Type-2 Duality Triangular Fuzzy Fractional Transportation Problem using Goal Programming Technique

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Abstract*: The authors focused goal programming technique for solving type-2 duality fuzzy fractional transportation problem by using interval-valued triangular intuitionistic fuzzy numbers. The proposed method solves three types of models in which variables are taken as type-2 fuzzy variables which have spring up to the fuzzy fractional transportation problem. In these models, duality is applied and a particular type of non-linear membership functions are used to resolve duality fractional transportation problem including fuzzy parameters. A numerical example for examining the performance of the proposed model is envisaged here.

Keywords: Duality, Fractional Transportation problem, Goal Programming, Type-2 Fuzzy set.

I. INTRODUCTION

The transportation problem is a particular kind of linear programming, which is associated with our day to day activities. It was initially introduced by Hitchcock [11] in 1941 and has independently treated by Koopmans Kantorovich. Because of its wide application in resolving problems including many product sources and destinations of products, this type of problems is called as a transportation problem. It is a special kind of network optimization problem. It plays an important role in logistics and supply chains.

The fractional programming has charmed the mindfulness of many researchers in the ancient period. It is a transportation problem whose decision parameters are fuzzy numbers. The main target is to find the transportation scheme which minimizes the whole fuzzy transportation cost. In the transportation problem, various quantities of a single homogeneous commodity are transported from one sending locations to other receiving locations with minimal costs or maximum profit. It is a distribution type problem, and it can be classified into groups based on the main objective and origin supply versus destination on demand. Transportation problem whose main objective is to minimize the cost of shipping goods is called minimizing also maximize the profit of shipping goods is called maximizing.

Goal programming was initiated by Charnes and Cooper in 1961. This has extensively applied to calculate different problems which involve multiple objectives. This necessitate decision maker to set an aspiration level for each good which can be a really tough task since it has several uncertainties. It is one of the powerful approach which has proposed for finding solutions to multi-objective operational research problems.

The idea of intuitionistic fuzzy set was launched by Atanassov [2],[3],[4] with vagueness or uncertainty. The main advantage of intuitionistic fuzzy sets is that both the degree of membership and non-membership of each element are included in the set. In recent years, it plays an indispensable role in decision making in a fuzzy environment. It is a tool in modelling real-life problems like financial services, sales analysis, product marketing, planning, manufacturing, transportation etc. Intuitionistic fuzzy numbers have a large impact in solving transportation problem to find the optimal solution in which the cost, supply and demand are fuzzy numbers. In the present study, we formulated a duality fractional transportation problem in which transportation costs are taken as numerical values and supply and demands are taken as triangular intuitionistic fuzzy numbers.

In optimization theory duality is the main objective of optimization problems which are arrived as two sections, one is primal and the other is dual. The solution of the dual problem gives a lower bound to the solution of the primal problem. The summarized optimal values of the primal and dual problems should not be equal. The duality has both theoretical and computational significance. Its concepts and relations are well known now for convex and concave programming problems.

In this piece of work, a goal programming technique for solving duality fuzzy fractional transportation problem by interval-valued triangular and trapezoidal intuitionistic fuzzy numbers by using a special type of non-linear (hyperbolic) membership functions. This paper is motivated by [17, 21]. In our research, we have attempted for the first time solving fractional transportation problem using duality method. This can be applied in several areas like fuzzy environment, fuzzy intuitionistic fuzzy environment etc. Few authors have handled the area based on the type-2 fuzzy environment. Due to higher order fuzziness sometimes we cannot handle type-1 fuzzy approaches. The transportation cost may differ in various places, which depends on time. In such cases, the type-2 fuzzy set is needed to tackle such data. In this paper, all the constraints and variables should be in type-2 fuzzy in behavior.

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In the proposed method the main formulation divides into three different models and idea of duality is applied everywhere. Also, the authors extended to solve fuzzy goal programming technique using hyperbolic membership function. In this, the objectives of the fractional goal programming formulation problem are transformed into fuzzy goals by providing an aspiration level to each item. Also, the constraint values are taken by using the conditions of interval-valued triangular intuitionistic fuzzy numbers. Lingo [14] software is used to solve the optimization problem.

