

# A Modified Method for Predicting Relative Permeability



Mohamed Ameen, Mahmoud Tantawy, Ahmed Gawish

**Abstract:** Evaluating the relative permeability data is very essential as all the reservoirs contain multiple fluid phases. In absence of lab measured data, many correlations were developed to capture an accurate formula for relative permeability data prediction. The objective of this work focuses on how to calculate the value of exponents incorporated into generalized Corey's correlation instead of using pre-assumed fixed values and use the estimated values of exponents to predict relative permeability data. A giant database of experimental results for 750 plugs, covering different types of reservoir rocks and fluid systems, was involved in the methodology development to test its validity and reliability. Relative permeability data prediction was performed for 750 plugs using the generalized Corey's correlation and the proposed methodology to estimate exponents. Predicted relative permeability data were compared to the collected actual experimental results and the prediction results of other common published correlations through extensive statistical analysis. Statistical analysis showed that the proposed methodology has significant reliability to predict relative permeability data.

**Keywords:** Corey's correlation, Exponents, Relative permeability, Relative permeability curve end points.

## I. INTRODUCTION

Various laboratory studies provided that the effective permeability for all reservoir phases relies greatly on phase saturation and reservoir wettability for that phase. The effective permeability value used into Darcy's equation depends on relative permeability value for the flowing fluids. Relative permeability is a giant effective controlling factor in history matching process for reservoir simulation and production optimization. Enhanced oil recovery methods use the relative permeability data to monitor fluid advance and movement while saturation change during flooding like for breakthrough time prediction. From relative permeability curve, we can know the residual oil saturation which affect the oil recovery factor and hence reservoir economic feasibility.

In the absence of lab measured data, many correlations were developed using regression techniques to capture an accurate formula for relative permeability data prediction.

## II. LITERATURE REVIEW

Most of the proposed correlations to predict relative permeability are exploiting the effective fluid saturation as input and an exponent that determines the equation nonlinearity. The effective phase saturation is defined by the following set of relationships:

$$S_o^* = \frac{S_o}{1-S_{wc}} \quad (1)$$

$$S_w^* = \frac{S_w - S_{wc}}{1-S_{wc}} \quad (2)$$

$$S_g^* = \frac{S_g}{1-S_{wc}} \quad (3)$$

### A. Common previous published relative permeability correlations:

Corey provided a clear generalized mathematical model for relative permeability data generation of the gas-oil and water-oil systems [3]:

$$k_{ro} = (k_{ro})_{swc} * \left[ \frac{1-S_w-S_{orw}}{1-S_{wc}-S_{orw}} \right]^n$$

$$k_{rw} = (k_{rw})_{sorw} * \left[ \frac{S_w-S_{wc}}{1-S_{wc}-S_{orw}} \right]^m \quad (4)$$

Wyllie and Gardner (1958) built his model upon his observation that, in some reservoir rocks, the correlation between the reciprocal of capillary pressure squared (1/Pc<sup>2</sup>) and the water effective saturation S<sub>w</sub><sup>\*</sup> is linear for a high range of saturation [3]. Torcaso and Wyllie (1958) proposed a simple expression to calculate k<sub>ro</sub> in a gas-oil system. The proposed model allows k<sub>ro</sub> calculation from k<sub>g</sub> measurements since k<sub>g</sub> lab determination experiments are easily performed, but k<sub>ro</sub> determination experiments are usually made with some difficulty. Having some petrophysical considerations, Pirson (1958) made some generalized relationships for calculating wetting and non-wetting phase relative permeability for both drainage and imbibition processes [3]. Mehdi Honarpour and L.F. Koederitz (1981) suggest correlations for two sets of rock samples with varying wettabilities: Sandstones, Limestone and dolomites through stepwise linear regression analysis that would mimic the relative permeability lab measurements using some inputs as phases saturations, absolute permeability, and porosity [4]. In 2001, M.N. Mohamad Ibrahim and L.F. Koederitz exploited performed linear regression analysis to develop their relative permeability prediction equations according to reservoir rock lithology and wettability [2].

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**B. Comments on previous published relative permeability correlations**

It was noticed that nearly all previously published relative permeability prediction correlations assume a fixed number for the exponent controlling the relation between effective fluid saturation and the values of fluids relative permeability ignoring effect of relative permeability end points on curve shape. From author's point of view, a giant discrepancy between the predicted and the actual measured in many cases can be due to that the correlation fixed value parameters invented through matching trials to certain group of samples and it is not mandatory that it can be applied for all samples with the same lithology and wettability.

