

# Intuitionistic Fuzzy IFP Ideals of N-Groups

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**Abstract:** In this paper the notion of intuitionistic fuzzy ideal of a module over a nearring is considered. We define the notion insertion of factors property (IFP) in case of intuitionistic fuzzy ideal and provide necessary examples and investigate some related properties.

**Keywords :** fuzzy ideal, IFP ideal, intuitionistic fuzzy ideal, Nearring, N-group.

## I. INTRODUCTION

Nearrings are generalized rings which are crucial in the nonlinear theory of group mappings. Nearrings are defined in a natural way. For a group  $(G,+)$ (not necessarily abelian), the set  $M(G) = \{f : G \rightarrow G\}$  together with component-wise addition and composition of mappings forms a nearring but not a ring. Nearrings doesnot require the commutativity of addition. An important type of nearrings obtained by considering the additive closure  $E(G)$  consists of all sums (or differences) of endomorphisms, which generalizes the concept of an endomorphism ring of an abelian group to the non-abelian case.

The notion fuzzy ideal of a nearring was studied by Abou-Zaid [1]. The substructures such as fuzzy subnearrings and several characterizations were obtained. Kim and Jun [5] introduced fuzzy N-subgroups of a nearring and obtained characterizations in terms of level ideals. Jun, Kim and Yon [18] introduced the notion of intuitionistic fuzzy ideal of a nearring and some related properties were investigated. Ideal theoretic structural results and examples connecting to lower and upper level cuts are also obtained. The notion of fuzzy IFP ideal was introduced by Satyanarayana and Syam Prasad [8] and they obtained several equivalent conditions for an fuzzy ideal to have the IFP. The concept of equiprime fuzzy and 3-prime fuzzy ideal with thresholds have been introduced and inter-relations have been proved along with basic isomorphism theorems. Kedukodi et.al. [19] have introduced c-prime fuzzy ideals of nearrings and proved the one-to-one correspondance involving the f-invariant c-prime fuzzy ideals of a nearring. In Kedukodi et.al. [17], the concept of equiprime fuzzy radical, c-prime, 3-prime ideals with necessary properties are also obtained.

Now we give the definition of nearrings as follows:

Pilz [7], Satyanarayana, Syam Prasad [16] A nearring is a generalized ring in which addition need not be abelian and only one distributive law is required. More precisely, the

binary operations  $+$  and  $\cdot$  satisfying the following conditions:

1.  $N$  is additive group (not necessarily abelian);
2.  $l.(m.n) = (l.m).n$  ;
3.  $(l + m) \cdot n = l.n + m.n$  for all  $l, m, n \in N$ .

The above definition is a right nearring, as it satisfies only right distributive property. Further, if  $0$  is the additive identity in  $N$  with  $n.0 = 0$  for all  $n \in N$ , then  $N$  is called zero-symmetric.

Let  $(I, +)$  be a subgroup of  $(N, +)$ . Then  $I$  is said to be an ideal of  $N$  if

- $(I, +)$  is a normal subgroup of  $(N, +)$
- $IN \subseteq I$
- $n(n' + i) - nn' \in I$  for any  $i \in I$  and  $n, n' \in N$

Note that  $I$  is a right ideal of  $N$  if  $I$  satisfies (i) and (ii) and  $I$  is a left ideal of  $N$  if  $I$  satisfies (i) and (iii)

Let  $(N, +, \cdot)$  be a nearring. A group  $(G, +)$  is said to be an  $N$ -group if there exists a mapping  $N \times G \rightarrow G$  satisfies :

- $(n + m)a = na + ma$  and
- $(nm)a = n(ma)$  for all  $a \in G$  and  $n, m \in N$ .

A fuzzy set  $\mu$  in a non empty set  $X$ , is a function  $\mu : X \rightarrow [0, 1]$ , and a complement of  $\mu$ , denoted by  $\bar{\mu}$ , in  $X$  given by  $\bar{\mu}(x) = 1 - \mu(x)$ .

