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Simulation of Crack Propagation in Some Self-Compacting Concrete Structures and Determination of Their Strength and Ductility Performances

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Abstract: ***A study on the crack propagation, strength and ductility performances of some self-compacting concrete (SCC) structures with and without steel fibers can be made through experiments. But testing is laborious. For large structure like dam, testing is impossible. Sometimes, there could be insufficient laboratory facilities. Under such circumstances, the non-linear analysis of concrete structures became a novel design tool. It employs the power of computer simulation using finite element method (FEM) based software to support the structural engineers.***

The ATENA software uses smeared crack approach and is based on the Bazant's crack band theory. The experimental total fracture energies of both SCC and steel fiber reinforced SCC (Steel fibers of length 25mm@ 0.6% by volume of mix) beams of strength M50 determined by RILEM's work of fracture method, corresponding material properties and tension softening behavior are used as input data in the software. The simulated load-deflection/CMOD curve could be used to access the strength and ductility performances of the structures. The strength is identified by peak load and ductility by extension of tail end of this curve beyond yield load. Hence, this paper presents an appropriate methodology to determine the performances by simulation.

Ductility is the desired property required for structures during earthquake. This can be improved by increasing the main steel reinforcement, incorporate fiber reinforced plastic (FRP) and steel tubes to impart it. But this could be uneconomical. In such cases, it is to be investigated that ductility can be improved by incorporating steel fibers of appropriate aspect ratio and volume in the mix.

The ductility ratio (μ) determined from the load-deflection/CMOD curves is further used to evaluate the response reduction factor(R) by an empirical formula as established in earlier investigation. The R used in base shear formula of IS 1893(Part-I)-2002 code considers the ductility property of the structures that are subjected to lateral base shear during earthquake.

Thus this study is carried out to identify the usefulness of simulation technique which is a new and robust tool to assess these performances of the structures using ATENA software that avoid the tedious testing procedures. This study investigates the influence of steel fibers in SFRSCC structures to improve their strength and ductility performances compared to SCC structures.

Keywords: ***applications, ductility performance, fracture parameters, crack pattern, simulation, strength performance.***

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I. INTRODUCTION

Under circumstances when it is not possible to test the performance of concrete structures in the laboratory due to their huge size and complex geometry, a new and robust tool such as computer simulation can be introduced for checking the performance of concrete structures. According to **V.Cervenka [1],** the simulation is considered as a virtual tool for testing of structures and can serve to find an optimal and cost effective design solution to support the structural engineers in design and development. It is a modern tool for optimization of structures to achieve the economy.

Also in cases which are not well covered by the code of practice provisions, the nonlinear analysis of concrete structures is a novel design tool since it employs the computer simulation. The finite element based failure analysis can take the advantages of rational theories such as fracture mechanics which makes possible a virtual testing of building structures under the designed loading and environmental conditions. The non-linear analysis of concrete structures is performed by using the finite element method (FEM) based ATENA software which it requires certain input data such as fracture parameters and material properties of concrete. The simulation can be performed by Smeared or Discrete Crack Approach of crack analysis. The smeared crack approach based upon Bazant's crack band model is adopted in the ATENA software **[2].** This approach is useful under circumstances where crack is not straight but tortuous and distributed cracking is possible especially for material like concrete.

In this study, the performances of some self-compacting concrete (SCC) and steel fiber reinforced self-compacting concrete (SFRSCC) structures such as pipes, corbels, deep beams, portal fames and dam of the given dimensions are proposed to be studied through simulation.

The experimental fracture parameters using various non-linear fracture models were evaluated by **Roesler et.al [3]-[5]** for M50 normal concrete and the same concrete incorporated with 40mm length polypropylene fibers through bending tests on three geometrically similar beams under three point loading.

These experimental fracture parameters for mode-I fracture were used as input data in ABAQUS software for predicting the experimental load-CMOD curves by simulation and to propose the crack models for the same materials.

Considering these investigations carried by **Roesler et.al[3]-[5]** as basis**,** the experimental fracture parameters of SCC beams**[6],[7]** and of SFRSCC beams **[8]** with hooked end steel fibers of length 25 mm added at 0.6% by the volume of the mix are evaluated. The fracture parameters such as total fracture energy, the tension softening character (exponential for SCC and SFRC for SFRSCC beams) and material properties were used as input data in ATENA software to predict the experimental load-CMOD curves through simulation and these curves are later used to characterize the bilinear crack model for SCC and tri-linear crack model for SFRSCC beams. The SFRC option used in software considers the portion of load CMOD curves after the first fiber pullout. The material properties are the tensile strength, poison's ratio, compressive strength and young's modulus.

Earlier to **Roesler's** above work, the nonlinear analysis of fracture mechanics was studied by many investigators. **Hillerborg [9]** proposed the first nonlinear theory of fracture mechanics to concrete known as Cohesive or Fictious Crack Model (FCM). The term fictitious is used to underline the fact that the portion of crack cannot be continuous with full separation of its faces as in a real traction free crack. **Hillerborg [10]** further applied the fictitious crack model to analyze the fracture in the fiber reinforced concrete (FRC) and to determine the fracture energy. The fracture process zone (FPZ) was modeled as a fictitious crack. The stress-displacement responses were obtained for the bulk material as well as the fracture zone. He suggested that the action of the fibers modifies the crack opening behavior and thus controls the stress- displacement response in the fracture zone. He also identified that the bond-slip properties of the composites, which in turn controls the crack opening behavior mainly depends upon the type (plain, deformed, etc.) and the fiber material (glass, steel, etc.). **A.E. Naaman and Visalvanich [11]** used the term 'pseudo plastic zone' to describe the zone where fibers provide bridging across the cracks. The model proposed assumed that the main portion of energy required during the fracture comes from fiber pullout in the pseudo plastic zone. **Yiu-Mai [12]** identified that the fracture mechanism of concrete and cement based composites containing notches and sharp cracks can be characterized by a cohesive zone that is formed at their tips. Cohesive Zone is a region where damages accumulate and fracture process occurs. This zone is known as fracture process zone (FPZ) in the crack**. V. Cervenka and R. Pukl [13]** performed the simulation for conventional vibrated concrete and fiber reinforced concrete beams by SBETA material modeling using the ATENA software **[2]**. The load displacement curves were obtained by a computer simulation using the program SBETA. **Chao Wang et.al [14]** investigated the fracture process of plain concrete and rubberized concrete by testing the beams of different sizes under three-point bending tests. They used ABAQUS software for the simulation.

SCC is different from normal concrete since it contains more powder compared to normal concrete. SCC is essential in congested members with heavy reinforcement due to its better deformability and segregation resistance. SCC has to satisfy the self-compaction when it is fresh state whereas strength and durability at harden state. Self-compaction can be achieved by using super plasticizer, limited aggregate content whereas strength and durability by using low w/c ratio, limiting the coarse aggregates content. An empirical mix design method presented **by Okamura and Ouchi [15]** are introduced as the guidelines for mix design in EFNARC **[16].** It is found that these design procedures are too complicated for practical implementation. Hence, the design procedure as suggested by **Nan Su, His and Chai [17]** is alternatively used. It is based on packing theory which starts with packing of all aggregates in the mix and later filling of aggregate voids with paste. It is easier to carryout and yields in less cement and filler material making concrete more economical. But the fresh mix is less workable at higher packing factors of 1.16 to 1.18, since it yields the less powder content. Thus SCC mix used in the research to cast 3 geometrically similar beams which are to be used to test to determine fracture energy is designed by Nan-Su method using smaller packing factor of 1.12. Such a mix was found to yield satisfactory results at fresh and harden state.

The hooked end steel fibers are added in SCC since these fibers significantly enhance the resistance against the cracking behavior of concrete. This is due to the reason that the steel material exhibits better strain hardening property. The presence and increase in steel fiber volume up to certain limit in concrete allows a significant increase in crack width of concrete thereby effectively enhance material toughness, ductility and the damage tolerance. The addition of steel fibers changes the failure mode of High Performance Concrete (HPC) specimens like SCC from complete damage to a somewhat ductile behavior. Due to very high strength and homogeneity, the SCC is very brittle. It can be made ductile by adding steel fibers. It is reported by **F. Papworth [18]** that the increase in compressive strength provided by steel fibers very rarely exceeds 25%. Both fiber volume (ranging from 30kg/m^3 to 120 kg/m^3) and aspect ratio(fiber length ranging between 30mm to 50mm) play an important role in increasing the flexural strength and toughness (calculated as per ASTM-C-1018 procedures) than compressive and split tensile strength.

The ductility is the essential property required for the structure subjected to lateral base shear during earthquake. It is responsible to reduce the transmitted force to the one that is sustainable. The ductility is the capacity of a material or structural component or entire structure to undergo a large deformation after its initial yield without any significant reduction in the yield strength later.