This paper is well ordered as follows: Section II explore the review of literature of the proposed problem. Section III briefly explains the preliminary background of the paper. Section IV scrutinize the problem formulation of fractional transportation problems, goal programming, fractional goal programming technique and fuzzy environment. Proposed method of the current study is explained in Section V. In Section VI, illustrative examples for triangular and trapezoidal intuitionistic fuzzy numbers were solved and the optimal solution is found out for both using duality approach. Results and discussions are given in Section VII. Finally conclusion is drawn out.

II. LITERATURE REVIEW

A large amount of researches have been studied fuzzy optimization which is one way or the other associates to this paper. Ebrahimnejad [10] developed a duality approach for solving bounded linear programming problems. Stancu-Minansian[20] explained fuzzy set approach for solving multiple objective linear fractional programming problem. Swarup [18] discussed duality in fractional programming. Zadeh [24] explained Fuzzy Sets and proposed a theory of fuzzy systems. Zimmermann [26] proposed fuzzy programming and linear programming with several objective functions and also explained toward a theory of fuzzy sysems. Chung [9] solved duality theory in fuzzy linear programming problems with fuzzy coefficients. A. Anju [5] solved Hexagonal Intuitionistic Fuzzy Fractional Transportation Problem Using Ranking and Russell's Method. Aneja and Nair [1] proposed a method to solve a bicriteria transportation problem. Mahdavi and Nasseri[16] discussed duality results and a dual simplex method for linear programming problems with trapezoidal fuzzy variables. D Zangiabadi and Maleki [25] applied fuzzy goal programming to the linear multiobjective transportation problem. Wahed [23] proposed interactive fuzzy goal programming for a multi-objective transportation problem. Chang [8] developed a fuzzy multi-objective fractional programming problems by goal programming approach. Liu and Chen [15] developed uncertain multi-objective programming and uncertain goal programming. Pal et al. [19] derived interval goal programming approach for solving multi-objective fuzzy goal programming with interval weights. Jahanshahloo et al. [12] discussed to find a solution for multi-objective linear fractional programming problem based on goal programming and data envelopment analysis. Stancu-Minasian [21] explained fractional programming in detail. Chakraborty et al., [7] has discussed a pair of linear primal dual programming using linear and exponential membership function using fuzzy programming approach and genetic algorithm approach. Bector and Chandra [6] discussed on duality in linear programming under fuzzy environment..Verdegay [22] briefly explained the fuzzy dual problem with the help of parametric linear programming and showed that the fuzzy primal and dual problems have the same fuzzy solution under some suitable conditions.

III. BASIC DEFINITION

In this section, some basic definitions and notions of fuzzy numbers are explained.

Definition 3.1 A triangular intuitionistic fuzzy number is denoted by $\tilde{A} = (a_1, a_2, a_3)(a_1', a_2, a_3')$ where $(a_1' \le a_1 \le a_2 \le a_3 \le a_3')$ with membership and non-membership functions are defined as follows:

$$\mu_{\tilde{A}}(x) = \begin{cases} \dfrac{x - a_1}{a_2 - a_1}, & for \quad a_1 \le x \le a_2 \\ \dfrac{a_3 - x}{a_3 - a_2}, & for \quad a_2 \le x \le a_3 \\ 0 \quad, & Otherwise. \end{cases}$$

$$\nu_{\tilde{A}}(x) = \begin{cases} \dfrac{a_1 - x}{a_2 - a_1}, & for \quad a_1 \le x \le a_2 \\ \dfrac{x - a_2}{a_3 - a_2}, & for \quad a_2 \le x \le a_3 \\ 1 \quad, & Otherwise. \end{cases}$$

Definition 3.2 *Type-2 fuzzy set is defined as the universe of all fuzzy sets in* $[0,1]$ *be* $G[0,1]$. *T in the universe X is a function defined as* $T : X \to [0,1]$.