**III. SCOPE OF WORK**

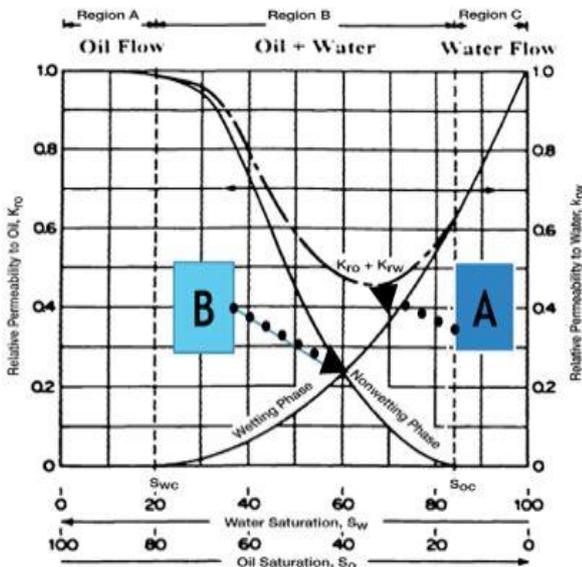
The objective of this work is to find a formula that can predict relative permeability data precisely through a modification applied to generalized Corey's correlation. The modification focuses on how to calculate the value of exponents incorporated into generalized Corey's correlation "m & n" instead of using pre-assumed fixed values.

**IV. METHODOLOGY HYPOTHESIS**

"Fig.1" represents a typical two-phase flow behavior for water-oil system [3]. It was noticed the following points:

- A. For the curve representing summation of oil and water relative permeability, the curve is minimized at point (A).
- B. The two curves of oil and water relative permeability are intersecting at point (B) which means that the relative permeability of the two phases are equal at this point.

Methodology hypothesis is that we can assume the water saturation at point (A) is the same as the saturation at point of intersection (B) as the difference @ X-axis ( $S_w$ ) is low. This hypothesis is just proposed for simplification and approximation but it is not strictly correct in many cases.



**Fig. 1. Typical two-phase relative permeability curves**

From the previous hypothesis, we can have two conclusions at the point of relative permeability curves intersection:

- 1. Oil relative permeability = Water relative permeability

Using the mathematical expression of generalized Corey's correlation for relative permeability:

$$(k_{ro})_{Swc} * \left[ \frac{1-s_w-s_{orw}}{1-s_{wc}-s_{orw}} \right]^{n_o} = (k_{rw})_{Sorw} * \left[ \frac{s_w-s_{wc}}{1-s_{wc}-s_{orw}} \right]^{n_w} \quad (5)$$

- 2. The first and second partial differentiation for  $(K_{r_o} + K_{r_w})$  with respect to  $(S_w)$  are equal to zero as it is "minimum value" for the curve.

The first partial differentiation:

$$\frac{\partial(k_{ro}+k_{rw})}{\partial s_w} = 0 \quad (6)$$

The second partial differentiation:

$$\frac{\partial^2(k_{ro}+k_{rw})}{\partial s_w^2} = 0 \quad (7)$$

**V. METHODOLOGY MATHEMATICAL DERIVATION**

Using the second conclusion "(6)" for the first partial differentiation:

$$\frac{\partial(k_{ro}+k_{rw})}{\partial s_w} = 0$$

Substituting by mathematical expression of Corey's "(4)" into "(6)":

$$(k_{ro})_{Swc} * \left[ \frac{1-s_w-s_{orw}}{1-s_{wc}-s_{orw}} \right]^{n_o-1} * \left[ \frac{-1}{1-s_{wc}-s_{orw}} \right] * n_o + (k_{rw})_{Sorw} * \left[ \frac{s_w-s_{wc}}{1-s_{wc}-s_{orw}} \right]^{n_w-1} * \left[ \frac{1}{1-s_{wc}-s_{orw}} \right] * n_w = 0 \quad (8)$$

Using the first conclusion "(5)" and by elimination the equal terms into "(8)" then:

$$\left[ \frac{1}{1-s_w-s_{orw}} \right] * n_o = \left[ \frac{1}{s_w-s_{wc}} \right] * n_w \quad (9)$$