A.K.H. Kwan and J.C.M. Ho [19] suggested to provide significant amount of confinement in the form of reinforcing steel or in the form of steel tube and fiber reinforced plastic (FRP) to improve the ductility of high strength concrete beams and columns.

To evaluate the flexural ductility and to avoid cumbersome nonlinear moment-curvature analysis, formulas for direct evaluation of the flexural ductility of beams and columns were established. The proposed formulas can predict the ductility of concrete beams and columns fair accurately.

It is also investigated by **P. Agarwal and M. Shrikhade [20]** that the toughness and ductility of the concrete members and structures can alternatively be improved by the addition of steel fibers in appropriate volume and aspect ratio in the mix.

The ductility of SFRSCC structure can be measured by a factor known as ductility ratio which is determined based on its displacements caused due to external load. The ductility ratio (μ) is the ratio of maximum to minimum deflection at yield load within which there is no significant loss of initial yielding in the load displacement/CMOD curves. The displacement ductility ratio or ductility factor (μ) of SFRSCC structures is used in the reduction of the required linear elastic strength of structure subjected to lateral shear during earthquake. The ductility ratio of the structures usually varies from 1 to 10. The New Zealand code proposes different design ductility factors for different structures. Its value is 1 for elastic structures whereas it ranges from 3 to 6 for ductile structures.

The ductility ratios can be related to the response reduction factor 'R' of the base shear formula in the IS 1893 code. The 'R' is based upon the fundamental natural period of frequency of earthquake ground motion. The incorporation of 'R' is an attempt to consider the structural ductility during earthquake. The IS 1893 code suggests R=1.5 for brittle material like unreinforced masonry wall building and as high value as 5 for more ductile structures like shear wall building. The 'R' value increases with the increase in structural ductility and its energy dissipation capacity. The 'R' value is assigned to different types of building structures based upon the empirical and semi empirical judgment and post building performance in the earthquake.

In this research, the ductility ratios evaluated from the simulated load displacement/CMOD curves for different structures. The response reduction factor R is established **[8]** empirically as $R = \sqrt{\mu^{1.5} - 1}$ based on tests and judgment.

Hence, the study is extended to use the input data determined for M50 SCC **[6],[7]** and SFRSCC mix **[8]** to evaluate the strength and ductility performances of some of the real-time structures such as pipes, dams, deep beams, corbels and portal frames through simulation. The simulation is performed to obtain the load-deflection/CMOD curves for these different structures subjected to mode-I fracture. These load deflection/CMOD curves are used to assess the fracture energy, ultimate (peak) load. The pre peak (hardening property) and post peak (softening property) behavior of the structures are used to measure the ductility property especially in SFRSCC structures. The peak load of the structure helps the designer to determine the strength performance of the structures. Due to incremental load on the structures during simulation, the initiation and propagation of the cracks in the structural model are noticed at critical sections. The crack propagation captured during simulation at different load steps in the structures could help to assess the extent of damage before the real structure on the ground is implemented.

II. RESEARCH SIGNIFICANCE

Like all concretes, the high performance concrete (HPC) has the disadvantage of rendering a lower ductility during earthquake. From the safety point of view, ductility should be regarded as crucial as strength. However, compared to strength analysis, ductility analysis is more difficult. A little attention is paid to ensure that the HPC structures would have sufficient ductility. In this regard, it should be noted that the current design codes, which do not provide any guidelines for ductility design, are not applicable to HPC structures. This study could prove that ductility can also be improved by steel fibers as by other methods such as increasing main reinforcement in the concrete etc.

Under such circumstances, this study is useful to measure the ductility and strength performances of structures by simulation using FEM based software. The experimental fracture parameters derived from nonlinear analysis and material properties of SCC and SFRSCC of given strength M50 can be used as input data in the software. The simulation avoids the testing for structure like dam and is tedious for other structures. Hence, a simple procedure with methodology can be introduced to study the strength and ductility performance of both structures. It is also helpful of identify the initiation, propagation of cracks at critical sections and thus the crack pattern in the structures due to external loading through simulation.

III. INPUT DATA FOR SIMULATION

The fracture parameters and material properties of SCC beams **[6], [7]** and SFRSCC beams **[8]** evaluated in the earlier investigations are to be used as INPUT data for simulation of the structures. These are given in **Table 1** below. The fracture parameters are evaluated through three point load bending tests on three geometrically similar size notched beams. The tests are performed using very stiff servo hydraulic deflection controlled universal testing machine of capacity 100 kN and clip gauge of maximum range 10mm.

Table-1: Material properties and fracture parameters for M50 concretes

Material property	For SCC	SFRSCC
28 days cube strength(f_c) in MPa	56.16	58.67
Young's modulus (E) in MPa	36700	39700
Poisson's ratio(μ)	0.184	0.186
Split tensile strength(f_t) in MPa	4.19	4.384

Size dependent total fracture energy based on load-CMOD curves as per RILEM's work of fracture method (Average of 3 companion beams)	G_F in N/m for SCC		G_{FRC} in N/m for SFRSCC	
	L-P-series	288.8	L-F-series	5569.00
	M-P-series	248.5	M-F-series	6622.03
	S-P-series	221.8	S-F-series	7680.67
Size independent specific fracture energy based on load-deflection curves for SCC beams			342.3 N/m	

The dimensions (length, depth, thickness) of 3 geometrically beams as suggested by the RILEM **[21]** for large, medium and small size beam series are 1100X240X100mm, 550X120X100mm and 275X60 X100mm respectively. The corresponding spans are 960mm, 480mm and 240mm respectively. Keeping thickness of all beams equal to 100mm, the beams will maintain the two dimensional homogeneity with the Span/Depth ratio is equal to 4.

The total fracture energies of both SCC and SFRSCC beams given in **Table 1** vary with depth of beam and hence are the size dependent. It should be scaled for the given size (or depth) of element/component/member of structure that is predominantly subjected to mode-I fracture. For corbel to the depth of cantilever beam equal to 800mm. For portal frame to the depth of beam equal to 400mm.For the SCC dam, the base length equal to 100m. For pipes, the outer diameters of the pipes are considered.

This is performed using the linear regression line equations derived for fracture energy in the earlier investigations **[6]-[8].** The equations (1) and (2) are for SCC and SFRSCC respectively and are given below.

i.e. $G_F = [3.722(D-60) + 221.8]\ N/m$ (1)

And, $G_{FRC} = [7600 - 12.22(D-60)]\ N/m$ (2)

Where, G_F and G_{FRC} are the total fracture energies for SCC and SFRCC respectively and D=depth of beam. However, these equations are used for beams of size greater than 60mm.

It is investigated **[6], [7]** that total fracture energy G_F calculated by RILEM's work of fracture method from experimental load-deflection curve is less than that obtained from load-CMOD curves. This ratio of $(G_F)v/(G_F)_{AV}$ is estimated to be 0.95 for all beams. However, the total fracture energies from equations (1) and (2) are based on load-CMOD curves. These values are to be modified by multiplying with a correction factor of 0.95, if the simulation of the structure required is based on load-deflection. The similar relation is assumed to exist even for SFRSCC and hence G_{FRC} values should be multiplied by 0.95.

Apart from above quantities, the software also needs the corresponding tension softening relation. The bilinear crack model proposed for SCC beams in the earlier investigation **[6], [7]** is exploited from exponentially decaying tension softening curve. For SCC structures, the simulation is performed using exponential tension softening curve which is the only option in ATENA software for plain mix. For SFRSCC structures **[8]**,

Table-2: Scaled experimental total fracture energies for M50 strength structures

Type of Structure	SCC structures		SFRSCC Structures	
	Scaled & modified experimental	Corresponding tension	Scaled & modified experimen	Corresponding tension
		total fracture energy in N/m	**softening curve**	**tal total fracture energy in N/m**	**softening curve**
Pipe	Ex 1	259.0	Exponential curve	---	SFRC
	Ex 2	288.8		5400.4	
	Ex3	333.5		---	
Dam		37418.0			
Deep beam		295.57		4433.84	SFRC
Corbel		470.06		5569.00	
Portal Frame		330.94		3272.94	

SFRC softening relation is used which considers the portion of load CMOD curves after the first fiber pullout. The estimated scaled total fracture energies and the corresponding tension softening curves are given in **Table 2** above.

IV. METHODOLOGY ADOPTED FOR STUDY

The following steps are used to study the crack propagation (pattern), strength and ductility performance of plain and reinforced SCC and SFRSCC structures through simulation.

