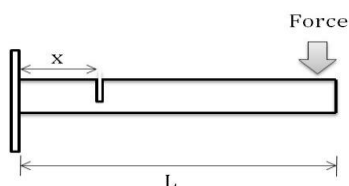


Fig. 2. Schematic diagram of cantilever beam with crack

For the comparative study, we intentionally made modification in the beam structure with crack over its surface for length of 0.5µm and width of 5µm and positioned at 30µm in x axis from the fixed end. Fig. 2 shows the schematic representation of the beam with crack.

V. SIMULATION RESULTS AND DISCUSSION

A. Analyses of device performance characteristics

For the above mentioned structures (i) healthy cantilever beam and (ii) cantilever beam with crack, was simulated for a constant Force (P) 2.52x10⁻⁶ N. when a force is applied over the surface of the cantilever beam it will deflect with respect to the amount of force applied. The healthy cantilever beam shows the total displacement of 2.34µm as shown in Fig. 3 whereas the cantilever beam with crack shows total displacement of 2.49 µm as shown in Fig. 4.

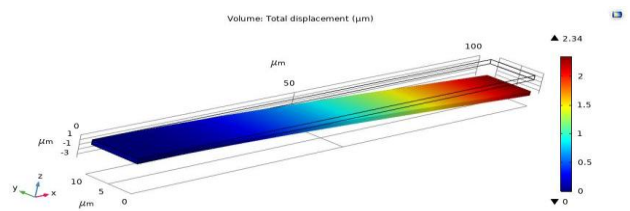


Fig. 3. Deflection of a healthy cantilever beam

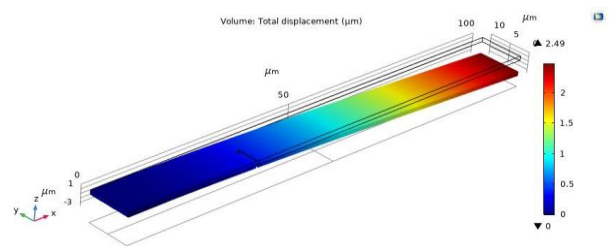


Fig. 4. Deflection of a cantilever beam with crack

The presence of crack has made its impact over the total displacement of the beam for constant load. Since micro cantilever beams were commonly used for sensing application, a small change in the device structure will reflect its sensitivity[21].

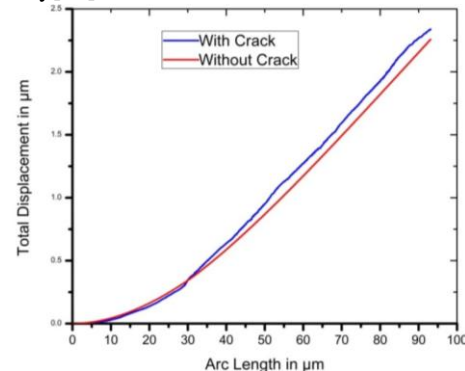
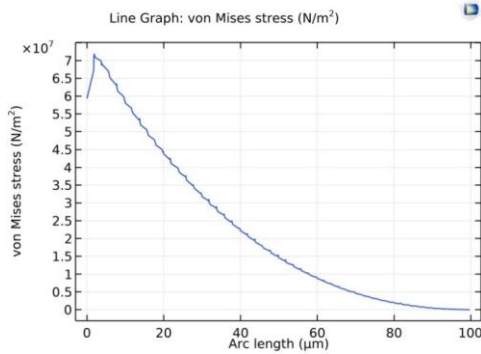


Fig. 5. Comparison of cantilever beam deflection

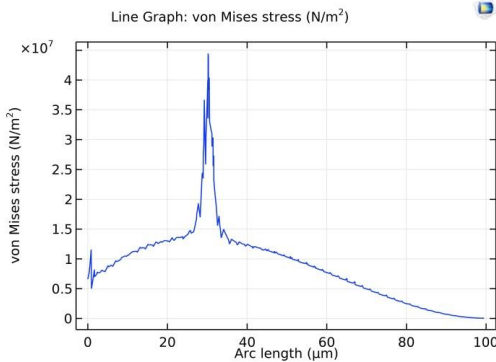
So it is very crucial to detect the damage occurred over the surface and to identify the defect location. Fig.5. illustrates the deformation of beam in terms of displacement in both structures. The device with crack tends to deflect more due to the changes occurred in its structure.



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(a)



(b)

Fig.6 Stress analysis of cantilever beam

Fig.6(a) represents the stress experienced by the beam without any defects. In such case, the stress will be maximum near the fixed end and it will gradually decrease. But in case of cracked beam the maximum stress will be at the cracked region. The stress will get distributed between the fixed end and the cracked region. Fig.6(b) clearly represents the peak stress at 30µm where the crack was developed. Fig.7 shows the pictorial representation of the stress around the cracked region which is encircled for clear indication. Beam will experience maximum stress at the fixed end and at the defect region.