By readjusting terms into "(9)":

$$\left[ \frac{s_w-s_{wc}}{1-s_w-s_{orw}} \right] = \frac{n_w}{n_o} \quad (10)$$

For simplifying we can assume that:

$$\frac{n_w}{n_o} = T \quad (11)$$

Substituting by "(11)" into "(10)":

$$\left[ \frac{s_w-s_{wc}}{1-s_w-s_{orw}} \right] = \frac{n_w}{n_o} = T$$

Using the second conclusion for the second partial differentiation "(7)":

$$\frac{\partial^2(k_{ro}+k_{rw})}{\partial s_w^2} = 0$$

Substituting by mathematical expression of Corey's correlation "(4)" into "(7)":

$$(k_{ro})_{Swc} * [(1-s_w-s_{orw})/(1-s_{wc}-s_{orw})]^{(n_o-2)} * [1/(1-s_{wc}-s_{orw})]^2 * n_o * (n_o-1) + (k_{rw})_{Sorw} * [(s_w-s_{wc})/(1-s_{wc}-s_{orw})]^{(n_w-2)} * n_w * (n_w-1) = 0$$

$$S_{orw})^{(n_w - 2)} * [1/(1 - S_{wc} - S_{orw})]^{2 * n_w * (n_w - 1)} = 0 \quad (12)$$

By elimination the equal terms into “(12)”:

$$\left[ \frac{S_w - S_{wc}}{1 - S_w - S_{orw}} \right]^2 = \frac{n_o * (n_o - 1)}{n_w * (n_w - 1)} \quad (13)$$

But we have from “(10)”:

$$\left[ \frac{S_w - S_{wc}}{1 - S_w - S_{orw}} \right] = \frac{n_w}{n_o}$$

Then using “(10)” and by elimination the equal terms into “(13)”:

$$\left[ \frac{S_w - S_{wc}}{1 - S_w - S_{orw}} \right] = \frac{(n_o - 1)}{(1 - n_w)} \quad (14)$$

But we have from “(11)”:

$$\left[ \frac{S_w - S_{wc}}{1 - S_w - S_{orw}} \right] = \frac{n_w}{n_o} = T \text{ \& } n_w = T * n_o$$

By substitution into “(14)” and eliminate the equal terms then:

$$\frac{(1 - T * n_o)}{(n_o - 1)} = T$$

By readjusting terms and substituting to get  $n_o$  value then get  $n_w$  by having  $n_w = T * n_o$ :

$$n_o = \frac{(1 + T)}{2 * T} \quad n_w = \frac{(1 + T)}{2} \quad (15)$$

Using regression analysis available in Excel “Solver-addin”, we can know value of water saturation at intersection point ( $S_{wx}$ ) that minimizes summation of oil and water relative permeability ( $Kr_o + Kr_w$ ) in “(16)” to get T-factor value then get exponents  $n_o$  &  $n_w$  values using the previous relations “(15)” that will be in relative permeability data prediction:

$$T = \left[ \frac{S_{wx} - S_{wc}}{1 - S_w - S_{orw}} \right]$$

$$k_{ro} + k_{rw} = (k_{ro})_{Swc} * \left[ \frac{1 - S_{wx} - S_{orw}}{1 - S_{wc} - S_{orw}} \right]^{\frac{(1+T)}{2 * T}} + (k_{rw})_{Sorw} * \left[ \frac{S_{wx} - S_{wc}}{1 - S_w - S_{orw}} \right]^{\frac{(1+T)}{2}} \quad (16)$$

For example: water-oil system

$$k_{ro} = (k_{ro})_{Swc} * \left[ \frac{1 - S_w - S_{orw}}{1 - S_{wc} - S_{orw}} \right]^{\frac{(1+T)}{2 * T}}$$

$$k_{rw} = (k_{rw})_{Sorw} * \left[ \frac{S_w - S_{wc}}{1 - S_w - S_{orw}} \right]^{\frac{(1+T)}{2}}$$

It is important to mention that the same derivation steps are valid for other fluid systems to get the exponents’ values and use them into calculating relative permeability values.

## VI. MODEL TUNING

The authors mentioned that methodology hypothesis relies on the assumption; the water saturation at point (A) is nearly the same as the saturation at point of intersection (B) as the difference @ X-axis ( $S_w$ ) is low “Fig.1”. The difference of

saturation for the two points can be negligible in some cases or significant in other cases depending on system wettability. The authors proposed an adjusting technique to overcome this shortage. The adjusting technique is to multiply the calculated exponents by a (lithology/wettability) dependent factor (A & B) as in “(17)”. Factors will be determined by having best prediction for our giant collected Kr dataset. Modified models with adjusting factors for different reservoir systems are found into appendix.