1. For the given dimensions of the structure and working loads, the structures are designed for reinforcement using the limit state method for the given grade of SCC material and steel. (In this study, SCC of grade M50 and steel of Fe 415 is used for the design)(Refer Table-4)
2. The 2D model of the structure with the given dimensions is developed in the software. Finite element mesh of suitable size and shape should be incorporated in it.(Refer Fig 4,5,8,10,15 and Fig 20 for all structures)
3. The experimental material properties, fracture parameters are to be evaluated for the given grade of SCC and SFRSCC beams and corresponding tension softening relations should be used as INPUT data in the software.(Refer Table-1)
4. The material properties to be used are the Young's modulus (E), Poisson's ratio (μ), Cube compressive strength (f_c) at 28 days, Tensile strength (f_t). .(Refer Table-1)
5. The size dependent total fracture energy based on load v/s CMOD curves should be scaled using regression equation (1) and (2) for SCC and SFRSCC structures respectively. This scaled value should be further modified using correction factor of 0.95 if, the simulation of the structure is based on load v/s deflection. The tension softening characters such as exponential for SCC and SFRC for SFRSCC beams are to be used. (Refer Table-2)
6. Alternatively, due to complex geometry of SCC structures, size independent specific fracture energy and corresponding tension softening relation can be used as INPUT data. This value is 342.3 N/m and given in **Table 1** above is evaluated **[6], [7]** for SCC beams by the method suggested by **Elices, M., et al. [22]** and **A. Ramachandramurthy et.al [23].**

Corresponding tension softening curve is the exponentially decaying with bilinear approximation as suggested by **Abdalla and Karihaloo [24]** and also by **A Ramachandramurthy et al. [25].** (Refer Table-1) (An example of corbel is undertaken in Section V-partF)

7. Simulation of the structures should be performed in the software under deflection control using the option **"prescribed deformation"**.

8. The deformations for successive increment of loads on the structure are stored in the software. The load-deflection /CMOD curves developed from these results are used to identify hardening (or pre peak) and softening (or post peak) behavior and to estimate the total fracture energies. The peak load of the curve for the particular structure is compared with the actual working load for which the structure is designed. This will help as a tool to the designer to optimize the structures before it is being implemented.

9. Ductility property can be assessed using load-deflection/CMOD curves of the structures.

10. Further, during the course of simulation, the crack initiation and propagation at critical sections at various stages of failure can be captured.

The performances of some of the SCC and SFRSCC structures are discussed as below.

V. ANALYSIS OF STRENGTH AND DUCTILITY PERFORMANCES OF STRUCTURES

A. Strength Analysis of SCC And SFRSCC Pipes

The current practice in the design of unreinforced concrete pipes is to assume that concrete is a linear elastic brittle material. As it is studied **[26]** earlier that unreinforced concrete pipes will fail in the following two different ways.

1. Bending (or beam) failure (**Fig 1.a**.)- In this case, the failure load is calculated using beam bending theory
2. Crushing (ring) failure (**Fig 1.b)-** The failure load in this case is calculated from the theory of elasticity of a thin ring.

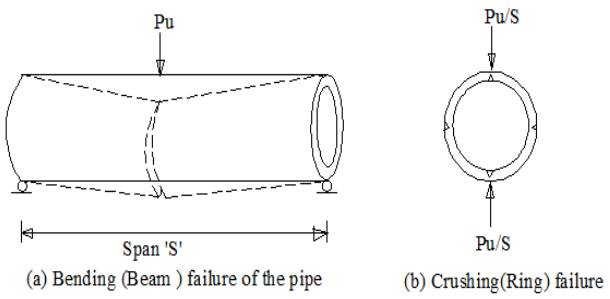


Fig.1. Modes of failures in pipes.

In bending failure, the pipe acts like a beam whereas in crushing failure, the pipe is loaded along its genetrix. Before it fails by crushing, fracture zones are formed in four places. At each place, a hinge rotation is formed. The relation between the rotation and corresponding moment is analyzed by Gustaffson **[27]**.

It is investigated **[26]** in fact that the pipes of diameter less than or equal to 300mm are known to fail by more often by bending rather than by crushing. This is particularly so because the longer pipes are usually used in practice to achieve economy. It is already proved that the bending strength is considerably lower than crushing strength.

The bending stress f_f corresponding to the ultimate load P_U as given by the beam theory is given in equation (3) below and it should be equal to tensile strength of concrete.

$$f_{fbending} = \left(\frac{M_U}{Z}\right) = \left\{\frac{\frac{P_u S}{4}}{\left[\frac{\pi}{32} d_o^3 \left[1-\left(\frac{d_i}{d_o}\right)^4\right]\right]}\right\} = \left\{\frac{8 P_U S}{\left[\pi d_o^3 \left[1-\left(\frac{d_i}{d_o}\right)^4\right]\right]}\right\} \quad (3)$$

Where, M_U=ultimate moment, Z= section modulus of pipe section, P_u= ultimate load, S=span of the pipe, d_i=internal diameter and d_o =outer diameter.

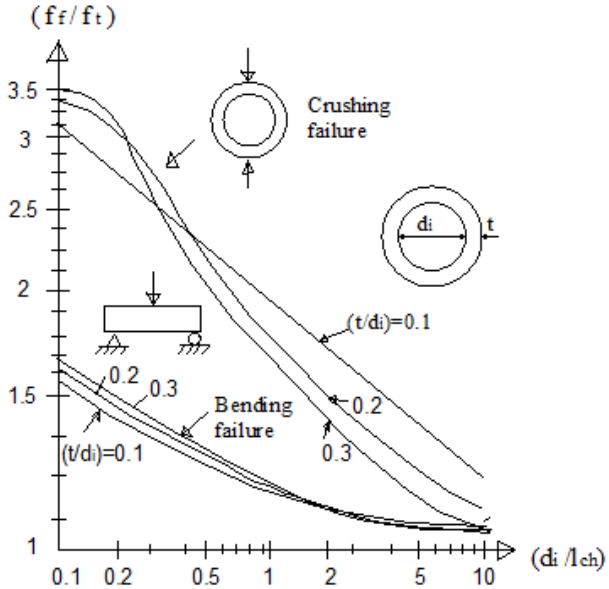


Fig.2. Result Analysis of Gustaffson for pipes

Similarly, the crushing stress $f_{fcrushing}$ corresponding to the line load $\left(\frac{P_U}{S}\right)$ at failure is given by the equation (4) below. According to the theory of curved beams, $f_{fcrushing}$ should be equal to tensile strength of concrete for very thin walled pipe.

$$f_{fcrushing} = \left[\left(\frac{6}{\pi}\right) \frac{\frac{P_U}{S}\left(1-\frac{d_i}{d_o}\right)}{d_o\left(1-\frac{d_i}{d_o}\right)^2}\right] \quad (4)$$

It is realized that the Gustaffson's **[26,][27]** result analysis shown in **Fig 2** above yield the realistic values as compared to the test results.

It is observed from **Fig 2** that the formal strength $(f_f)$is much higher for crushing failure than that for the bending failure and it is much more size dependent for crushing failure. This is due to the fact that the structure is statically indeterminate. The high size dependency also means that the f_f is very sensitive to the value of fracture energy(G_F), which for the crushing failure is as important as the concrete strength.

Elfgren [28] identified the advantage with such a design. It is that the safety against failure becomes more even than the design performed by conventional methods, where the flexural strength is assumed to be constant irrespective of loading case and size of the pipes. Due to this advantage, Scandinavian pipes were designed according to Gustaffson's method.

Design Example of the SCC Pipes

Using the Gustafson's concept, the three design examples for SCC pipes of the different dimensions are carried out. For all the examples, the length and thickness of pipe are 1m and 30 mm respectively.

Example 1:
Given data:
Internal diameter $d_i = 100$ mm, Thickness(t) = 30mm, Length of pipe = 1m
Solution:
Outer diameter d_o = (100+ 2*30) =160mm
For size D (=outer diameter d_o) =160mm,
Using equation (1),

$$G_F = [3.722(D - 60) + 221.8]\ \text{N/m} \qquad (5a)$$

$$= [3.722 * (160 - 60) + 221.8] = 259\frac{N}{m} = 0.259\ \text{N/mm}$$

Characteristic length $l_{ch} = \left(\frac{EG_F}{f_t^2}\right)$

$$= \left[\frac{(0.369 * 10^5 * 0.259)}{4.19^2}\right] = 544.375\ \text{mm}$$

$$\left(\frac{d_i}{l_{ch}}\right) = \left(\frac{100}{544.375}\right) = 0.184 \approx 0.18$$

$$\text{and } \left(\frac{t}{d_i}\right) = \left(\frac{30}{100}\right) = 0.30 \qquad (5b)$$

From the graph given in **Fig 2**,

$$f_{f_{bending}} = (1.55 f_t) = (1.55 * 4.19) = 6.4945\ \text{MPa}$$

$$f_{f_{crushing}} = (3.3 f_t) = (3.30 * 4.19) = 13.827\ \text{MPa}$$

$$(f_{f_{bending}} = 6.4945\ \text{MPa}) = \left(\frac{\left(\frac{P_U S}{4}\right)}{\left(\frac{\pi}{32}\right) D_o^3 \left[1 - \left(\frac{D_i}{D_o}\right)^4\right]}\right)$$

$$= \left(\frac{\left(\frac{P_U * 1000}{4}\right)}{\left(\frac{\pi}{32}\right) 160^3 \left[1 - \left(\frac{100}{160}\right)^4\right]}\right)$$

Hence, $P_U = 8742\ \text{N} = 8.72\ \text{kN}$ (5c)

$$(f_{f_{crushing}} = 13.827\ \text{Mpa}) = \left[\left(\frac{6}{\pi}\right)\frac{\frac{P_U}{S}\left(1 - \frac{D_i}{D_o}\right)}{D_o\left(1 - \frac{D_i}{D_o}\right)^2}\right]$$

$$= \left[\left(\frac{6}{\pi}\right)\frac{\frac{P_U}{1000}\left(1 - \frac{100}{160}\right)}{160 * \left(1 - \frac{100}{160}\right)^2}\right]$$

Hence $P_U = 98996.96\ \text{N} = 99\ \text{kN}$ (5d)

Similarly, the results for other two examples of the different pipe diameters are presented in **Table 3** below.