Let  $\mu$  be a fuzzy set in  $N$ . Then  $\mu$  is called a fuzzy ideal of  $N$  with thresholds  $\alpha, \beta \in [0, 1]$ ,  $\alpha < \beta$  if it satisfies ,

- $\alpha \vee \mu(x - y) \geq \beta \wedge \min \{ \mu(x), \mu(y) \}$
- $\alpha \vee \mu(xy) \geq \beta \wedge \min \{ \mu(x), \mu(y) \}$
- $\alpha \vee \mu(y + x - y) \geq \beta \wedge \mu(x)$
- $\alpha \vee \mu(xy) \geq \beta \wedge \mu(y)$
- $\alpha \vee \mu(x(y + i) - xy) \geq \beta \wedge \mu(i)$  for all  $x, y, i \in N$ .

Note that  $\mu$  is a fuzzy left ideal of  $N$  if it satisfies (i), (iii) and (iv) and  $\mu$  is a fuzzy right ideal of  $N$  if it satisfies (i), (iii) and (v). we call  $\mu$  as a fuzzy left ideal of  $N$ . If  $\mu$  satisfies (i), (ii) then we call it as a fuzzy sub- nearring of  $N$ .

**Definition 1.1.** ( K. Atanassov[2] ) Let  $X$  be a non empty set. An intuitionistic fuzzy set (IFS)  $A$  is of the form  $A = \{x, \mu_A(x), \gamma_A(x) : x \in X\}$  where the functions  $\mu_A : X \rightarrow [0, 1]$  and  $\gamma_A : X \rightarrow [0, 1]$  denotes the degree of membership and the degree of non-membership, respectively, and  $0 \leq \mu_A(x) + \gamma_A(x) \leq 1, x \in X$ . we shall use the symbol  $A = (\mu_A, \gamma_A)$  for the IFS  $A = \{x, \mu_A(x), \gamma_A(x) : x \in X\}$

## II. INTUITIONISTIC FUZZY IDEALS

**Definition 2.1.** An IFS  $A = (\mu_A, \gamma_A)$  in  $N$  with thresholds  $\alpha, \beta \in [0, 1]$ ,  $\alpha < \beta$  is called an intuitionistic fuzzy sub near-ring of  $N$  if

- (IF1)  $\alpha \vee \mu_A(x - y) \geq \beta \wedge \min \{ \mu_A(x), \mu_A(y) \}$

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$$\alpha \vee (\gamma_A(x - y)) \leq \beta \wedge \max \{ \gamma_A(x), \gamma_A(y) \}$$

$$(IF2) \alpha \vee \mu_A(xy) \geq \beta \wedge \min \{ \mu_A(x), \mu_A(y) \} \text{ and}$$

$$\alpha \vee (\gamma_A(xy)) \leq \beta \wedge \max \{ \gamma_A(x), \gamma_A(y) \}, \text{ for all } x, y \in N.$$

**Example 2.2.** Define addition and multiplication on  $N = \{1, m, n, o\}$  as follows:

+	1	m	n	o
1	1	m	n	o
m	m	1	o	n
n	n	o	m	1
o	o	n	1	m

.	1	m	n	o
1	1	1	1	1
m	1	1	1	m
n	1	1	1	n
o	1	1	1	o

Then  $(N, +, \cdot)$  is a near-ring. We define an IFS  $A = (\mu_A, \gamma_A)$  by  $\mu_A(n) = \mu_A(o) = 0.3, \mu_A(m) = 0.6, \mu_A(1) = 1$  and  $\gamma_A(n) = \gamma_A(o) = 0.5, \gamma_A(m) = 0.25, \gamma_A(1) = 0$ . Then  $A$  is an intuitionistic fuzzy sub near-ring of  $N$  with thresholds  $\alpha, \beta \in [0, 1], \alpha < \beta$

The following Lemma is straightforward.

**Lemma 2.3.** An IFS  $A = (\mu_A, \gamma_A)$  in  $N$  is an intuitionistic fuzzy sub near-ring of  $N$  if  $\alpha \vee \min \{ \mu_A(x - y), \mu_A(xy) \} \geq \beta \wedge \max \{ \mu_A(x), \mu_A(y) \}$  and  $\alpha \vee \min \{ \gamma_A(x - y), \gamma_A(xy) \} \leq \beta \wedge \max \{ \gamma_A(x), \gamma_A(y) \}$  for all  $x, y \in N, \alpha, \beta \in [0, 1], \alpha < \beta$ .