For example: water-oil system

$$k_{ro} = (k_{ro})_{Swc} * \left[ \frac{1 - S_w - S_{orw}}{1 - S_{wc} - S_{orw}} \right]^{A * \frac{(1+T)}{2 * T}}$$

$$k_{rw} = (k_{rw})_{Sorw} * \left[ \frac{S_w - S_{wc}}{1 - S_w - S_{orw}} \right]^{B * \frac{(1+T)}{2}}$$

## VII. DATABASE BUILDING

A giant database of experimental results for 750 plugs was involved in the methodology development to test its validity and reliability. Data was collected from available SCAL reports on the internet, SPE papers published until 2017 and reports from some petroleum companies handled without locating data. Nearly 80% of results were used for modelling and 20 % were used for model testing, validation and comparison to other common correlations. In case of water-oil system, data is sorted according to wettability using “modified craig’s rule” (see table. I). In case of gas systems, it is assumed that gas is always the non-wetting phase. Table II shows the collected data after sorting and the number of results used for modelling or validation.

Table-I: Modified Craig’s wettability rule [1]

Rock Wettability	$S_{wc}$	$S_w$ at which $k_{rw}^*$ and $k_{ro}^*$ are equal	$k_{rw}^*$ at $S_w = 100 - S_{orw}$ (fraction)
Strongly Water-Wet:	$\geq 15\%$	$\geq 45\%$	$\leq 0.07$
Water-Wet:	$\geq 10\%$	$\geq 45\%$	$0.07 < k_{rw}^* \leq 0.3$
Oil-Wet:	$\leq 15\%$	$\leq 55\%$	$\geq 0.5$
Intermediate: (Mixed-Wet)	$\geq 10\%$	$45\% \leq S_w \leq 55\%$	$> 0.3$
	$\leq 15\%$	$45\% \leq S_w \leq 55\%$	$< 0.5$

## VIII. STATISTICAL ANALYSIS [5]

A statistical comparison was performed between model’s results and three common correlations: Corey’s correlation, Honarpour’s correlation and Pirson’s correlation [3] [4]. Although there are numerous recent correlations for relative permeability prediction but the three previous mentioned correlations were selected for statistical comparison due to their common usage and proven reliability in many reservoir engineering cases for many years. Table III shows the statistical comparison for

Table-II: Collected data after sorting

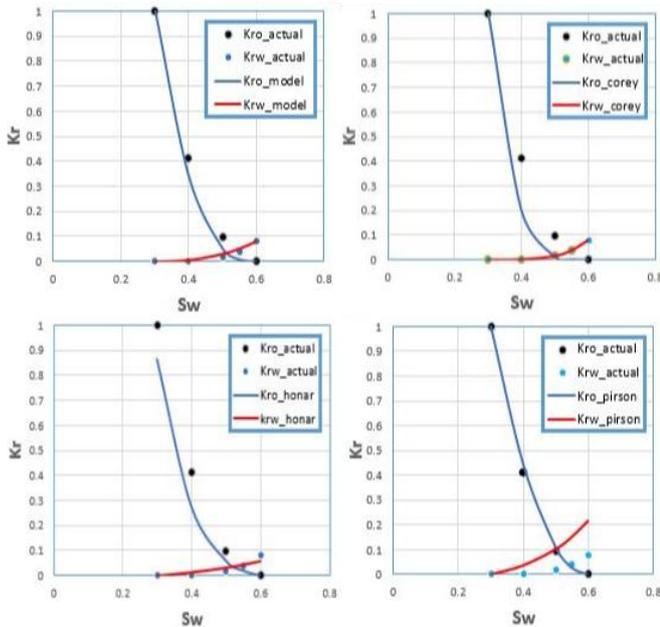
System	Litho	Wett	Total no.	Mod data	Valid Data
Water & Oil	SS	WW	300	250	50
	LS		35	28	7
	Dolo		17	14	3
	SS	OW	44	36	8