Table- 3: Comparison of the results for SCC pipes design examples

Ex no	Internal dia (D_i) in mm	Thick-ness of pipe in mm	External dia (D_o) in mm	Length of pipe in mm	P_U in kN	
					By bend-ing	By crush-ing
1	100	30	160	1000	8.72	99.00
2	180		240		21.50	53.86
3	300		360		45.20	17.75

It is evident from **Table 3** that for the SCC pipes of particular length and thickness, as its internal diameter of pipe increases, the load carrying capacity of pipe under bending increases whereas under crushing the load capacity decreases.

It is also identified that Gustaffson's approach is successful for SCC pipes but not applicable to SFRSCC pipes due to its large values of G_{FRC} and l_{ch}.In such cases, the ATENA software can successfully be used to identify the failure pattern in SFRSCC pipes by simulation using the large value of G_{FRC} and other parameters as input data.

Comparison of Load-CMOD Curves for SCC and SFRSCC Pipes

The performance of the SCC and SFRSCC pipes under crushing along its genetrix is analyzed by the software. The **Example 2** is modeled in the software. The finite elements of suitable size and shape are incorporated in the pipe. The simulation is performed using parameters needed as INPUT data in the software and results are shown in **Fig 3** below.

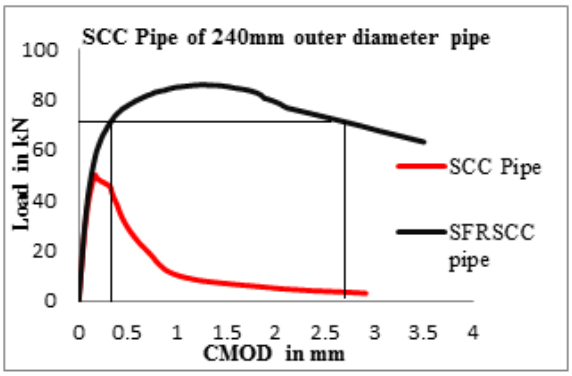


Fig.3. Load-CMOD curve for SCC and SFRSCC pipes

The peak load of SCC pipe from the graph is 50.08kN under crushing. But from Gustaffson's approach, it is 53.86 kN. This proves that ultimate loads for SCC pipe obtained from both approaches are almost same.

Similarly, the peak load for SFRSCC pipe is 85.83kN.Thus there will be increase in peak load of these pipes more than 70% compared to that of SCC pipe. This increase is associated with the increase the strain hardening property of SFRSCC pipe compared to SCC pipe by extending the curve up to large amount of deflection before the pipe reaches the peak load. This indicates that the steel fibers in SCC are responsible to increase crushing load capacity to a considerable extent.

The ductility ratio of SFRSCC pipes is determined from **Fig 3**. The yield load from the graph is 72 kN. At this yield load, the ductility of these pipes are calculated as follows,

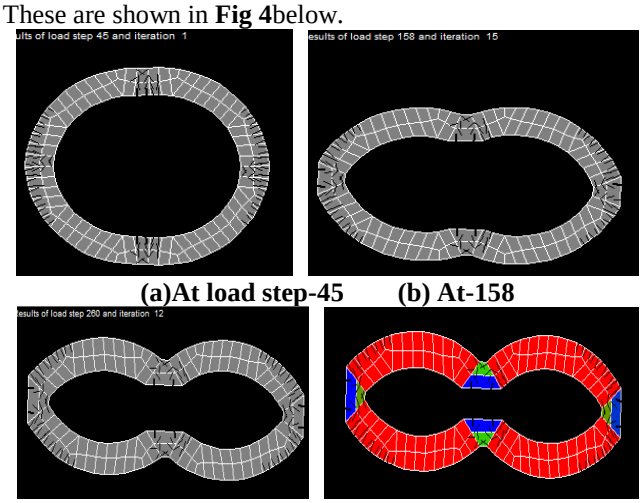
$$\text{Ductility ratio}(\mu) = \frac{\text{Maximum deflection at yield load}(u_{max})}{\text{Deflection at yield load}(u_y)}$$

$$= \frac{2.65\text{mm}}{0.40\text{mm}} = 6.625 \qquad (6)$$

There will be considerable increase of ductility in SFRSCC pipe as identified by its post peak behavior in **Fig 3**. Thus there is the advantage to use SFRSCC pipe as elevated pipes above the ground since they are efficient in absorbing the large amount of energy and lateral base shear during earthquake.

Study of Crack Pattern in SCC Pipes

These are shown in **Fig 4**below.



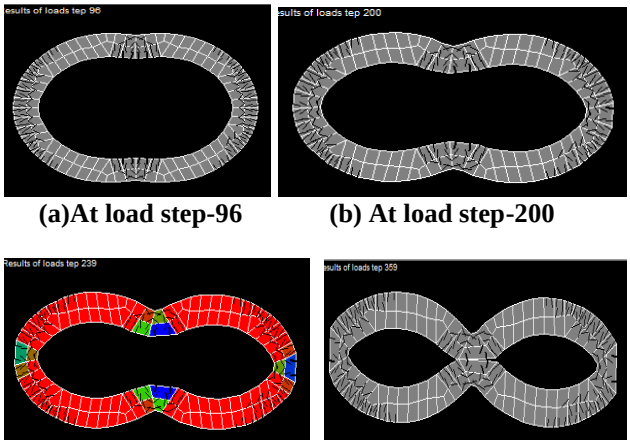
(a)At load step-45 **(b) At-158**

(c) At load step-260 **(d) At load step-280**

Fig.4. Crack pattern in SCC pipes at different load steps

Study of Crack Pattern in SFRSCC Pipes

Similarly these are shown in **Fig 5**below.



(a)At load step-96 **(b) At load step-200**

(c)At load step-239 **(d) At load step-359**

Fig.5. Crack pattern in SFRSCC pipes at different load steps

It is evident from **Fig 4** and **Fig 5** that the cracks are initiated in two horizontal crack zones (or hinge rotations) and are formed from inside surface of the pipes whereas cracks are formed at the two vertical fracture zones from outside surface of the pipe.

B. Study of Crack Pattern in Dams

The cracks in dams are due to the following two reasons.

1. Due to temperature gradient arising during the hydration of mass concrete.
2. Due to reservoir loads, earthquakes and temperature variations at the surface arising in its service.

The investigation of cracked dams has the following objectives.

1. Assessment of safety
2. Numerical simulation of crack initiation mechanism
3. Prediction of propagation and stability of the observed crack with respect to any future action
4. Evaluation of effectiveness of repair methods.

This is achieved for a SCC dam of the dimensions given in **Fig 6** below by simulation.

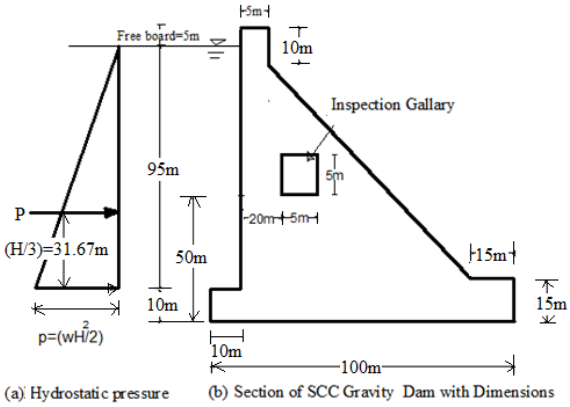


Fig.6. Cross-section of dam

Load -Deflection Curve For SCC Dam

The load deflection graph shown in **Fig 7** is developed using the software results for horizontal displacement occurred due to increment of load in the same direction. From the graph, the peak load that the SCC dam will take is 42,835 kN.

