International Journal of Recent Technology and Engineering (IJRTE)
ISSN: 2277-3878, Volume-8, Issue-6, March 2020

Using Soft Set Relations and Mappings of Kernels and Closures

K. Geetha, N. Anitha

Abstract: ***Abstract: In this paper, the views of anti-reflexive kernel, symmetric kernel, reflexive closure, and symmetric closure of a SS relation are initially presented, respectively. Then, their correct calculation formulae and a few laws are received. Finally, SS relation function and inverse SS relation functions are introduced, and some related conditions are discussed.***

Keywords: Soft sets (SS) relations Kernels Closures SS relation functions Inverse SS relation functions. In this paper using SS means (Soft set).

I. INTRODUCTION

To clear up difficult problems in finances, business and atmosphere, we cannot well use standard devices since of exceptional kinds of incomplete expertise. An extensive collection of systems such as probability opinion, theory of fuzzy, hard set impression, ambiguous set idea and the interval arithmetic are well recognized and often. Suitable calculated events for exhibiting uncertainty. However, what those theories can deal with is simply a right part of uncertainty. Every of these systems has its characteristic glitches as talked about by Molodtsov [1]. The purpose for those troubles is, likely, the insufficiency of the parameterization device of these theories. Molodtsov [1] introduced the idea of SS as a kind different mathematical scheme for commerce with worries that is unfastened from the problems that have stricken the present hypothetical methods. This idea has tested valuable in lots of special fields along with choice making [2]–[7], statistics analysis [8], forecasting [9], simulation [10], and assessment of complete satisfactory [11]. The concept and primary residences of SS idea had been provided in [1],[12]. In the classical SS theory, a state of affairs can be composite in the actual domain since of the fuzzy countryside of the parameters. With this influence of opinion, the standard soft sets protracted to fuzzy soft sets [13]-[15], IFSSs, indistinct soft sets [18], interval valued fuzzy soft sets [19], and IIVFSs [20], respectively. Up to the existing, SS idea has been carried out to some algebra structures: groups [21],[22], semirings [23], rings [24], algebras,d-algebras ,ordered semigroups and BL-algebras. Xiao et al. Proposed the perception of one of a kind disjunctive soft sets and deliberate some of its processes. Gong et al.deliberate the bijective SS with its processes. Ontology-primarily based SS idea became supplied In [4], Çağman and Enginoğlu studied products of soft sets and recently, Babitha and Sunil proposed SS relations and lots of associated concepts are discussed. In the existing paper, we try to conduct a further observe alongside this line. This paper is a try to widen the theoretical components of soft sets again. In order to refresh the essential ideas of set concept we check with the relaxation of this paper is organized as follows. The following phase in short recollects the notions of soft sets and SS family members. In Section 3, we outline anti-reflexive kernel and symmetric kernel of a SS relation, respectively. Results involving them are obtained. Section four offers the principles of reflexive closure and symmetric closure of a SS relation, and proves some theorems based on them. In Section five, we suggest SS relation functions and inverse SS relation functions, and a few related law are mentioned. The final section summarizes the conclusions and gives a few focusses for intention of research.

2. Preliminaries

Definition 2.1. Let U be a preliminary universe set and E be a of parameters. Let $P\ (U)$ denote the power set of U and $A \subset E$. A pair (F, A) is known as a SS over U, wherein F a function is given by $F\colon A \to P\ (U)$.

Definition 2.2. Let $E = \{e_1, e_2, e_3, \dots e_n\}$ be a set of parameters. The NOT set of E denoted by means of $\sim E$ is defined by, $\sim E = \{\sim e_1, \sim e_2, \sim e_3, \dots \sim e_n\}$, wherein $\forall i$, $\sim e_i$ = not $\sim e_i$.

Definition 2.43. The complement of a SS (F, A) is denoted by using $(F, A)^c$ and is defined through $(F, A)^c$ = $(F^c, \sim A)$, in which $F^c\colon \sim A \to \mathcal{P}(U)$ is a function given by $F^c(\sim\delta) = U - F(\delta)$, for all $\sim\delta \in \sim A$.