## A Modified Method for Predicting Relative Permeability

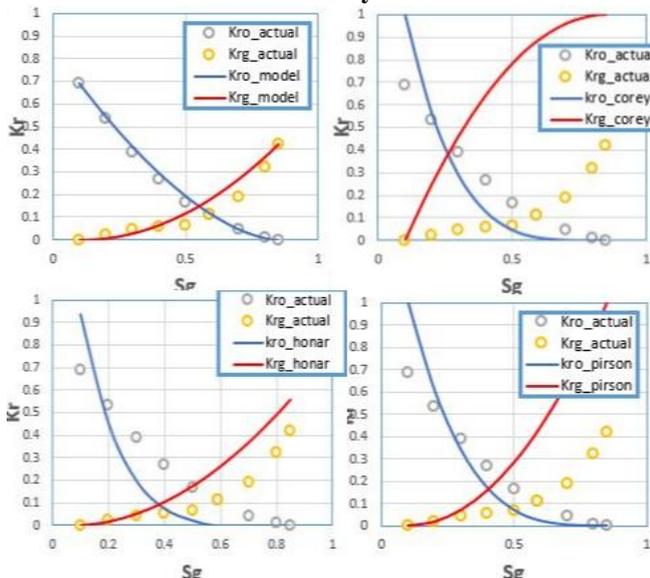
	LS	MW	16	13	3
	Dolo		5	4	1
	SS		59	48	11
	LS		30	24	6
	Dolo		13	11	2

Gas & Oil	SS	OW	156	126	30
	LS		16	13	3
	Dolo		16	13	3
Gas & Water	SS	WW	28	22	6
	LS		1	1	0
Gas & Condensate	SS	CW	19	15	4
	LS		3	3	0
<b>Total</b>			<b>758</b>	<b>621</b>	<b>137</b>

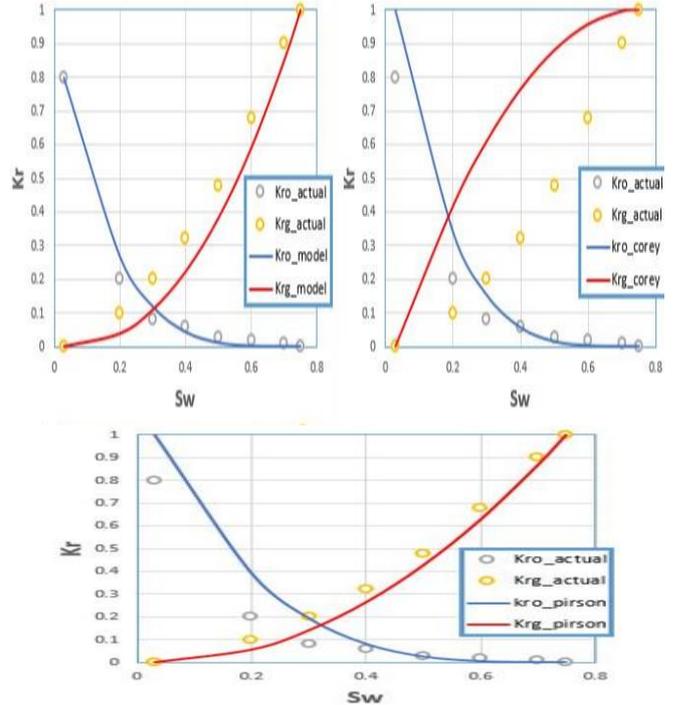
different reservoir fluid systems. For gas/condensate system, previous correlations gas/oil models were used as previous correlations did not consider models for gas/condensate system specifically. Fig.2, 3, 4 & 5 show examples for results comparison for different reservoir systems.



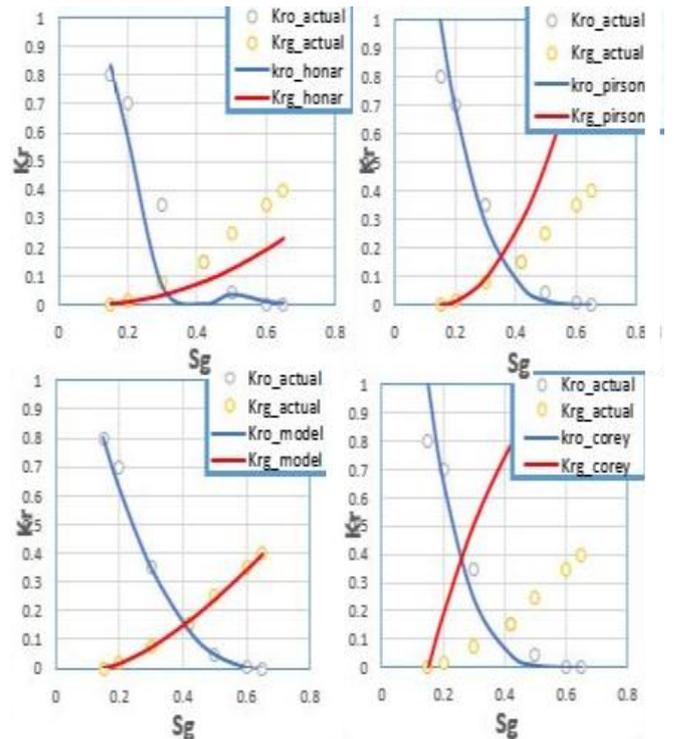
**Fig.2. Results comparison for water-oil water wet sandstone system**