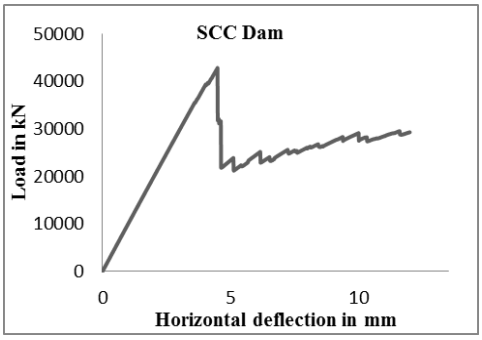


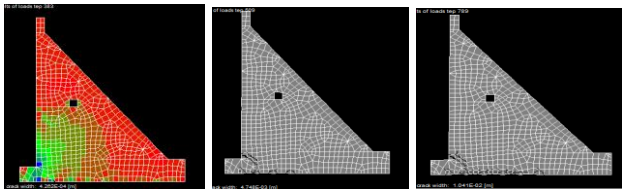
Fig.7. Load v/s Horizontal deflection for SCC dam

According to the theory of fracture mechanics, the SCC dam behaves as brittle structure following the principles of linear elastic fracture mechanics (LEFM) theory. This is due to the small size of fracture process zone (FPZ) occurring at the crack tip as compared to the huge size of the dam. Such a behavior of dam is identified in the graph by a sudden drop in the load at a particular deflection.

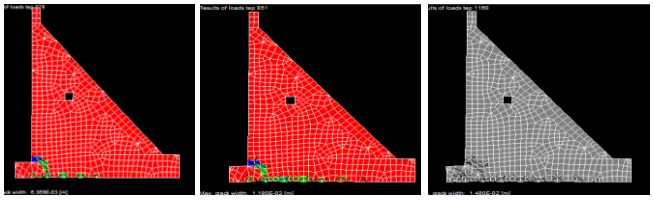
However, the use of steel fibers in dams to improve the ductility is discouraged since dam is large in size and affects the economy. The dam is water retaining structure that may lead to the corrosion of steel fibers.

Crack Pattern in SCC Dams

The crack propagations in SCC dam during simulation at different load steps are given in **Fig 8** below.



(a)At load step 263 **(b) At load step 509** **(c)At load step 578**



(d)At load step 789 (e) At load step851 (f) At load step 1100

Fig.8.Crack pattern in SCC dam at different load step

It is observed from **Fig 8** that the cracks are initiated at the junction between heel end and upstream vertical wall of the dam for the critical case when the reservoir is full. The width of the cracks and its propagation goes on increases as the incremental load increases on the dam. This study helps the designer to identify the failure pattern before it is being implemented as a real structure.

C.Deep Beams

These are the special beams provided under circumstances where the shear force is more predominant than the bending moment. It is investigated [27] that the behavior of deep beams is influenced by shear span to depth (a/d) ratio

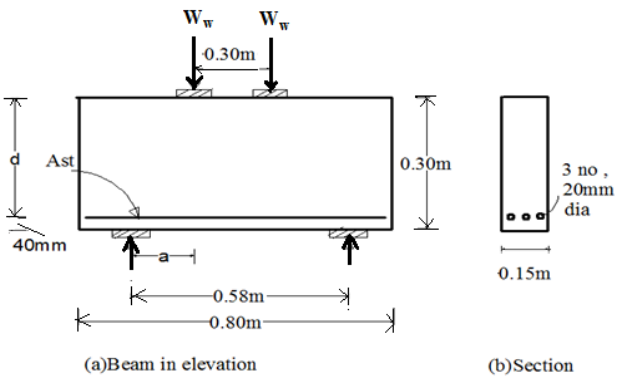


Fig.9.Details of deep beam

Failure pattern change considerably when this ratio is around 1.50.The addition of hooked end fibers improves both strength and ductility of these beams. The dimensions of beam are shown in **Fig 9.** The mesh arrangement in it is shown in **Fig 10** above.

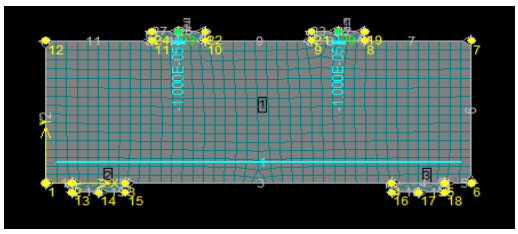


Fig.10.Finite element mesh in deep beam

Load-Deflection Graph

The reinforcement in the deep beam is designed numerically for vertical load W=350 kN. The simulated results are used to draw the graph shown in **Fig 11** below.

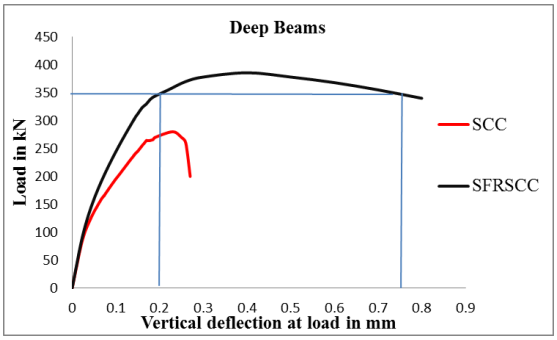


Fig.11. Load v/s Deflection graph for deep beam

The simulated peak loads for the SCC and SFRSCC deep beams are found to be 280.07 kN and 385.65 kN respectively. The increase in peak load in SFRSCC beam is 37.70% as compared to SCC beams.

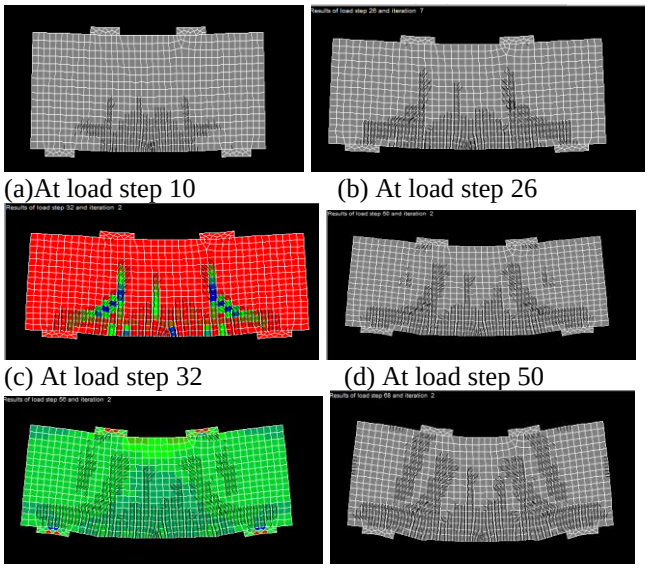
The ductility for SFRSCC deep beams at the yield load $(P_y) = 350$ kN is measured as follows,

$$\text{Ductility ratio}(\mu)\text{for SFRSCC beam} = \frac{0.76\text{mm}}{0.20\text{mm}} = 3.8 \qquad (7)$$

The steel fibers improve the ductility. Its strain hardening property is responsible to increase the peak load of SFRSCC beams compared to SCC beams and are able yield the peak load even greater than the design load of 350 kN.

Study of Crack Pattern for SCC Deep Beam

The crack pattern in SCC and SFRSCC deep beam are shown in **Fig 12** and **Fig 13** respectively below.



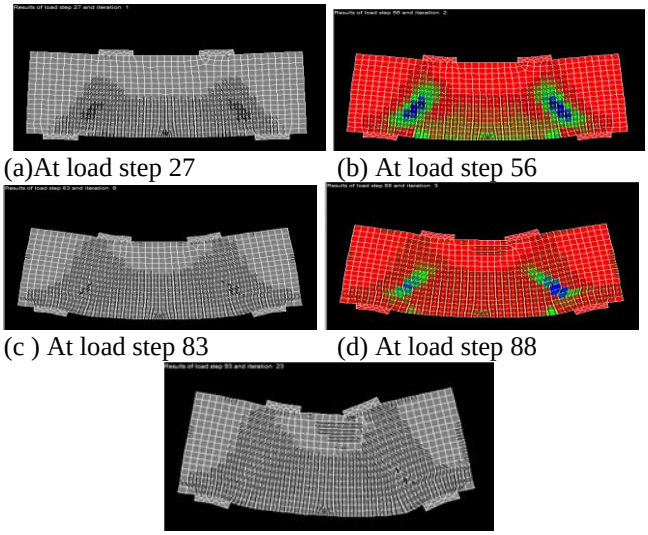
(a)At load step 10 (b) At load step 26

(c) At load step 32 (d) At load step 50

(e) At load step 56 (f) At load step 68

Fig.12. Crack pattern in SCC deep beam at various load steps

Study of Crack Pattern for SFRSCC Deep Beam



(a)At load step 27 (b) At load step 56

(c) At load step 83 (d) At load step 88

(e) At load step 93

Fig.13. Crack pattern in SFRSCC deep beam at various load steps

It is observed from **Fig 12** and **Fig 13** that cracks due to shear occurring diagonally between vertical load and nearer support are more predominant compared to cracks due moment that occur at the bottom in the middle for beams.