Definition.2.4.A SS is called null SS and it is indicated by ϕ that is $F(\delta) = \phi$ for all $\delta \in A$. A SS is said to be complete set under U and it is denoted by U, $F(\delta) = U$, for all $\delta \in A$.

Revised Manuscript Received on February 01, 2020.
* Correspondence Author
K. Geetha*, Research scholar Periyar University PG Extension center-Dharmapuri . Email:k geetagenesh@gmail.com
Dr. N. Anitha, Department of Mathematics , Periyar University PG Extension center-Dharmapuri , India . Email: anithaarenu@gmail.com

Theorem2.5.The two parameter set A and B then the following are result are hold

(a) $\sim(\sim A) = A$

(b) $\sim(A \cup B) = \sim A \cup \sim B,$

(c) $\sim(A \cap B) = \sim A \cap \sim B.$

Definition.2.6. A union of two soft sets (F, A) and (G, B) over the common place universe U is the SS(H, C), where $C = A \sqcup B$, and for all $e \in C$,

$$H(e) = \begin{cases} F(e), e \in A - B \\ G(e), e \in B - A \\ F(e) \sqcup G(e), e \in A \sqcap B \end{cases}$$

Definition.2.7. A intersection of two soft sets (F, A) and (G, B) over the common place universe U is the SS(H, C), where$C = A \sqcap B$, and for all$e \in C$,

$$H(e) = \begin{cases} F(e), e \in A - B \\ G(e), e \in B - A \\ F(e) \sqcap G(e), e \in A \sqcap B \end{cases}$$

Definition.2.8. The Cartesian product of (F, A) and (G, B) as defined as $(F, A) \times (G, B) = (H, A \times B)$, where $H : A \times B \to \wp(U \times U)$ and $H(x, y) = F(x) \times G(y)$, where $(a, b) \in A \times B$. i.e., $H(x, y) = \{(c_i, c_j)$; where $c_i \in F(x)$ and $c_j \in G(y)\}$. It can be extended for n value.

Example 2.9. Take the SS(F, A) which defines the ''cost of the cars'' and the SS(G, B) which defines the ''desirability of the cars''.

Assume that $U = \{c_1, c_2, c_3, c_4, c_5, c_6, c_7, c_8, c_9, c_{10}\}$,

A = {very expensive; expensive; economy} and

B = {beautiful; moderate; economy}.

Let F (very costly) = $\{c_2, c_4, c_7, c_8\}$,

F (Expensive) = $\{c_1, c_3, c_5\}$,

F (Cheap) = $\{c_6, c_9, c_{10}\}$, and

G (Beautiful) = $\{c_2, c_3, c_7\}$,

G (Moderate) = $\{c_6, c_5, c_8\}$,

G (Economy) = $\{c_6, c_9, c_{10}\}$.

Now $(F, A) \times (G, B) = (H, A \times B)$ where a classic element will appearance like

H (Very expensive, beautiful) = $\{c_2, c_4, c_7, c_8\} \times \{c_2, c_3, c_7\}$

$= \{(c_2, c_2), (c_2, c_3), (c_2, c_7), (c_4, c_2), (c_4, c_3), (c_4, c_7), (c_7, c_2), (c_7, c_3), (c_7, c_7), (c_8, c_2), (c_8, c_3), (c_8, c_7)\}$.

Definition 2.10. SS relation from (F, A) to(G, B) is a soft subset of $(F, A) \times (G, B)$ over U.

3. Relation of kernels in soft set

Definition.3.1. Let (F, A) be a SS then R be the soft relation is defined by

(a) $S_1(x, y) \in R$, for all $x \in A \Rightarrow R \text{ is } Reflexive$

(b) $s_1(x, y) \to S_2(y, x)$, for all $x, y \in A \times A \Rightarrow R \text{ is symmetric}.$

Definition 3.2. Let (F, A) SSand R be a SS relation, then

(a) If $F(x) \times F(x) \in R, \forall x \in A \Rightarrow R$ is $reflexive$

(b) If $F(x) \times F(y) \in R \Rightarrow F(y) \times F(x) \in R, \forall (x, y) \in A \times A \Rightarrow R$ is symmetric.