**Fig.3. Results comparison for gas-oil limestone system**



**Fig.4. Results comparison for gas-water sandstone system**



**Fig.5. Results comparison for gas-condensate limestone system**

Table-III: Statistical comparison analysis results

System	Lithology	Wettability	R-Square				Mean Absolute Error			
			Modified Corey	Corey	Honarpour	Pirson	Modified Corey	Corey	Honarpour	Pirson
Water-Oil	SS	WW	0.93	0.89	0.94	0.95	0.04	0.06	0.06	0.07
	LS	WW	0.97	0.94	0.95	0.97	0.01	0.02	0.02	0.04
	Dolo	WW	0.96	0.78	0.83	0.84	0.01	0.01	0.02	0.03
	SS	OW	0.971	0.953	0.973	0.968	0.01	0.02	0.02	0.05
	LS	OW	0.94	0.92	0.97	0.98	0.01	0.02	0.03	0.02
	Dolo	OW	0.99	0.93	0.97	0.99	0.00	0.01	0.01	0.02
	SS	MW	0.965	0.920	0.948	0.951	0.01	0.02	0.01	0.01
	LS	MW	0.86	0.85	0.90	0.89	0.03	0.03	0.04	0.03
	Dolo	MW	0.97	0.90	0.96	0.95	0.01	0.02	0.01	0.02
Gas-Oil	SS	**	0.97	0.84	0.94	0.98	0.01	0.05	0.17	0.02
	LS	**	0.94	0.83	0.88	0.96	0.06	0.19	0.15	0.09
	Dolo	**	0.98	0.86	0.99	0.99	0.01	0.07	0.11	0.03
Gas - water	SS	**	0.86	0.76	N/A	0.90	0.01	0.02	N/A	0.01
	LS	**	0.99	0.92	N/A	0.98	0.01	0.20	N/A	0.13
Gas - cond	SS	**	0.92	N/A	N/A	N/A	0.06	N/A	N/A	N/A
	LS	**	0.95	N/A	N/A	N/A	0.05	N/A	N/A	N/A
Average			0.95	0.88	0.94	0.95	0.02	0.05	0.05	0.04

**IX. CONCLUSION**

1. Unlike previously published correlations, the methodology honors not only empirical background but also mathematical derivation based on experimental observations that were not exploited before.
2. A mathematical relation was invented to calculate the relative permeability relation exponents instead of assuming them.
3. The mathematical modification allows variation of relative permeability relation exponents with variation of end points that can be related to permeability and porosity distribution especially in numerical reservoir simulation studies.
4. The proposed model considers gas/condensate systems unlike previously published common relative permeability prediction correlations.
5. A giant data base of actual experimental relative permeability measurements “750 datasets” was used to test model validity and reliability.
6. Statistical analysis was made for model results comparison to actual data and three other common correlations; Corey, Honarpour and Pirson. Statistical analysis showed that the proposed methodology has significant reliability to predict relative permeability data.

7. The proposed model is valid to be used for almost all reservoir fluid systems, lithologies and wettabilities.

**MODEL PROGRAM AVAILABILITY**

The model macro enabled excel sheet can be downloaded through the below link:  
[https://drive.google.com/file/d/1tx\\_JjVrgrfd\\_5Vl61uCK-p0kSYh3MdEV/view?usp=drivesdk](https://drive.google.com/file/d/1tx_JjVrgrfd_5Vl61uCK-p0kSYh3MdEV/view?usp=drivesdk)

**NOMENCLATURE**

- kro Oil relative permeability
- krg Gas relative permeability
- krw Water relative permeability
- k Absolute permeability by air
- Lith Reservoir lithology
- Wet Reservoir wettability
- Phi Porosity
- Slc liquid total critical saturation
- (kro)Swc Oil relative permeability at connate-water saturation
- (kro)Sgc Oil relative permeability at critical gas saturation