D.Corbels

The corbels are considered as special types of cantilever deep beams. They are often used in the industrial buildings to transfer the heavy beam reaction to the columns. Corbels are typically reinforced with main reinforcement at its top with vertical stirrups to resist the shear force. Steel fibers are tried as replacement to vertical stirrups.

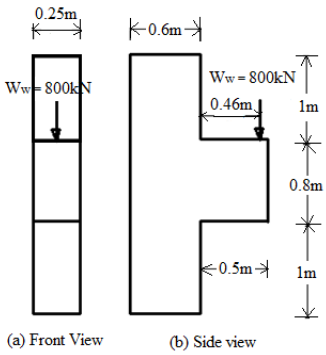


Fig.14.Dimensions details Fig.15. Mesh arrangement with reinforcement in corbel

The past investigation **[29]** shows that the failure is catastrophic, if no shear reinforcement is provided in the corbel. The addition of fibers changes the mode of failure to gradual and improves ductility. A fiber volume fraction of 1.4% was found to improve both strength and ductility.

A corbel for the dimensions shown in **Fig 14** is considered for study by simulation. The **Fig 15** shows the arrangement of mesh in the corbel.

Load-Deflection Curve for Corbel

The reinforcement in corbels was designed for working load of 800 kN. The simulation results are represented as graph in **Fig 16** below.

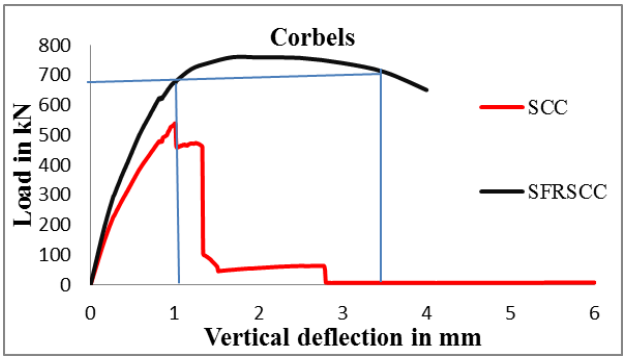


Fig.16.Load v/s Vertical deflection curves

The peak load for SCC corbel is 540.32 kN whereas for SFRSCC corbel, it is 760.24 kN. There will be increase in peak load by 40.70 % in SFRSCC corbel compared to SCC corbel. Comparing these simulated peak loads with the design load of 800 kN, it is found that both the corbels are unable to reach the design load.

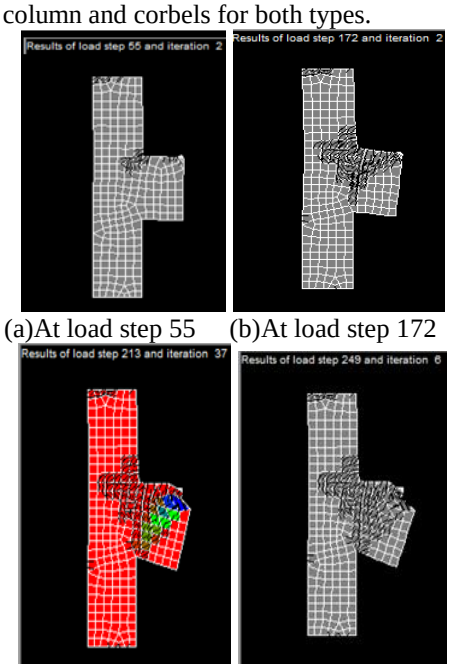
The ductility due to steel fibers in SFRSCC corbel at the yield load of $P_y = 680$ kN is as follows.

$$\text{Ductility ratio}(\mu) = \left(\frac{3.8\text{mm}}{1.05\text{mm}}\right) = 3.62 \quad (8)$$

Study of Crack Pattern in SCC Corbel

The simulated crack patterns are shown in **Fig 17** and **Fig 18** below for SCC and SFRSCC corbel respectively.

It is observed that from above figures that the cracks will initiate and propagate in the upper corner at the junction between column and corbels for both types.

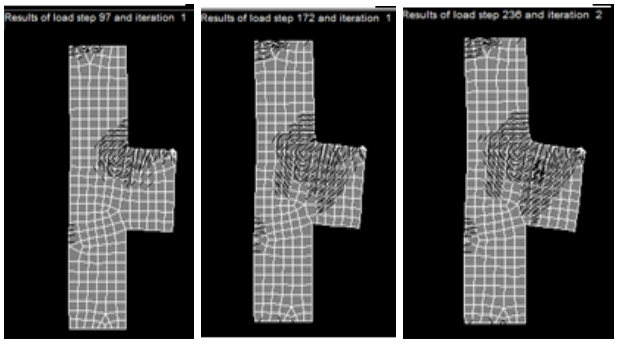


(a)At load step 55 (b)At load step 172

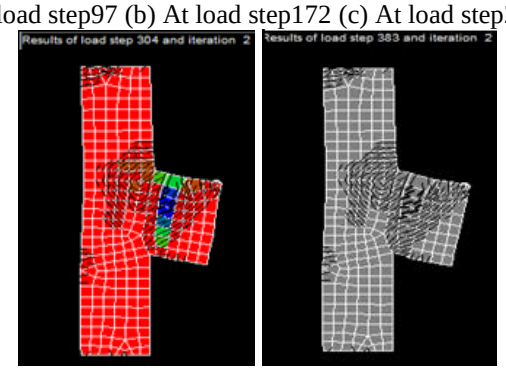
(c) At load step 213 (d) At load step 249

Fig.17.Crack pattern in SCC corbel at various load steps

Study of Crack Pattern in SFRSCC Corbel



(a)At load step97 (b) At load step172 (c) At load step236



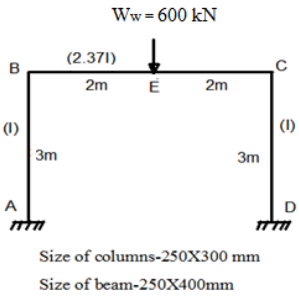
(d) At load step 304 (e) At load step 383

Fig.18.Crack pattern in SFRSCC corbel at various load steps

E. Portal Frame

In a portal frame, major attention is to be given for studying the beam-column connection subjected to hogging bending moment that tends to close the corner between the beam and column. In beam column connections, steel fibers provide necessary ductility to induce flexural failure rather than sudden shear failure. This aspect is extremely important for structures located in earthquake prone zones. The fiber reinforced structures show less damage at junctions, greater initial stiffness, less cracking, larger moment and shear capacities.

A portal frame of the dimensions as shown in **Fig 19** is considered. The steel reinforcement in the frame is designed for the working load of 600 kN. The arrangement of mesh in the frame is shown in **Fig 20** below.



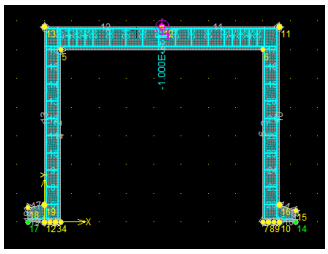


Fig.19.Dimensions of portal frame Fig 20. Mesh arrangement

Load Deflection Curve for the Frames

From the graph shown in **Fig 21**, the peak load for SCC and SFRSCC portal frames are 540.32 kN and 729.78 kN respectively. There will be increase in peak by 35.06 % in SFRSCC frame compared to SCC frame. If these loads are compared with the design load of 600 kN, only SFRSCC frame is able yield peak load greater than design load.

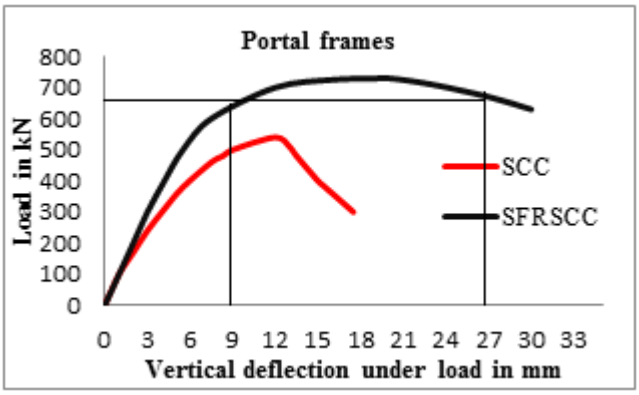


Fig.21.Load v/s Deflection curves for SCC and SFRSCC portal frames

The ductility for SFRSCC frame at yield load of $P_y = 650$ kN is as follows,

$$\text{Ductility ratio}(\mu) = \left(\frac{27\text{mm}}{8.5\text{mm}}\right) = 3.18 \qquad (9)$$

The member ductility of beam in the frame as calculated above expression (9) can further be used to determine the structural ductility (μ_b) of a single storey portal frame. For the portal frame subjected to lateral shear during earthquake, the $\mu_b$is established in terms of ratio of stiffness of beam and column of the frame as given by the expression (10) below. The μ_b can later be used to judge the response reduction factor (R) values used in base shear formula of IS Code 1893 (Part-I)-2002. However, for multistoried building frames, the R can be established by push over tests.