IV. PROBLEM FORMULATION

A. Fuzzy Fractional Transportation Problem [2]

The fuzzy fractional transportation problem is a special kind of optimization problem. It plays an important role in engineering, business, finance, economics, supply management etc. for reducing cost and improving service. For instance, a company has m storehouses and n retail stores. If we want to ship a single commodity from the storehouses to the stores. Each storehouse has a level of supply and demand. The total supply of the commodity from storehouse i is A_i, and the total demand for the product at store j is B_j. Let C_{ij} and D_{ij} be, respectively, the transportation cost and the obtained profit per unit of the product from storehouse i to store j, and the values of α and β are given fixed costs. In this fractional programming paper, numerator and denominator may be represented as cost and profit [18].

Let x_{ij} be the amount of commodities to be transported from storehouse i to store j. The mathematical explanation is given as

$$\begin{cases} Minimize\ Z = \dfrac{\sum_{i=1}^{m}\sum_{j=1}^{n} \tilde{C}_{ij}\, x_{ij} + \alpha}{\sum_{i=1}^{m}\sum_{j=1}^{n} \tilde{D}_{ij}\, x_{ij} + \beta} \\ subject\ \ to \\ \sum_{j=1}^{n} x_{ij} \le \tilde{A}_i,\ i = 1,2,\ldots m \\ \sum_{i=1}^{m} x_{ij} \le \tilde{A}_j,\ j = 1,2,\ldots n \\ x_{ij} \ge 0 \quad \forall i, j. \end{cases}$$

B. Goal Programming

Goal programming is a limb of multi-objective optimization, which in turn is a branch of multi-criteria decision analysis. It is an extension or generalization of linear programming to handle many conflicting objective measures. The fuzzy goals are categorized by its membership functions which is altered to fuzzy flexible membership goals by establishing negative and positive deviational variables and allocating elevated membership value to each. The major aim is to minimize the deviations between the achievement of goals $Z^q(x)$ and aspiration levels G_1 and G_2. A mathematical behavior of goal programming is given below:

$$\begin{cases} Minimize\ Z^q(x) = \dfrac{\sum_{i=1}^{m}\sum_{j=1}^{n} [C^q{}_{Lij}, C^q{}_{Rij}]x_{ij} + \alpha - G_1}{\sum_{i=1}^{m}\sum_{j=1}^{n} [D^q{}_{Lij}, D^q{}_{Rij}]x_{ij} + \beta - G_2} \\ subject\ \ to \\ \sum_{j=1}^{n} x_{ij} \le A_i = [S_{Li}, S_{Ri}],\ i = 1,2,\ldots m \\ \sum_{i=1}^{m} x_{ij} \le \tilde{A}_j = [t_{Lj}, t_{Rj}],\ j = 1,2,\ldots n \\ x_{ij} \ge 0 \quad \forall i, j.,\ x \in F \end{cases}$$

where $Z^q(x)$ is the linear function of the q^{th} goal, G_q is the aspiration level of the q^{th} goal. For solving the goal programming approach the function is taken as $Z^q(x) = D_q^{\ +} - D_q^{\ -} + G_q$. Then the achievement function can be derived as,

$$\begin{cases} Minimize\ Z^q(x) = \sum_{i=1}^{m}\sum_{j=1}^{n} (D_q^{\ +} - D_q^{\ -}) \\ \dfrac{\sum_{i=1}^{m}\sum_{j=1}^{n} [C^q{}_{Lij}, C^q{}_{Rij}]x_{ij} + \alpha - G_1}{\sum_{i=1}^{m}\sum_{j=1}^{n} [D^q{}_{Lij}, D^q{}_{Rij}]x_{ij} + \beta - G_2} \\ subject\ \ to \\ \dfrac{[C^q{}_{Lij}, C^q{}_{Rij}]x_{ij} - G_1}{[D^q{}_{Lij}, D^q{}_{Rij}]x_{ij} - G_2} = D_q^{\ +} - D_q^{\ -},\ q = 1,2,\ldots Q \\ x \in F(F\ is\ a\ \ feasible\ set) \\ D_q^{\ +} - D_q^{\ -} \ge 0,\ q = 1,2,\ldots,Q. \end{cases}$$