**Lemma 2.4.** If an IFS  $A = (\mu_A, \gamma_A)$  in  $N$  satisfies the condition (IF1), then

- (i)  $\alpha \vee \mu_A(0) \geq \beta \wedge \mu_A(x)$  and  $\alpha \vee \gamma_A(0) \geq \beta \wedge \gamma_A(x)$ .
- (ii)  $\mu_A(-x) = \mu_A(x)$  and  $\gamma_A(-x) = \gamma_A(x)$  for all  $x \in N$  and  $\alpha, \beta \in [0, 1], \alpha < \beta$ .

**Proof.** (i) for any  $x \in N,$

$$\alpha \vee \mu_A(0) = \alpha \vee \mu_A(x - x) \geq \beta \wedge \min \{ \mu_A(x), \mu_A(x) \}$$

$$= \beta \wedge \mu_A(x) \text{ and}$$

$$\alpha \vee \gamma_A(0) = \alpha \vee \gamma_A(x - x) \leq \beta \wedge \max \{ \gamma_A(x), \gamma_A(x) \}$$

$$= \beta \wedge \gamma_A(x)$$

(ii). By using (i) we get

$$\mu_A(-x) = \mu_A(0 - x) \geq \min \{ \mu_A(0), \mu_A(x) \} = \mu_A(x)$$

And  $\gamma_A(-x) = \gamma_A(0 - x) \leq \max \{ \gamma_A(0), \gamma_A(x) \} = \gamma_A(x)$  for all  $x \in N$ . Since  $x \in N$  is arbitrary, we conclude that  $\mu_A(-x) = \mu_A(x)$  and  $\gamma_A(-x) = \gamma_A(x)$  for all  $x \in N$ .

We have considered  $\alpha = 0, \beta = 1$  in the following proposition.

**Proposition 2.5.** If an IFS  $A = (\mu_A, \gamma_A)$  in  $N$  satisfies the condition (IF1), then

- (i)  $\mu_A(x - y) = \mu_A(0)$  implies  $\mu_A(x) = \mu_A(y)$ .
- (ii)  $\gamma_A(x - y) = \gamma_A(0)$  implies  $\gamma_A(x) = \gamma_A(y)$ , for all  $x, y \in N$ .

**Proof.** (i) Let  $x, y \in N$  be such that  $\mu_A(x - y) = \mu_A(0)$ . Now  $\mu_A(x) = \mu_A(x - y + y) \geq \min \{ \mu_A(x - y), \mu_A(y) \} = \min \{ \mu_A(0), \mu_A(y) \} = \mu_A(y)$ .

Similarly,  $\mu_A(y) \geq \mu_A(x)$  and so  $\mu_A(x) = \mu_A(y)$ .

- (iii) Suppose  $\gamma_A(x - y) = \gamma_A(0)$  for all  $x, y \in N$ . Then  $\gamma_A(x) = \gamma_A(x - y + y) \leq \max \{ \gamma_A(x - y), \gamma_A(y) \} = \max \{ \gamma_A(0), \gamma_A(y) \} = \gamma_A(y)$ .

In a similar way  $\gamma_A(y) \geq \gamma_A(x)$  and so  $\gamma_A(x) = \gamma_A(y)$ .

**Definition 2.6.** Let  $t \in [0, 1]$  and a fuzzy set  $\mu$  in a non-empty set  $X$ . Then the upper  $t$ -level cut  $\mu$  is defined as  $U(\mu : t) = \{x \in N \mid \mu(x) \geq t\}$ , and the lower  $t$ -level cut  $\mu$  is defined as  $L(\mu : t) = \{x \in N \mid \mu(x) \leq t\}$  is called lower  $t$ -level cut of  $\mu$ .

**Example 2.7.** Consider the nearring given in Example 2.2,  $I = \{a, b\}$  a subnearring of  $N$ . Then define  $A = (\mu_A, \gamma_A)$ , the IFS in  $N$  as follows.

$$\mu_A(x) = \begin{cases} 0.2 & \text{if } x \in I \\ 0.1 & \text{otherwise} \end{cases}$$

$$\gamma_A(x) = \begin{cases} 0.3 & \text{if } x \in I \\ 0.5 & \text{otherwise} \end{cases}$$

Then  $A = (\mu_A, \gamma_A)$  is an intuitionistic fuzzy sub near-ring of  $N$  and  $U(\mu_A : 0.2) = I = L(\gamma_A : 0.3)$ .