Definition 3.3. Let (F, A) and R be the soft relation then R is anti-reflexive if $F(x) \times F(x) \notin R, \forall x \in A.$

Definition 3.4. Let I be a SS relation on (F, A). If $\forall x, y \in A$ and $\neq y$, $F(x) \times F(x) \in I$, but $F(x) \times F(y) \notin I$, then I is called to be identity SS relation.

Definition 3.5. Let (F, A) soft relation and the inverse of a SS relation R on denoted as R^{-1} is defined by

$R^{-1} = \{F(y) \times F(x) / F(x) \times F(y) \in R\}$.

Definition 3.7. Let (F, A) and R, S be two SS relations on $\forall x, y \in A$, if $F(x) \times F(y) \in R\,H \Rightarrow F(x) \times F(y) \in S$, then we call $R \subset S\ (or\ R \leq S)$.

Theorem 3.8. Let R, S be two SS relations on (F, A). Then

(a) R is symmetric iff $R = R^{-1}$.

(b) $(R^c)^{-1} = (R^{-1})^c$.

(c) $(R^{-1})^{-1}$ = R, $(R^c)^c$ = R.

(d) $R \sqcup S \sqsupseteq R, R \sqcup S \sqsupseteq S$.

(e) $R \sqcap S \subset R, R \sqcap S \sqsubseteq S$.

(f) $R \sqsubseteq S, H \Rightarrow R^c \sqsubseteq S^{-1}$.

(g) If $P \sqsupseteq S$ and $P \sqsupseteq R$, then $P \sqsupseteq R \sqcup S$.

(h) If $P \sqsubseteq S$ and $P \sqsubseteq R$, then $P \subset R \sqcap S$.

(i) If $R \sqsubseteq S$, then $R \sqcup S = S$ and $R \sqcup S = R$.

(j) $(R \sqcup S)^{-1} = R^{-1} \sqcup S^{-1}$, $(R \sqcap S)^{-1} = R^{-1} \sqcap S^{-1}$.

(k) $(R \sqcup S)^c = R^c \sqcup S^c$, $(R \sqcap S)^c = R^c \sqcap S^c$.

Proof. Clearly, (c) to (i) obviously true . We only show (a), (b), (j), and (k).

(a) $'' \Rightarrow '' \forall F(x) \times F(y) \in R$, by the symmetry of R, we have $F(x) \times F(y) \in R$, then $F(x) \times F(y) \in R^{-1}$. So $R \sqsubseteq R^{-1}$. Equally, $\forall F(x) \times F(y) \in R^{-1}$, then $F(y) \times F(x) \in R$. By the symmetry of R , $F(x) \times F(y) \in R$. So $R^{-1} \sqsubseteq R$. Thus $R = R^{-1}. '' \Leftarrow H$ Assume $R = R^{-1}$. $\forall x, y \in A$, if $F(x) \times F(y) \in R$, then $F(x) \times F(y) \in R^{-1}$, so $F(y) \times F(x) \in R$. Hence R is symmetric.

(b)
$\forall F(x) \times F(y) \in (R^c)^{-1}, x, y \in A$, $then\ F(y) \times F(x) \in R^c$
, which implies that $F(y) \times F(x) \notin R$. Thus $F(x) \times F(y) \notin R^{-1}$, which implies that $(x) \times F(y) \in (R^c)^{-1}$. Hence $(R^c)^{-1} \sqsubseteq (R^c)^{-1}$. In opposition, $\forall F(x) \times F(y) \in (R^c)^{-1}, x, y \in A$, then
$F(x) \times F(y) \notin R^{-1}$, which implies that $F(y) \times F(x) \notin$ R. So $F(y) \times F(x) \in R^c$, which implies that $F(x) \times F(y) \in (R^c)^{-1}$. Hence $(R^c)^{-1} \sqsubseteq (R^c)^{-1}$. Therefore $(R^c)^{-1} = (R^c)^{-1}$.