V. PROPOSED METHOD

The proposed method is the best method to find the optimal solution of type-2 duality fuzzy fractional transportation problem with interval valued intuitionistic fuzzy numbers having supply and demand which are real numbers and transportation cost $\dfrac{C_{ij}}{D_{ij}}; (i = 1,2,\ldots,m); (j = 1,2,\ldots,n)$ from i^{th} source to j^{th} destination. Different types of type-2 fuzzy fractional transportation models are given below

1) Type-2 Fuzzy Fractional Transportation Model for Triangular Fuzzy Numbers

The formulation for type-2 fuzzy fractional transportation for triangular fuzzy number is given as

$$\begin{cases} Minimize\ Z = (Z_1, T_{(Z_2, Z_{3)}}) = \dfrac{\sum_{i=1}^{m}\sum_{j=1}^{n} a_{ij} T(b_{ij}, c_{ij}) \otimes (x_{ij}, T(y_{ij}, z_{ij}))}{\sum_{i=1}^{m}\sum_{j=1}^{n} d_{ij} T(e_{ij}, f_{ij}) \otimes (x_{ij}, T(y_{ij}, z_{ij}))} \\ subject\ \ to \\ \sum_{j=1}^{n} (x_{ij}, T(y_{ij}, z_{ij})) \le a_i^{\ 1}, T(b_i^{\ 1}, c_i^{\ 1}),\ i = 1,2,\ldots m \\ \sum_{i=1}^{m} (x_{ij}, T(y_{ij}, z_{ij})) \le a_j^{\ 2}, T(b_i^{\ 2}, c_i^{\ 2}),\ j = 1,2,\ldots n \\ x_{ij} \ge 0,\ 0 \le y_{ij} \le 0.5,\ 0.5 \le z_{ij} \le 1 \quad \forall i, j. \end{cases}$$

The conditions for y_{ij} and z_{ij} are given to maintain the consistency in the solutions. By splitting the above problem into co-ordinate wise, again, we will get three different crisp solutions. Now we are going to consider the first co-ordinate x_{ij} in model-I.

Model-I

In this model we are going to consider the co-ordinate x_{ij} and we have to minimize Z_1. The formulation is given as follows.

$$\begin{cases} Minimize\ Z_1 = \dfrac{\sum_{i=1}^{m}\sum_{j=1}^{n} a_{ij} x_{ij}}{\sum_{i=1}^{m}\sum_{j=1}^{n} e_{ij} x_{ij}} \\ subject\ \ to \\ \sum_{j=1}^{n} x_{ij} \le a_i^{\ 1},\ i = 1,2,\ldots m \\ \sum_{i=1}^{m} x_{ij} \le a_j^{\ 2},\ j = 1,2,\ldots n \\ x_{ij} \ge 0 \quad \forall i, j. \end{cases}$$

Here we want to maximize $T_{(Z_2, Z_3)}$, for that we have to minimize Z_2 and maximize Z_3.

Model-II

Now we are going to consider the co-ordinate y_{ij} we obtain its fractional transportation as given below in Model-II. Here y_{ij} corresponds to the minimum point.

$$\begin{cases} Minimize\ Z_2 = \dfrac{\sum_{i=1}^{m}\sum_{j=1}^{n} b_{ij} y_{ij}}{\sum_{i=1}^{m}\sum_{j=1}^{n} f_{ij} y_{ij}} \\ subject\ to \\ \quad \sum_{j=1}^{n} y_{ij} \le b_i^{\,1},\ i = 1,2,...m \\ \quad \sum_{i=1}^{m} y_{ij} \le b_j^{\,2},\ j = 1,2,...n \\ \quad y_{ij} \ge 0 \quad \forall i, j. \end{cases}$$

Model-III

Now consider the third co-ordinate z_{ij}, we attain the Model-III as given below. In this z_{ij} corresponds to the maximum point.