In general we get the following theorem

**Theorem 2.8.** Let  $I$  be a sub near-ring of  $N$  and let  $A = (\mu_A, \gamma_A)$  be an IFS in  $N$  with thresholds  $\alpha, \beta \in [0, 1]$  where  $\alpha < \beta$  defined by

$$\mu_A(x) = \begin{cases} t_o & \text{if } x \in I \\ t_1 & \text{otherwise} \end{cases}$$

$$\gamma_A(x) = \begin{cases} s_o & \text{if } x \in I \\ s_1 & \text{otherwise} \end{cases}$$

for all  $x \in N$ , and  $t_i, s_i \in [0, 1]$  such that  $t_o > t_1, s_o < s_1$  and  $t_i + s_i \leq 1$  for  $i = 0, 1$ . Then  $A = (\mu_A, \gamma_A)$  is an intuitionistic fuzzy sub near-ring of  $N$  with thresholds  $\alpha < t_i, s_i < \beta, i = 0, 1$  and  $U(\mu_A : t_o) = I = L(\gamma_A : s_o)$ .

**Proof.** Let  $x, y \in N$ .

Case (i): If  $x \notin I$  or  $y \notin I$ . Then clearly the conditions (IF1) and (IF2) of the definition holds.

Case (ii): If  $x, y \in I$  then since  $I$  is a subnearring,  $x - y \in I, xy \in I$ . Therefore

$$\alpha \vee \min \{ \mu(x-y), \mu(xy) \} = \alpha \vee t_o = t_o \text{ (since } \alpha < t_o < \beta)$$

$$= \beta \wedge t_o$$

$$= \beta \wedge \max \{ t_o, t_o \}$$

$$= \beta \wedge \max \{ \mu(x), \mu(y) \}.$$

Similarly  $\alpha \vee \max \{ \gamma(x-y), \gamma(xy) \} = \beta \wedge \min \{ \gamma(x), \gamma(y) \}$ . Hence by Lemma 2.3, we conclude that  $A = (\mu_A, \gamma_A)$  is an intuitionistic fuzzy sub near-ring of  $N$  with thresholds  $\alpha, \beta$ .

**Definition 2.9.** An IFS  $A = (\mu_A, \gamma_A)$  in  $N$  is called intuitionistic fuzzy ideal of  $N$  if

- (i)  $A = (\mu_A, \gamma_A)$  is an intuitionistic fuzzy sub near-ring of  $N$ .
- (ii)  $\alpha \vee \mu_A(x) = \beta \wedge \mu_A(y + x - y)$  and  $\alpha \vee \gamma_A(x) = \beta \wedge \gamma_A(y + x - y)$
- (iii)  $\alpha \vee \mu_A(xy) \geq \beta \wedge \mu_A(y)$  and  $\alpha \vee \gamma_A(xy) \leq \beta \wedge \gamma_A(y)$
- (iv)  $\alpha \vee \mu_A((x + i)y - xy) \geq \beta \wedge \mu_A(i)$  and  $\alpha \vee \gamma_A((x + i)y - xy) \leq \beta \wedge \gamma_A(i)$  for all  $x, y, i \in N$ . If

$A = (\mu_A, \gamma_A)$  satisfies (i), (ii) and (iii) then it is called an intuitionistic fuzzy left ideal of  $N$ . If  $A = (\mu_A, \gamma_A)$  satisfies (i), (ii) and (iv) then it is called an intuitionistic fuzzy right ideal of  $N$ .

Now we define IF ideal of an  $N$ -group  $G$

**Definition 2.10.** Let  $G$  be an  $N$ -group, then a IF subset  $(\mu, \gamma)$  of  $G$  is said to be a IF ideal of  $G$  if it satisfies

- (i)  $\mu(g_1 - g_2) \geq \min \mu(g_1), \mu(g_2)$ ,  $\gamma(g_1 - g_2) \leq \max \{\gamma(g_1), \gamma(g_2)\}$
- (ii)  $\mu(n + g - n) \geq \mu(g)$ ,  $\gamma(n + g - n) \leq \gamma(g)$
- (iii)  $\mu(ng) \geq \mu(g)$ ,  $\gamma(ng) \geq \gamma(g)$
- (iv)  $\mu(n(g + g') - ng) \geq \mu(g')$ ,  $\gamma(n(g + g') - ng) \leq \gamma(g')$  for all  $g, g' \in G, n \in N$ .