(j) $\forall x, y \in A, F(x) \times F(y) \in (R \sqcup S)^{-1} \Leftrightarrow F(y) \times F(x) \in R \sqcup S \Leftrightarrow F(y) \times F(x) \in R$

or

$F(y) \times F(x) \in S \Leftrightarrow F(x) \times F(y) \in R^{-1}$

or $F(x) \times F(y) \in S^{-1} \Leftrightarrow F(x) \times F(y) \in R^{-1} \sqcup S^{-1}$. So $(R \sqcup S)^{-1} = R^{-1} \sqcup S^{-1}$. The proof of $(R \sqcap S)^{-1} = R^{-1} \sqcap S^{-1}$ is similar.

(k) $\forall x, y \in A, F(x) \times F(y) \in (R \sqcup S)^c \Leftrightarrow F(x) \times F(y) \notin R \cup S \Leftrightarrow F(x) \times F(y) \Leftrightarrow$ R and $F(x) \times F(y) \notin S \Leftrightarrow F(x) \times F(y) \in R^c$ and $F(x) \times F(y) \in S^c \Leftrightarrow F(x) \times F(y) \in R^c \sqcap S^c$. So $(R \sqcup S)^c = R^c \sqcap S^c$. The proof of $(R \sqcap S)^c = R^c \sqcup S^c$ is like

Definition 3.9. Let (F, A) be fuzzy soft se and let R be a SS relation on (F, A).

(a) The maximal anti-reflexive SS relation $\sqsubseteq R$ is called anti-reflexive kernel of R, denoted by $AR(R)$.

(b) The maximal symmetric SS relation $\sqsubseteq R$ is called symmetric kernel of R, denoted by $S(R)$.

Theorem 3.10. Let (F, A) SS and R be a SS relation. Then

(1)) $AR(R) = R \cap I^c$. Hence we find a function (called AR kernel operator)

$$AR: SSR(F, A) \rightarrow SSR(F, A).$$

(2) $S(R) = R \sqcup R^{-1}$. Hence we find a function (called S kernel operator)

$$S: SSR(F, A) \rightarrow SSR(F, A).$$

Proof. (1) We know that by the above theorem 3.8 (e), $R \sqcap I^c \subset R$ and $R \sqcap I^c \subset I^c$.

Step 1: $\forall x \in A$, by the definition of I , $F(x) \times F(x) \in I$, so $F(x) \times F(x) \notin I^c$. Hence $F(x) \times F(x) \notin R \sqcap I^c$, i.e. $R \sqcap I^c$ is a AR SS relation on (F, A) .

Step 2: If H is a AR SS relation on (F, A) and $H \subset R$. Then $H \subset I^c$. Hence $H \subset R \sqcap I^c$

So $AR(R) = R \sqcap I^c$.

(2) We know that by the above theorem (j) and (c), $(R \sqcap S)^{-1} = R^{-1} \sqcap R^{-1} = R^{-1} \sqcap R = R \sqcap R^{-1}$.

That is $R \sqcap R^{-1}$ is a symmetric SS relation on (F, A) by Theorem 3.8(a). Also $R \sqcap R^{-1} \subset R$. On the other hand, if H is a symmetric SS relation on (F, A) and $H \subset R$. By Theorem 3.8(g), $H^{-1} \subset R^{-1}$. Then by Theorem 3.8(a) and (i),

$H = H^{-1} \subset R \sqcap R^{-1}$. So $S(R) = R \sqcap R^{-1}$

Theorem 3.11. The AR kernel operator ar has the following properties:

(1) $AR(m) = m, AR(I^c) = I^c$.

(2) $\forall R \in SSR(F, A), AR(R) \subset R$.

(3) $\forall R, S \in SSR(F, A), AR(R \sqcup S) = AR(R) \sqcup AR(S), AR(R \sqcap S) = AR(R) \sqcap AR(S)$.

(4) $\forall R, S \in SSR(F, A), if\ R \subset S$, Then $AR(R) \subset AR(S)$.

(5) $\forall R \in SSR(F, A), AR(AR(R)) = AR(R)$.

Proof. (1) By the anti-reflexivity of m and I^c, clearly, $AR(m) = m, AR(I^c) = I^c$.

(2) $\forall R \in SSR(F, A)$, By Theorems 3.10(1) and 3.8(f), $AR(R) = R \sqcup I^c \subset R$.