$$\begin{cases} Maximize\ Z_3 = \dfrac{1}{mn} \dfrac{\sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij} z_{ij}}{\sum_{i=1}^{m}\sum_{j=1}^{n} g_{ij} z_{ij}} \\ subject\ to \\ \quad \sum_{j=1}^{n} z_{ij} \ge nc_i^{\,1},\ i = 1,2,...m \\ \quad \sum_{i=1}^{m} z_{ij} \ge mc_j^{\,2},\ j = 1,2,...n \\ \quad 0.5 \le z_{ij} \le 1 \quad \forall i, j. \end{cases}$$

2) Duality Fractional Transportation

This paper is motivated by Fractional transportation by [21]. The above models are explained in terms of duality. The existing paper introduced two types of dual problems for fractional transportation problem. The first dual problem was introduced by Swarup in 1954, and the second by Stancu Minasian and Teghem in 1918. In this current study we are considering the Stancu Minasian dual problem. Consider the following fractional transportation problem in terms of primal problem.

$$\begin{cases} Minimize\ f(X) = \dfrac{\sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij} x_{ij}}{\sum_{i=1}^{m}\sum_{j=1}^{n} d_{ij} x_{ij}} \\ subject\ to \\ \quad \sum_{j=1}^{n} x_{ij} \le a_i,\ i = 1,2,...m \\ \quad \sum_{j=1}^{n} x_{ij} \ge b_j,\ j = 1,2,...n \\ \quad x_{ij} \ge 0,\ \forall i, j. \end{cases}$$

Note that $\sum_{j=1}^{n} d_{ij} x_{ij} > 0$ for all feasible solutions. A necessary condition for a primal problem to have the solution is $\sum_{i=1}^{m} a_i \ge \sum_{j=1}^{n} b_j$. Now the dual problem can be defined as,

$$\begin{cases} Maximize\ g(U) = \dfrac{\sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij} x_{ij}}{\sum_{i=1}^{m}\sum_{j=1}^{n} d_{ij} x_{ij}} \\ subject\ to \\ \quad \sum_{j=1}^{n} x_{ij}\, \tilde{1} \approx \tilde{a}_i,\ i = 1,2,...m \\ \quad \sum_{j=1}^{n} x_{ij}\, \tilde{1} \approx \tilde{b}_j,\ j = 1,2,...n \\ \quad x_{ij} \ge 0,\ \forall i, j. \end{cases}$$

with $\sum_{i=1}^{m} \tilde{a}_i \approx \sum_{j=1}^{n} \tilde{b}_j$ where the values of $\tilde{c}_{ij}$ and $\tilde{d}_{ij}$ are calculated by using the formula i.e.;

$$(a_i)^1 = a_i + \frac{1}{16}\left[(\alpha_{4i} - \alpha_{3i}) + \left(4 - \left(\frac{3\gamma}{\delta}\right)\right)(\alpha_{2i} - \alpha_{1i})\right],$$

$$(b_j)^1 = b_j + \frac{1}{16}\left[(\beta_{4j} - \alpha_{3j}) + \left(4 - \left(\frac{3\gamma}{\delta}\right)\right)(\beta_{2j} - \beta_j)\right]$$

we convert firstly the constraints into single valued constraints. Again by using LINGO software we obtain the optimal solution and optimal values. Now substituting these optimal values in the constraint, then the dual problem becomes.

$$\begin{cases} Maximize\ g(U) = \dfrac{\sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij} U_{ij}}{\sum_{i=1}^{m}\sum_{j=1}^{n} d_{ij} U_{ij}} \\ subject\ to \\ \quad (-r_i + sj) - c_{ij}\left[\sum_{i=1}^{m}\sum_{j=1}^{n} d_{ij} U_{ij}\right] + d_{ij}\left[\sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij} U_{ij}\right] \le 0,\ \forall i, j \\ \quad -\sum_{i=1}^{m} a_i r_i + \sum_{j=1}^{n} b_j s_j \ge 0. \end{cases}$$

Now the dual linear problem has the form

$$\begin{cases} Maximize\ G(R,S) = -\sum_{i=1}^{m} a_i r_i + \sum_{j=1}^{n} b_j s_j \\ subject\ to \\ \qquad (-r_i + s_j) \le t_{ij} \quad \forall i, j. \end{cases}$$

Then by using the LINGO software [14], we get the n of the dual problem. Now according to direct duality theorem by [21] we get the optimal solution. By solving dual problem if we are getting the same solutions then we can say that it is not a basic feasible solution, or else if we are getting different solutions then we can say that it is a basic feasible solution. This we are going to apply in the above three models. After that we found out the goal programming approach for solving duality fractional transportation problem using a special type of hyperbolic membership function for each models separately.