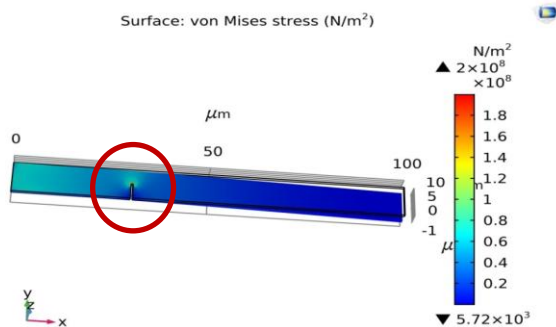
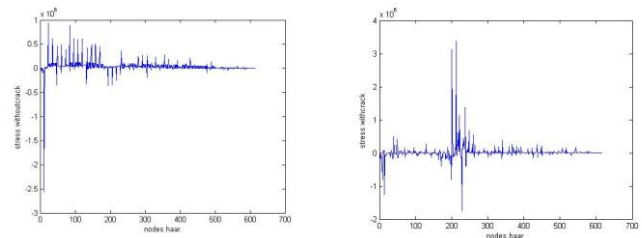


Fig.7 Stress over cracked portion

B. Wavelet Analysis

The obtained results were analyzed using discrete wavelet transform with various mother wavelets like Haar, Bior3.5 and Daubechies. Selection of wavelet function is another important factor to analyze and produce efficient result. The wavelet coefficients will respond with peaks or ridges when

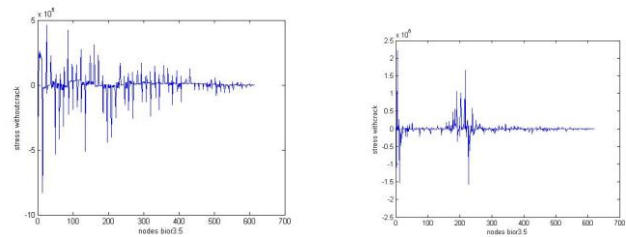
discontinuity or with sudden change in data. The response of wavelet coefficients for various wavelet functions were carried out for obtained data's. Fig. 8 represents the variation in coefficient for both modes using Haar wavelet.



(a) Healthy Beam

(b) Cracked Beam

Fig. 8 wavelet coefficient for Haar wavelet



(a) Healthy Beam

(b) Cracked Beam

Fig. 9 wavelet coefficient for Bi-orthogonal wavelet Bior 3.5

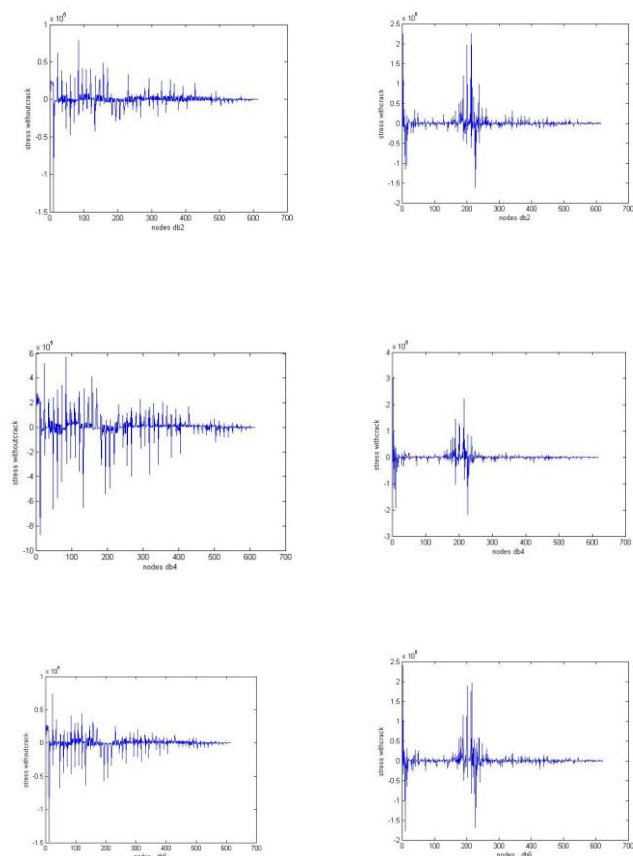
The beam length of 100µm is differentiated around 600 nodes and the analysis was carried out. Fig. 9 represents the variation in coefficient using Bior 3.5 wavelet.

Similarly Fig. 10 shows the variations in wavelet coefficients using Daubechies wavelet which is more sensitive to damage or defect over the structure. Since this wavelet function were well proved to provide accurate results, here the wavelet coefficients shows peaks accurately between the nodes 200-250, where the crack were actually present.

VI. CONCLUSION

The present work describes the accurate and efficient method for detecting the cracks present in micro cantilever beam using various wavelet functions. Location of cracks or any other defects and its location can be accurately identified. Once again we had proved that the wavelet type analysis provides the best results for crack identification. This method satisfies the importance of detecting the cracks or defects in micro structured devices. Among various wavelet functions, Daubechies wavelet coefficients respond well with the perturbation caused over the structure due to the crack. This proposed method shows the effectiveness of wavelet analysis for damage detection.

This type of novel approach can be applied for various device structures of all dimensions and applications. In our future work, the proposed approach will be applied to identify the multiple cracks present in the similar micro structured cantilever beam.



(a) Healthy Beam (b) Cracked Beam
Fig.10 wavelet coefficient for various Daubechies wavelet

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