For a single storey portal frame, the relationship between the beam ductility (μ) and system ductility (μ_b) as given in the literature **[20]** is as follows.

$$\mu_b = \left[\frac{\mu + \frac{k_b}{k_c}}{1 + \frac{k_b}{k_c}}\right] \qquad (10)$$

Where, k_b and $k_c$are the stiffness of beam and column of the single storey portal frame respectively.

Consider an example as follows,

The size of beam is 250X400mm and column is 250X300mm.

$$k_b = \text{Relative stiffness of beam} = \left(\frac{I}{L}\right) = \left(\frac{I}{L_b}\right)$$

$$= \left[\frac{\frac{(0.25X0.40^3)}{12}m^4}{4m}\right] = 3.333X10^{-4}m^3$$

Where, I=Moment of Inertia of beam about horizontal centroidal axis,

L_b = span of the beam

$$k_c = \text{Relative stiffness of column} = \left(\frac{I}{L}\right) = \left(\frac{I}{L_c}\right)$$

$$= \left[\frac{\frac{(0.25X0.30^3)}{12}m^4}{2.9m}\right] = 1.940X10^{-4}m^3$$

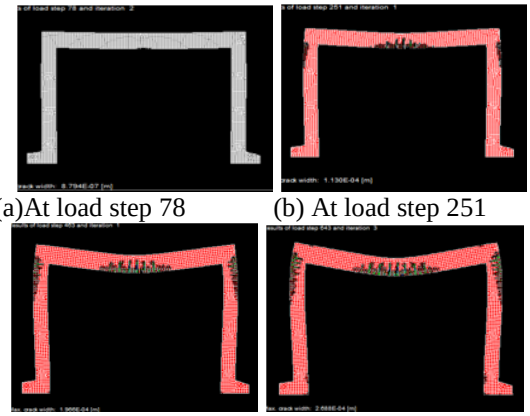
Where, I=Moment of Inertia of column about XX axis L_c = height of column.

$$\left(\frac{k_b}{k_c}\right) = \left(\frac{3.333X10^{-4}m^3}{1.940X10^{-4}m^3}\right) = 1.718 \qquad (11)$$

And, $\mu_b = \left[\frac{3.18+1.718}{1+1.718}\right] = 1.80$ (12)

The ductility ratio of the portal frame reduces to 1.80 whose beam ductility ratio was 3.18.This indicates that ductility ratio of portal frame is always less compared to that of its beam member. Hence, lower value of R is expected for single storey frame.

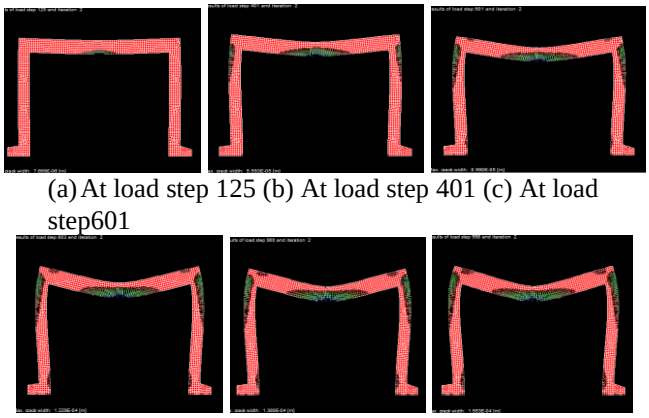
Study of Crack Pattern in SCC Portal Frame



(a)At load step 78 (b) At load step 251

(c)At load step 403 (d) At load step 643

Fig.22.Crack pattern in SCC portal frame at various load steps



(a) At load step 125 (b) At load step 401 (c) At load step601

(d) At load step 803 (e) At load step 900 (f) At load step 998

Fig.23.Crack pattern in SFRSCC portal frame at various load steps

F. Simulation of SCC corbel using size independent fracture energy and corresponding tension softening law

In the above studies, the size dependent total fracture energy is scaled to the size of the component of the structure predominantly subjected to mode-I fracture.

Since, these structures are having complex geometry, it is proposed to use size independent specific fracture energy evaluated for SCC mix in our earlier investigation **[6],[7]** using the method suggested **[22], [23]** and the corresponding tension softening curve **[24], [25]** corresponding to specific energy for plain mix as bi-linear softening curve that is derived from exponentially decaying softening curve.

Hence, a SCC corbel is simulated using size dependent fracture energy of 342.3 N/m, exponentially decaying tension softening curve and material properties of SCC beams as input data in the ATENA software,. This simulated curve is compared with that obtained for scaled size dependent fracture energy of 470.06 N/m. The comparison of both simulated curves is shown **Fig 24** below.

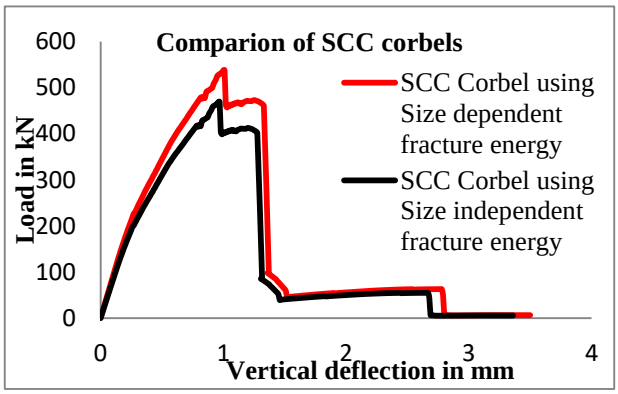
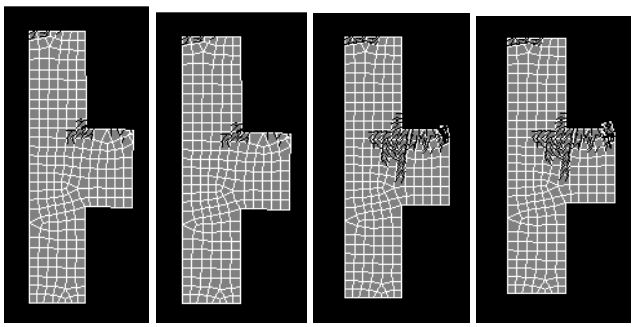


Fig.24. Comparison of simulation curves for SCC corbels

The peak load for SCC corbel which was 540.32 kN at the vertical deflection of 1.01mm using scaled fracture energy is reduced to 469.46 kN at the vertical deflection of 0.9696mm when the size independent fracture energy is used in the software. Hence, there will be decrease in peak load by 13.11%. And the peak load of 469.46 kN is far below than the design load of 800 kN.

Crack propagation captured at various load steps during simulation



(a)At load step55 (b) At 70 (c) At 140 (d) At 205

Fig.25. Crack pattern in SCC corbel at various load steps

VI. RESULTS AND DISCUSSIONS

All the above results are presented in **Table 4** below.

Table-4: Design load, peak load, ductility ratio and R value for structures.

Type of struct	Design load In kN	Peak load from Simulation in kN		Ductility property for SFRSCC structures		
		SCC	SFRSCC	Yield load in kN	Duct ratio	$R = \sqrt{(\mu^{1.5}-1)}$
Pipe	53.86	50.08	85.83	72	6.625	---

Dam	50,000	42835	--	--	--	---
Deep beam	350	280.07	385.65	350	3.8	2.53
Corbel	800	540.32	760.24	680	3.62	2.43
Corbel*		469.46				
Portal frame	600	540.32	729.78	650	3.18 for beam	2.16
					1.80 for frame	1.19

Corbel*-- SCC Corbel simulated using size independent specific fracture energy and the corresponding exponentially (bi-linear) decaying curve.

It is observed from the **Table 4** that the simulated peak loads for reinforced SCC structures are lower than the design load. The SFRSCC structures are only able to yield the peak loads nearer or more than the design load.

The simulated peak load for SCC pipe is nearer to design load determined from the Gustaffson's theory. For SFRSCC pipes, the simulated peak loads are far greater than the design loads.

The increase in peak loads of SFRSCC structures such as pipe, deep beam, corbel and portal frame compared to that of SCC structures are 71.39%, 37.7%, 41.27% and 35.06% respectively.

The increase in peak loads of SFRSCC structures from simulation greater than the design loads for pipe, deep beam, corbel and portal frame are 58.76 %, 10.19%, -4.98% and 21.63% respectively.

Similarly, the decrease in the peak load in SCC structures compared to the design loads for the same structures taken in same order are 7.26%,19.98%,32.74% and 9.95% respectively.