(3) $\forall R, S \in SSR(F, A)$, by Theorem 3.10(1), $AR(R \sqcup S) = (R \sqcup S) \sqcap I^c = (R \sqcup I^c) \sqcup (S \sqcup I^c) = AR(R) \sqcup AR(S)$, $AR(R \sqcap S) = (R \sqcap S) \sqcap I^c = (R \sqcap I^c) \sqcap (S \sqcap I^c) = AR(R) \sqcap AR(S)$.

(4) $\forall R, S \in SSR(F, A), R \subset S$, by (3) and Theorem 3.8(e) and (j), $AR(S) = AR(R \sqcup S) = AR(R) \sqcup AR(S) \supset AR(R\quad)$. (5) $\forall R \in SSR(F, A)$, by Theorem (1), $AR(R) = R \sqcap I^c$.

Hence $AR(AR(R)) = AR(R \sqcap I^c) = (R \sqcap I^c) \sqcap I^c = R \sqcap I^c = AR(R)$.

4. Soft set relations on Closures

In this segment, we are able to introduce the concepts of reflexive closure and symmetric closure of a SS relation, and take a look at their properties.

Definition 4.1. Let (F, A) be a SS and R be the relation. The minimum reflexive SS relation containing R is referred to as reflexive closure of R, denoted by means of $R(R)$.

Definition 4.2. Let (F, A) and let R be a SS relation. The $'' \wedge ''$ (minimum) symmetric SS relation containing R is referred to as symmetric closure of R, denoted by means of S(R).

Theorem 4.3. Let (F, A) and R be a SS relation Then

$R^*(R) = R \cup I$. Hence we find a function (named reflexive closure operator)

$R^*: SSR(F, A) \to SSR(F, A)$.

$S(R) = R \sqcup R^{-1}$. Hence we find a functions (named as symmetric closure operator)

$S^*: SSR(F, A) \to SSR(F, A)$.

Proof. (1) By Theorem 3.8(d), $R \sqcup I \supset R$. $\forall x \in A, F(x) \times F(x) \in I \subset R \sqcup I$, so $R \sqcup I$ is reflexive. On the other hand, if T is a reflexive SS relation on (F, A) and $H \supset R$. By the reflexivity of $H, H \supset I$, thus by Theorem 3.8(h), we have $H \supset R \sqcup I$. So $R(R) = R \sqcup I$. By Theorem 3.8(j), $(R \sqcup R^{-1})^{-1} = R^{-1} \sqcup (R^{-1})^{-1} = R^{-1} \sqcup R = R \sqcup R^{-1}$, i.e. $R \sqcup R^{-1}$ is a symmetric SS relation on (F, A), and $R \sqcup R^{-1} \supset R$ by Theorem 3.8(4). If T is a symmetric SS relation on (F, A) and $H \supset R$. By Theorem 3.8(6), $H^{-1} \supset R^{-1}$. According to Theorem 3.8(a) and (g), $H = H^{-1} \supset R \sqcup R^{-1}$. So $S(R) = R \sqcup R^{-1.}$

5. Functions on Soft set relations

In this part, we will bring together the ideas of SS relation functions and inverse SS relation functions, and argue some connected properties.

Definition 5.1. Let (F, A) to (G, B) be a soft relation and R be a SS relation. Then the domain of R (dom R) is stated as the SS(D, A_1) where $A_1 = \{x \in A \mid H(x, y) \in R$ for some $y \in B\}$ and $D(x_1) = F(x_1)\ \forall x_1 \in A_1$.

The range of R (ran R) is defined as the SS(RG, B_1), where $B_1 \subset B$ and $B_1 = \{y \in B \mid H(x, y) \in R$ for some $x \in A\}$ and $RG(y_1) = G(y_1)\ \forall y_1 \in B_1$.

Definition 5.2. Let (F, A) and (G, B) be two nonempty soft sets. Then a SS relation 'f'' from (F, A) to (G, B) is called a SS function if every element in domain has a unique element in the range. If $F(x) f G(y)$ then we write $f(F(x)) = G(y)$.

Definition 5.3. A SS function f from (F, A) to (G, B) is called

(1) $(1-1)$ if $F(x) \neq F(y)$ implies $f(F(x)) \neq f(F(y))$, i.e. each element of the RNG $''f''$ appearances closely once in the function.