## A Modified Method for Predicting Relative Permeability

( $k_{rw}$ ) $S_{orw}$   
 Water relative permeability at the residual oil saturation  
 ( $k_{rc}$ ) $S_{cc}$   
 Condensate relative permeability at critical gas saturation  
 $S_{orw}$  Residual oil saturation in the water-oil system  
 $S_{org}$  Residual oil saturation in the gas-oil system  
 $S_{gc}$  Critical gas saturation  
 $n_o$  Oil exponent on relative permeability curves  
 $n_w$  Water exponent on relative permeability curves  
 $n_g$  Gas exponent on relative permeability curves  
 $n_c$  Condensate exponent on relative permeability curves  
 $T$  T-factor in the modified model  
 $S_{w\_x}$   
 Value of water saturation at the intersection point  
 $A$  Adjusting factor in the modified model  
 $B$  Adjusting factor in the modified model  
 $R^2$  Coefficient of determination  
 $MAE$  Mean absolute error

### APPENDIX

Modified models for different reservoir rock and fluid systems:

#### A. Water-Oil system:

Water-wet sandstone

$$k_{ro} = (k_{ro})_{Swc} * \left[ \frac{1-S_w-S_{orw}}{1-S_{wc}-S_{orw}} \right]^{3.5 * \frac{(1+T)}{2+T}}$$

$$k_{rw} = (k_{rw})_{Sorw} * \left[ \frac{S_w-S_{wc}}{1-S_{wc}-S_{orw}} \right]^{1.6 * \frac{(1+T)}{2}}$$

Oil-wet sandstone

$$k_{ro} = (k_{ro})_{Swc} * \left[ \frac{1-S_w-S_{orw}}{1-S_{wc}-S_{orw}} \right]^{4 * \frac{(1+T)}{2+T}}$$

$$k_{rw} = (k_{rw})_{Sorw} * \left[ \frac{S_w-S_{wc}}{1-S_{wc}-S_{orw}} \right]^{2.03 * \frac{(1+T)}{2}}$$

Mixed-wet sandstone:

$$k_{ro} = (k_{ro})_{Swc} * \left[ \frac{1-S_w-S_{orw}}{1-S_{wc}-S_{orw}} \right]^{3.8 * \frac{(1+T)}{2+T}}$$

$$k_{rw} = (k_{rw})_{Sorw} * \left[ \frac{S_w-S_{wc}}{1-S_{wc}-S_{orw}} \right]^{1.46 * \frac{(1+T)}{2}}$$

Water-wet limestone:

$$k_{ro} = (k_{ro})_{Swc} * \left[ \frac{1-S_w-S_{orw}}{1-S_{wc}-S_{orw}} \right]^{2.5 * \frac{(1+T)}{2+T}}$$

$$k_{rw} = (k_{rw})_{Sorw} * \left[ \frac{S_w-S_{wc}}{1-S_{wc}-S_{orw}} \right]^{1.13 * \frac{(1+T)}{2}}$$

Oil-wet limestone:

$$k_{ro} = (k_{ro})_{Swc} * \left[ \frac{1-S_w-S_{orw}}{1-S_{wc}-S_{orw}} \right]^{4.5 * \frac{(1+T)}{2+T}}$$

$$k_{rw} = (k_{rw})_{Sorw} * \left[ \frac{S_w-S_{wc}}{1-S_{wc}-S_{orw}} \right]^{0.96 * \frac{(1+T)}{2}}$$

Mixed-wet limestone:

$$k_{ro} = (k_{ro})_{Swc} * \left[ \frac{1-S_w-S_{orw}}{1-S_{wc}-S_{orw}} \right]^{4.35 * \frac{(1+T)}{2+T}}$$

$$k_{rw} = (k_{rw})_{Sorw} * \left[ \frac{S_w-S_{wc}}{1-S_{wc}-S_{orw}} \right]^{0.86 * \frac{(1+T)}{2}}$$

Water-wet Dolomite:

$$k_{ro} = (k_{ro})_{Swc} * \left[ \frac{1-S_w-S_{orw}}{1-S_{wc}-S_{orw}} \right]^{3.5 * \frac{(1+T)}{2+T}}$$

$$k_{rw} = (k_{rw})_{Sorw} * \left[ \frac{S_w-S_{wc}}{1-S_{wc}-S_{orw}} \right]^{0.87 * \frac{(1+T)}{2}}$$

Oil-wet Dolomite:

$$k_{ro} = (k_{ro})_{Swc} * \left[ \frac{1-S_w-S_{orw}}{1-S_{wc}-S_{orw}} \right]^{4 * \frac{(1+T)}{2+T}}$$

$$k_{rw} = (k_{rw})_{Sorw} * \left[ \frac{S_w-S_{wc}}{1-S_{wc}-S_{orw}} \right]^{0.92 * \frac{(1+T)}{2}}$$