VI. NUMERICAL EXAMPLE

	D_1	D_1	D_1	S_i
S_1	$\frac{(2,T_{(0.1,0.6)})}{(2,T_{(0.1,0.6)})}$	$\frac{(3,T_{(0.4,0.6)})}{(4,T_{(0.3,0.75)})}$	$\frac{(4,T_{(0.15,0.7)})}{(5,T_{(0.25,0.65)})}$	$(10,T_{(0.15,0.8)})$
S_2	$\frac{(2,T_{(0.25,0.9)})}{(1,T_{(0.250.9)})}$	$\frac{(1,T_{(0.3,0.75)})}{(5,T_{(0.25,0.7)})}$	$\frac{(3,T_{(0.25,0.6)})}{(6,T_{(0.15,0.7)})}$	$(16,T_{(0.35,0.7)})$
S_3	$\frac{(4,T_{(0.15,0.4)})}{(2,T_{(0.35,0.7)})}$	$\frac{(3,T_{(0.1,0.7)})}{(1,T_{(0.25,0.6)})}$	$\frac{(3,T_{(0.25,0.7)})}{(4,T_{(0.1,0.75)})}$	$(14,T_{(0.15,0.6)})$
D_j	$(15,T_{(0.35,0.7)})$	$(13,T_{(0.15,0.8)})$	$(12,T_{(0.05,0.5)})$	

The problem is solved by using four different types of models given below using trapezoidal fuzzy numbers.

Model-I

$$Minimize = \frac{2x_{11} + 3x_{12} + 4x_{13} + 2x_{21} + x_{22} + 3x_{23} + 4x_{31} + 3x_{32} + 3x_{33} + \alpha}{3x_{11} + 4x_{12} + 5x_{13} + x_{21} + 5x_{22} + 6x_{23} + 2x_{31} + x_{32} + 4x_{33} + \beta}$$

$$\begin{aligned} & subject\ to \\ & x_{11} + x_{12} + x_{13} \le [(3,11,12;0.4)(1,11,14;0.6)] \\ & x_{21} + x_{22} + x_{23} \le [(13,15,20;0.4)(10,15,23;0.6)] \\ & x_{31} + x_{32} + x_{33} \le [(18,12,25;0.4)(16,12,26;0.6)] \\ & x_{11} + x_{21} + x_{31} \le [(11,15,26;0.4)(8,15,27;0.6)] \\ & x_{12} + x_{22} + x_{32} \le [(9,13,17;0.4)(6,13,20;0.6)] \\ & x_{13} + x_{23} + x_{33} \le [(10,12,14;0.4)(9,12,16;0.6)] \\ & \quad x_{ij} \ge 0. \end{aligned}$$

By using the equations the constraints can be changed into

$$Minimize = \frac{2x_{11} + 3x_{12} + 4x_{13} + 2x_{21} + x_{22} + 3x_{23} + 4x_{31} + 3x_{32} + 3x_{33} + \alpha}{3x_{11} + 4x_{12} + 5x_{13} + x_{21} + 5x_{22} + 6x_{23} + 2x_{31} + x_{32} + 4x_{33} + \beta}$$

$$\begin{aligned} & subject\ to \\ & x_{11} + x_{12} + x_{13} \le 10 \\ & x_{21} + x_{22} + x_{23} \le 16 \\ & x_{31} + x_{32} + x_{33} \le 14 \\ & x_{11} + x_{21} + x_{31} \le 15 \\ & x_{12} + x_{22} + x_{32} \le 13 \\ & x_{13} + x_{23} + x_{33} \le 12 \\ & \quad x_{ij} \ge 0. \end{aligned}$$