**Definition 2.11.** An IF ideal  $(\mu, \gamma)$  of  $N$  is said to have IFP if for any  $a, b \in N$ ,  $\mu(anb) \geq \mu(ab)$  and  $\gamma(anb) \leq \gamma(ab)$ , for all  $n \in N$ .

**Definition 2.12.** An IF ideal  $(\mu, \gamma)$  of an  $N$ -group  $G$  is said to have IFP if for every  $a \in N, g \in G$ ,  $\mu(ang) \geq \mu(ag)$  and  $\gamma(ang) \leq \gamma(ag)$ , for all  $n \in N$ .

**Example 2.13.** Consider the example of the nearring given in 2.2 Define an IFS  $(\mu, \gamma)$  as follows:  
 $\mu(n) = \mu(0) = 0, \mu(m) = 0.33, \mu(1) = 1$  and  $\gamma(n) = \gamma(0) = 1, \gamma(m) = 0.33, \gamma(1) = 0$   
Then  $A$  is an intuitionistic fuzzy sub near-ring and an intuitionistic fuzzy left(right) ideal of  $N$ . Further it can be verified that  $(\mu, \gamma)$  is a intuitionistic fuzzy IFP ideal of  $N$  over  $N$ .

**Theorem 2.14.** Let  $G$  be an  $N$ -group, if an IFS  $A = (\mu, \gamma)$  is an intuitionistic fuzzy ideal of  $G$ , then the sets  $G_\mu = \{x \in G : \mu(x) = \mu(0)\}$  and  $G_\gamma = \{x \in G : \gamma(x) = \gamma(0)\}$  are ideals of  $G$ , for thresholds  $\alpha, \beta$ .

**Proof.** Let  $x, y \in G_\mu$ . Then  $\alpha \vee \mu(x - y) \geq \beta \wedge \min\{\mu(x), \mu(y)\} = \mu(0)$ . Now  $\alpha \vee \mu(0) \geq \beta \wedge \mu(x)$  for all  $x \in G$ , it follows that  $\alpha \vee \mu(x - y) \geq \beta \wedge \mu(0)$ , and so  $x - y \in G_\mu$ . For any  $x \in G_\mu$  and  $n \in N$ , we have  $\alpha \vee \mu(n + x - n) \geq \beta \wedge \mu(x) = \beta \vee \mu(0)$  and so  $n + x - n \in G_\mu$ . Thus  $(G_\mu, +)$  is a normal subgroup of  $(G, +)$ . Let  $n \in N$  and  $g \in G_\mu$ . Then  $\alpha \vee \mu(ng) \geq \beta \wedge \mu(g) = \beta \wedge \mu(0)$  hence  $ng \in G_\mu$ . Now let  $n \in N$  and  $g, g' \in G_\mu$ . Then  $\alpha \vee \mu(n(g + g') - ng) \geq \beta \wedge \mu(g') = \alpha \vee \mu(0)$ , which implies that

$\alpha \vee \mu(n(g + g') - ng) \geq \beta \wedge \mu(0)$  and hence  $n(g + g') - ng \in G_\mu$ . Hence  $G_\mu$  is an ideal of  $G$ . Now we prove that  $G_\gamma$  is an ideal of  $G$ , let  $x, y \in G_\gamma$ . Now  $\alpha \vee \gamma(x - y) \leq \beta \wedge \max\{\gamma(x), \gamma(y)\} = \beta \wedge \gamma(0)$  and so  $\alpha \vee \gamma(x - y) = \beta \wedge \gamma(0)$  and so  $x - y \in G_\gamma$ . Let  $x \in G_\gamma$  and  $n \in N$ . Then  $\alpha \vee \gamma(n + x - n) = \beta \wedge \gamma(n) = \alpha \vee \gamma(0)$  and thus  $n + x - n \in G_\gamma$ . This shows that  $(G_\gamma, +)$  is a normal subgroup of  $(G, +)$ . For any  $n \in N$  and  $g \in G_\gamma$  we get  $\alpha \vee \gamma(ng) \geq \beta \wedge \gamma(g)$ , hence  $\alpha \vee \gamma(ng) = \beta \wedge \gamma(0)$ . Therefore  $ng \in G_\gamma$ . For any  $n \in N$  and  $g, g' \in G_\gamma$ , we have  $\alpha \vee \gamma(n(g + g') - ng) \leq \beta \wedge \gamma(g') = \gamma(0)$  which implies that  $\alpha \vee \gamma(n(g + g') - ng) = \beta \wedge \gamma(0)$ , hence  $n(g + g') - ng \in G_\gamma$ . This completes the proof.