Oil-wet Dolomite:

$$k_{ro} = (k_{ro})_{Swc} * \left[ \frac{1-S_w-S_{orw}}{1-S_{wc}-S_{orw}} \right]^{3.9 * \frac{(1+T)}{2+T}}$$

$$k_{rw} = (k_{rw})_{Sorw} * \left[ \frac{S_w-S_{wc}}{1-S_{wc}-S_{orw}} \right]^{1.84 * \frac{(1+T)}{2}}$$

Gas-Oil system:

Sandstone:

$$k_{ro} = (k_{ro})_{Sgc} * \left[ \frac{1-S_g-S_{lc}}{1-S_{gc}-S_{lc}} \right]^{3.54 * \frac{(1+T)}{2+T}}$$

$$k_{rg} = (k_{rg})_{Sorg} * \left[ \frac{S_g-S_{gc}}{1-S_{gc}-S_{lc}} \right]^{1.76 * \frac{(1+T)}{2}}$$

$$S_{lc} = S_{wc} + S_{org}$$

Limestone:

$$k_{ro} = (k_{ro})_{Sgc} * \left[ \frac{1-S_g-S_{lc}}{1-S_{gc}-S_{lc}} \right]^{1.85 * \frac{(1+T)}{2+T}}$$

$$k_{rg} = (k_{rg})_{Sorg} * \left[ \frac{S_g-S_{gc}}{1-S_{gc}-S_{lc}} \right]^{1.8 * \frac{(1+T)}{2}}$$

$$S_{lc} = S_{wc} + S_{org}$$

Dolomite:

$$k_{ro} = (k_{ro})_{Sgc} * \left[ \frac{1-S_g-S_{lc}}{1-S_{gc}-S_{lc}} \right]^{5.4 * \frac{(1+T)}{2+T}}$$

$$k_{rg} = (k_{rg})_{Sorg} * \left[ \frac{S_g-S_{gc}}{1-S_{gc}-S_{lc}} \right]^{1.53 * \frac{(1+T)}{2}}$$

$$S_{lc} = S_{wc} + S_{org}$$

Gas-Water system:

Sandstone:

$$k_{rw} = (k_{rw})_{Sgc} * \left[ \frac{1-S_g-S_{wc}}{1-S_{gc}-S_{wc}} \right]^{3.74 * \frac{(1+T)}{2+T}}$$

$$k_{rg} = (k_{rg})_{Swc} * \left[ \frac{S_g - S_{gc}}{1 - S_{gc} - S_{wc}} \right]^{2.43 * \frac{(1+T)}{2}}$$

Limestone:

$$k_{rw} = (k_{rw})_{Sgc} * \left[ \frac{1 - S_g - S_{wc}}{1 - S_{gc} - S_{wc}} \right]^{3.74 * \frac{(1+T)}{2 * T}}$$

$$k_{rg} = (k_{rg})_{Swc} * \left[ \frac{S_g - S_{gc}}{1 - S_{gc} - S_{wc}} \right]^{2.43 * \frac{(1+T)}{2}}$$

Gas-condensate system:

Sandstone:

$$k_{rc} = (k_{rc})_{Sgc} * \left[ \frac{1 - S_g - S_{lc}}{1 - S_{gc} - S_{lc}} \right]^{4.46 * \frac{(1+T)}{2 * T}}$$

$$k_{rg} = (k_{rg})_{Scc} * \left[ \frac{S_g - S_{gc}}{1 - S_{gc} - S_{lc}} \right]^{1.785 * \frac{(1+T)}{2}}$$

$$S_{lc} = S_{wc} + S_{cc}$$

Limestone:

$$k_{rc} = (k_{rc})_{Sgc} * \left[ \frac{1 - S_g - S_{lc}}{1 - S_{gc} - S_{lc}} \right]^{2.66 * \frac{(1+T)}{2 * T}}$$

$$k_{rg} = (k_{rg})_{Scc} * \left[ \frac{S_g - S_{gc}}{1 - S_{gc} - S_{lc}} \right]^{1.22 * \frac{(1+T)}{2}}$$

$$S_{lc} = S_{wc} + S_{cc}$$

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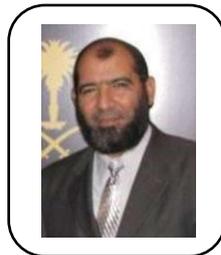
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