The ductility property is noticed only for reinforced SFRSCC structures. For reinforced SCC portal frames and corbels, ductility is low compared to SFRSCC structures. The ductility ratio for SFRSCC pipe, deep beam, corbel and portal frame are 6.625, 3.80, 3.62 and 3.18 respectively. This shows that the pipes behave more ductile under external load than the other structures.

The lower R values for SFRSCC structures such as deep beam, corbel and portal frame lies from 2 to 3 indicates that these are in position to transform their behavior from elastic to ductile. This will help the designer to adopt the means to improve further their ductility.

VII. CONCLUSIONS

- The study identifies the usefulness of the fracture parameters derived from the nonlinear analysis of fracture mechanics for simulation of reinforced SCC and SFRSCC structures using FEM based software to identify the crack pattern and ductility property of the structures.
- The SFRSCC structures are only able to experience the peak load nearer or more than the design load. A large extent of strain hardening before peak load occurs is observed in these structures.
- The Gustafson concept is successful only for SCC pipes and is unsuccessful for SFRSCC pipes due to its high total fracture energy (G_{FRC}). In such case, ATENA software can successfully be used.
- The SCC dam due to its large size will behave as brittle material obeying LEFM theory. In SCC and SFRSCC deep beams, the cracks due to shear force are more predominant than the bending moment. In case of single storey SFRSCC portal frame, the system ductility (μ_b) is smaller the beam ductility ratio.
- In case of complex SCC structures, size independent specific fracture energy can alternatively be used as input data for simulation instead of scaling down the size dependent fracture energy to the size of predominantly subjected to mode-I fracture.
- Hence, it is concluded that the steel fibers of length 25mm and volume at 0.6% by volume of mix can be used to improve strength and ductility in the SFRSCC structures even though it can be improved by other means as stated earlier. The methodology adopted for simulation in this study can be brought into general procedure to measure the strength and ductility performances of the structures which could avoid tedious test procedures. The ductility property is observed only for SFRSCC structures. The response reduction factors (R) of the SFRSCC structures are useful to design if they are exposed to lateral base shear during earthquake. The R for deep beam, corbel and portal frame ranges from 2 to 3 which indicate that their behavior can be transformed from elastic to ductile behavior which is the desirable property of quake resistant structure. However, this study could be extended further to confirm the present simulation results through experiments provided there are sufficient laboratory facilities are available

NOTATIONS USED

L-P series	Larger size-Plain SCC beam series
M-P series	Medium size-Plain SCC beam series
S-P series	Smaller size-Plain SCC beam series
L-F-series	Large size-Fiber-Beam specimen series
M-F-series	Medium size-Fiber-Beam specimen series
S-F-series	Small size-Fiber-Beam specimen series

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Conflict of Interest:

On behalf of all authors, my-self the corresponding author states that there is no conflict of interest.

REFERENCES

1. Vladimir Cervenka., Computer Simulation of Failure of Concrete Structures for Practice, First Fib Congress 2002 Concrete Structures in Twenty First Century, October 13-19, Osaka, Japan.
2. ATENA software handbook
3. Jeffery, Roesler.,Glaucio.H.Paulino., Kyoungsoo, Park. and Cristian, Gaedick., "Concrete fracture prediction using bilinear softening", Cement and concrete composites, Volume 29,2007, pp 300-312.
4. Jeffery, Roesler, Glaucio,H., Paulino,Kyoungsoo, Park and Cristian, Gaedick., "Determination of the kink point in the bilinear softening model for concrete", Engineering fracture mechanics, Volume 75, 2008, pp 3806-3818.
5. Jeffery,Roesler., Glaucio,H.,Paulino, Kyoungsoo, Park., and Cristian Gaedick. "Cohesive fracture model for functionally graded fiber reinforced concrete", Cement and concrete research, Volume-40, March-2010, pp 956—965.
6. H.Vijayakumar and Sakey Shamu., "Fracture model prediction for self-compacting concrete using bi-linear softening" The international reviewer (ISSN 2395-1575), Volume 4, Issue 2, July- December 2017, pp.29-34.
7. Vijayakumar, Halakatti. "Study of fracture parameters of self-compacting concrete" International Journal of Technical Innovation in Modern Engineering and Science (IJTIMES) (ISSNe-2455-2585), Vol-5, Issue-12, December 2019, pp24-43.
8. Vijayakumar, Halakatti. "Modeling of fracture of steel fiber reinforced self-compacting concrete" International Journal of Technical Innovation in Modern Engineering and Science (IJTIMES) (ISSNe-2455-2585) , Vol-5, Issue-12, December 2019, pp44-63.
9. Shah.S.P., Swartz.S.E and Ouyang.C, A Text Book on Fracture Mechanics of Concrete: Application Of Fracture Mechanics to Concrete, Rock Other Quasi-Brittle Material. Published by John wiley and sons, (edition=1995), pp- 110-120.
10. Hillerborg,A., "Analysis of fracture by means of the fictitious crack model particularly for fiber reinforced concrete" The international journal of cement composites,Vol-2,No-4,1980,pp-177-184.
11. Visalvanich, K and Naaman, A.E., "Fracture model for fiber reinforced concrete" ACI materials Journal,No,80-14,1983,pp128-138.
12. Yiu Wing Mai, Cohesive Zone and Crack Resistance (R) Curves of Cementaceous Materials and Their Fiber Reinforced Composites Engineering Fracture Mechanics Volume-69, 2002, pp-219-234.
13. Cervenka, V and Pukl, R, Sbeta Analysis of Size Effect in Concrete Structures, Proceedings of the Second International Conference on Fracture Mechanics of Concrete Structures,(FRAMCOS 2): 1387-1396, Ed. F.H. Wittmann, AEDIFICATIO,ETH Zurich,1995, Switzerland.
14. Chao Wang, Yamei Zhang and Zhe Zhao., Fracture Process Of Rubberized Concrete By Fictitious Crack Model And AE Monitoring, Computers and Concrete, Volume 9, No- 1(2012),pp, 51-61.
15. Okamura, H. And Ouchi, M.,"Self Compacting Concrete", Journal of Advanced Concrete Technology, Volume 1, No 1, April 2003, pp 5−15.
16. EFNARC (European Federation of Natural Trade Associations Representing Producers and Applicators Of Specialist Building Product), Specifications and Guidelines for Self-Compacting Concrete, February-2002, Hampshire, UK.
17. Nan Su, Hsu, K.C. and Chai, H.W., A Simple Mix Design for Self-Compacting Concrete, Cement and concrete Research, Volume 31(2001), pp1799−1807.
18. Frank Papworth, Use of Steel Fibers In Concrete, Presented at The Concrete Institute of Australia, NSW Branch 1997, pp 1−10.
19. A.K.H. Kwan and J.C.M. Ho "Ductility Design of High-Strength Concrete-Beams and Columns" Advances in Structural Engineering Vol. 13 No. 4, 2010, pp-651-664
20. Pankaj, Agarwal., and Manish Shrikhade "A text book on "Earthquake resistant design of structures" PHI Publishers, New Dehli,ISBN-81-203-2892-2,Edition-September 2006,pp 345.
21. Shah.S.P., Swartz.S.E and Ouyang.C, A text book on fracture mechanics of concrete: application of fracture mechanics to concrete, rock other quasi-brittle material. Published by John Wiley and Sons, pp-211.
22. M. Elices, G.V. Guinea, J. Planas "Measurement of the fracture energy using three point bend tests: Part 3-Influence of cutting the P-δ tail" Materials and Structures, 1992, 25, pp. 327-334.
23. A, Ramachandramurthy.,B,L,Karihaloo., Nagesh, R, Iyer and B.K. Raghuprasad "Determination of Size Independent Specific Fracture Energy of Concrete Mixes by Two Methods", Cement And Concrete Research, 2013, Volume-50, pp 19-25.
24. H,M,Abdalla and B,L,Karihaloo.," A method for constructing the bilinear tension softening diagram of concrete corresponding to its true fracture energy", Magazine of Concrete Research,2004,56,No 10,December,pp 597-604.
25. A, Ramachandramurthy., B,L,Karihaloo., Nagesh, R, Iyer and B.K.Raghuprasad "Bilinear tension softening diagrams of concrete mixes corresponding to their size-independent specific fracture energy", Construction and Building Materials,47(2013),pp 1160-1166.
26. Bushan, L, Karihaloo.," Fracture mechanics and structural concrete" ,Published by Longman scientific and technical ,England, First published 1995,pp 208-281.
27. Gustaffsson. P.J and Dahlblom (1996),Fracture mechanics in the design of concrete pipes, Report TVSM -7212 (in Swedish), Division of structural mechanics, Lund University, Sweden.
28. Elfgren, L.," Fracture mechanics of concrete structures from theory to application", Published by Chapman and hall, pp 315-377.
29. Perumalsamy, N, Balaguru, A text book "Fiber reinforced cement composites" McGraw hill Inc, ISBN 0-07-112967-7, International edition 1992, pp 464.

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