**Proposition 2.15.** Let  $(\mu, \gamma)$  be an intuitionistic fuzzy IFP ideal of  $G$ . Then the following are equivalent

- (i)  $(\mu, \gamma)$  has IFP.
- (ii)  $(\mu_t, \gamma_s)$  has IFP for all  $s, t \in (\alpha, \beta]$ .

**Proof.** (i)  $\Rightarrow$  (ii)  
Let  $t, s \in (\alpha, \beta]$  and  $a \in N, g \in G$  such that  $ag \in \mu_t$ . Take  $n \in N$ , since  $(\mu, \gamma)$  has IFP,  $\alpha \vee \mu(ang) \geq \beta \wedge \mu(ag) \geq \beta \wedge t = t$  (since  $t < \beta$ )  $\Rightarrow \alpha \vee \mu(ang) \geq t$  and hence  $ang \in \mu_t$ . Let  $\gamma(ag) = s$ . Then  $\alpha \vee \gamma(ang) \leq \beta \wedge \gamma(ag) \geq \beta \wedge s = s$  This implies  $\gamma(ang) \leq s \Rightarrow ang \in \gamma_s$ . Hence  $(\mu_t, \gamma_s)$  has IFP for all  $s, t \in (\alpha, \beta]$ .  
(iii)  $\Rightarrow$  (i)

In a contrary way suppose that  $(\mu, \gamma)$  does not have IFP. Then there exists  $a \in N, g \in G$  and  $n \in N$  such that  $\alpha \vee \mu(ang) < \beta \wedge \mu(ag)$  and  $\alpha \vee \gamma(ang) > \beta \wedge \gamma(ag)$ . Now choose  $t, s \in (\alpha, \beta]$  such that  $\alpha \vee \mu(ang) < t < \beta \wedge \mu(ag)$  and  $\alpha \vee \gamma(ang) > s > \beta \wedge \gamma(ag)$ .  $\Rightarrow \mu(ang) < t < \mu(ag)$  and  $\gamma(ang) > s > \gamma(ag)$ . This shows that  $ang \notin \mu_t, ang \notin \gamma_s$  which is a contradiction to the fact that  $(\mu_t, \gamma_s)$  has IFP.

**Definition 2.16.** An IF ideal  $(\mu, \gamma)$  of  $N$  is called IF c-prime if for all  $a, b \in N$ ,

$$\alpha \vee \max \{ \mu(a), \mu(b) \} \geq \beta \wedge \mu(ab) \text{ and } \alpha \vee \min \{ \gamma(a), \gamma(b) \} \leq \beta \wedge \gamma(ab)$$

**Example 2.17.** Consider the nearring  $N = (Z_8, +, \cdot)$  where  $Z_8$  is nearring of integers addition module 8; and the multiplication table is defined below.

.	0	1	2	3	4	5	6	7
0	0	0	0	0	0	0	0	0
1	0	1	0	1	0	1	1	0
2	0	2	0	2	0	2	2	0
3	0	3	0	3	0	3	3	0
4	4	4	4	4	4	4	4	4
5	4	5	4	5	4	5	5	4
6	4	6	4	6	4	6	6	4
7	4	7	4	7	4	7	7	4

Define IF ideal  $(\mu, \gamma)$  of  $N$  as follows

$$\mu(x) = \begin{cases} 0.7 & \text{if } x \in \{0,1,2,3\} \\ 0.3 & \text{if } x \in \{4,5,6,7\} \end{cases}$$

$$\gamma(x) = \begin{cases} 0.2 & \text{if } x \in \{0,1,2,3\} \\ 0.6 & \text{if } x \in \{4,5,6,7\} \end{cases}$$

Then the following conditions are satisfied.