(2) Onto if RNG $f = (G, B)$.

(3) if f $1-1$ onto

Definition 5.4. Let f be a SS function from (F, A) to (G, B),

International Journal of Recent Technology and Engineering (IJRTE)
ISSN: 2277-3878, Volume-8, Issue-6, March 2020

The SS relation function induced by f, meant by the notation $f \rightarrow$, is a functions from $SSR(F,A)$ to $SSR(G,B)$ that maps R to $f \rightarrow (R)$, where $f \rightarrow (R)$ is defined by, $f \rightarrow (R)$ =

$$\{f\,(F(x_1)) \times f\,(F(x_2)) \mid F(x_1) \times F(x_2) \in R\}.$$

CONCLUSION

Further we can apply interval valued fuzzy soft set, Intuitionistic interval valued fuzzy soft relations and proves some theorems.

REFERENCES

1. D. Molodtsov, SS theory first results, Comput. Math. Appl. 37 (1999) 19–31.
2. N. Çağman, S. Enginoğlu, Soft matrix theory and its decision making, Comput. Math. Appl..
3. A.R. Roy, P.K. Maji, A fuzzy SStheoretic approach to decision making problems, J. Comput. Appl. Math. 203 (2) (2007) 412–418.
4. Y. Zou, Z. Xiao, Data analysis approaches of soft sets under incomplete information, Knowl Based Syst. 21 (8) (2008) 941–945.
5. Z. Xiao, K. Gong, Y. Zou, A combined forecasting approach based on fuzzy soft sets, J. Comput. Appl. Math. 228 (1).
6. S.J. Kalayathankal, G.S. Singh, A FS flood alarm model, Math. Comput. Simulatio.
7. W. Xu, J. Ma, S. Wang, G. Hao, Vague soft sets and their properties, Comput. Math. Appl.
8. X.B. Yang, T.Y. Lin, J.Y. Yang, Y. Li, D. Yu, Comb. Of IVF set an soft set, Comput. Math.
9. Y. Jiang, Y. Tang, Q. Chen, H. Liu, J. Tang, IVIFSS and their properties, Comput. Mat Ap.
10. [F. Feng, Y.B. Jun, X.Z. Zhao, Soft semirings, Comput. Math. Appl. 56 (2008) 2621–2628.
11. U. Acar, F. Koyuncu, B. Tanay, Soft sets and soft rings, Comput. Math. Appl. 59 (11).
12. Y.B. Jun, K.J. Lee, A. Khan, Soft ordered semigroups, Math. Logic Q. 56 (1) (2010) 4250.
13. J. Zhan, Y.B. Jun, Soft BL-algebras based on fuzzy sets, Comput. Math. Appl. 59 (6)
14. Z. Xiao, K. Gong, S. Xia, Y. Zou, Exclusive disjunctive soft sets, Comput. Math. Appl. 59
15. K. Gong, Z. Xiao, X. Zhang, The bijective SSwith its operations, Comput. Math. Appl.
16. [Y. Jiang, Y. Tang, Q. Chen, J. Wang, S. Tang, Extending soft sets with description logics
17. M.I. Ali, F. Feng, X. Liu, W.K. Min, M. Shabir, On some new operations in SS theory,
18. K. Qin, Z. Hong, On soft equality, J. Comput. Appl. Math. .
19. A. Kharal, B. Ahmad, Mappings on FS classes, Adv. Fuzzy Syst.

AUTHORS PROFILE

K. Geetha –Research scholar Periyar University PG Extension center,Dharmapuri

Dr. N. Anitha Designation : Assistant Professor Educational Qualification : M.Sc.M.Phil.,PGDCA., Ph.D.,Official Address : Department of Mathematics, Periyar University PG Extension Centre, Dharmapuri Teaching Experience : 15 Years & 4 Months Research Experience : 9 Years, 7 Ph.D under Guidance 15 student she was guide M.Phil programme . Area of Specialization Fuzzy Algebra. Area of Research: Ring Theory she as published 37 articles. She has published 4 Books, Papers Presented in 10 Conferences/ seminars and Participated in Workshop/Seminar/Symposium Training Programme : Special Lectures Delivered 10