$\alpha \vee \max \{ \mu(a), \mu(b) \} \geq \beta \wedge \mu(ab)$  and  
 $\alpha \vee \min \{ \gamma(a), \gamma(b) \} \leq \beta \wedge \gamma(ab)$  where  $0.2 < \alpha < \beta < 0.8$ ,  
 and for all  $a, b \in Z_8$ .

Hence  $(\mu, \gamma)$  is a  $c$ -prime IF ideal of  $(Z_8, +, \cdot)$ .

**Proposition 2.18.** Let  $(\mu, \gamma)$  be an intuitionistic fuzzy ideal of  $N$ . then  $(\mu, \gamma)$  is a  $c$ -prime fuzzy ideal of  $N$  if and only if its upper level cut  $\mu_t, t \in [0, \mu(0)]$  and the lower level cut  $\gamma_s, s \in [\gamma(0), 1]$  are  $c$ -prime ideals of  $N$ .

Proof. Note that by theorem 3.14 of Jun, Kim and Yon[18] it is clear that  $(\mu, \gamma)$  is an intuitionistic fuzzy ideal if and only if the upper level cut  $\mu_t$  and the lower level cut  $\gamma_s$  are ideals of  $N$  for all  $t \in [0, \mu(0)]$  and  $s \in [\gamma(0), 1]$ . Now suppose  $(\mu, \gamma)$  be intuitionistic fuzzy  $c$ -prime ideals of  $N$ . Take  $t \in [0, \mu(0)]$  and  $s \in [\gamma(0), 1]$ ,  $a, b \in N$  such that  $ab \in \mu_t$  and  $ab \in \gamma_s$ . This implies  $\mu(ab) \geq t$  and  $\gamma(ab) \geq s$ . Since  $(\mu, \gamma)$  is intuitionistic fuzzy  $c$ -prime, we have

$\alpha \vee \max \{ \mu(a), \mu(b) \} \geq \beta \wedge \mu(ab) \geq \beta \wedge t = t$  and  
 $\alpha \vee \min \{ \gamma(a), \gamma(b) \} \leq \beta \wedge \gamma(ab) \leq \beta \wedge s = s$ , for all the threshold  $\alpha, \beta \in [0, 1]$  with  $\alpha < \beta$ . This implies that  $a \in \mu_t$  or  $b \in \mu_t$  and  $a \in \gamma_s$  or  $b \in \gamma_s$ .

Hence the level cuts  $\mu_t$  and  $\gamma_s$  are  $c$ -prime ideals of  $N$ . To prove the converse, if possible suppose that there exists  $a, b \in N$  and  $\alpha, \beta \in [0, 1], \alpha < \beta$  such that

$\alpha \vee \max \{ \mu(a), \mu(b) \} < \beta \wedge \mu(ab)$  and  
 $\alpha \vee \min \{ \gamma(a), \gamma(b) \} > \beta \wedge \gamma(ab)$ . Now choose  $t$  and  $s$  such that  
 $\alpha \vee \max \{ \mu(a), \mu(b) \} < t < \beta \wedge \mu(ab)$  and  
 $\alpha \vee \min \{ \gamma(a), \gamma(b) \} > s > \beta \wedge \gamma(ab)$ .

This implies that  $ab \in \mu_t$  but neither  $a \in \mu_t$  nor  $b \in \mu_t$  and also  $ab \in \gamma_s$  but neither  $a \in \gamma_s$  nor  $b \in \gamma_s$  which is a contradiction to the converse hypothesis. Hence  $(\mu, \gamma)$  is an intuitionistic fuzzy  $c$ -prime ideal of  $N$ .

### III. CONCLUSION

We have considered right nearrings, and the  $N$ -group, the modules over a nearring. Fuzzy aspects of ideals of nearrings have been studied by many authors. In this paper we have defined the notion insertion of factors property (IPF) in case of intuitionistic fuzzy ideals of nearrings. We have provided suitable example and a characterization for intuitionistic fuzzy IPF ideal of an  $N$ -group. Further we have also defined IF  $c$ -prime ideal of a nearring and provided elementary results.

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