This is the primal problem we have taken and by solving we get $x_{11} = 15, x_{22} = 13, x_{23} = 3, x_{33} = 9$ is the optimal value and its optimal solution is $Z_1 = 0.4817$. Again, the dual problem is

$$Minimize\ Z_1(U) = \frac{2u_{11} + 3u_{12} + 4u_{13} + 2u_{21} + u_{22} + 3u_{23} + 4u_{31} + 3u_{32} + 3u_{33} + \alpha}{3u_{11} + 4u_{12} + 5u_{13} + u_{21} + 5u_{22} + 6u_{23} + 2u_{31} + u_{32} + 4u_{33} + \beta}$$

$$\begin{aligned} & subject\ to \\ & -p_1 + q_1 + u_{12} - 2u_{13} + 4u_{21} - 7u_{22} \\ & -3u_{23} + 8u_{31} + 7u_{32} + u_{33} \le 0 \\ & -p_1 + q_2 + u_{11} + u_{13} + 5u_{21} - 11u_{22} \\ & -6u_{23} + 10u_{31} + 9u_{32} \le 0 \\ & -p_1 + q_3 - 2u_{11} - u_{12} + 6u_{21} - 15u_{22} \\ & -9u_{23} + 12u_{31} + 11u_{32} - u_{33} \le 0 \\ & -p_2 + q_1 - 4u_{11} - 5u_{12} - 6u_{13} - 9u_{22} \\ & -9u_{23} + u_{32} - 5u_{33} \le 0 \\ & -p_2 + q_2 + 7u_{11} + 11u_{12} + 15u_{13} + 9u_{21} \\ & +9u_{23} + 18u_{31} + 15u_{32} + 6u_{33} \le 0 \\ & -p_2 + q_3 + 3u_{11} + 6u_{12} + 9u_{21} - 9u_{22} \\ & +18u_{31}15u_{32} + 6u_{33} \le 0 \\ & -p_3 + q_1 - 8_{11} - 10u_{12} - 12u_{13} - 18u_{22} \\ & -18u_{23} + 2u_{32} - 10u_{33} \le 0 \\ & -p_3 + q_2 - 7_{11} - 9u_{12} - 11u_{13} - u_{21} \\ & -14u_{22} - 15u_{23} - 2u_{31} - 9u_{33} \le 0 \\ & -p_3 + q_3 - u_{11} - u_{13} + 5u_{21} \\ & -11u_{22} - 6u_{23} + 10u_{31} + 9u_{32} \le 0 \end{aligned}$$

$-10p_1 - 16p_2 - 14p_3 - 15q_1 - 13q_2 - 12q_3 \le 0$.

The dual fractional programming has the form

$$\begin{aligned} Maximize\ Z_1(U) = &-10p_1 - 16p_2 - 14p_3 \\ &-15q_1 - 13q_2 - 12q_3 \\ subject\ \ to\ \ & \\ &-p_1 + q_1 \le 91 \\ &-p_1 + q_2 \le 176 \\ &-p_1 + q_{31} \le 261 \\ &-p_2 + q_1 \le 249 \\ &-p_2 + q_2 \le -231 \\ &-p_2 + q_3 \le 18 \\ &-p_3 + q_1 \le 498 \\ &-p_3 + q_2 \le 413 \\ &-p_3 + q_3 \le 176 \\ &p_1, p_2, p_3, q_1, q_2, q_3 \ge 0 \end{aligned}$$

The optimal solution of the dual fractional programming problem is given as $p_1 = 6, p_2 = 231, p_3 = 0, q_1 = 0, q_2 = 2, q_3 = 0$ and the optimal value of the objective function is $Z_1(U) = 156$. Next in this goal programming approach is used for solving duality fuzzy fractional transportation problem using a special type of hyperbolic membership function. For this we are considering he right and center values of the objective function. Positive deviational variables are only taken into consideration and negative deviational variables are taken as zero. The problem was solved by the linear interactive global optimization LINGO[14] software and the optimal compromise solution is presented as follows $Z_1(U) = 156, D_q^+ = 1.25, 1.208, D_q^- = 0$ and $\phi = 0.1$.

Model -II

$$Minimize\ Z_2 = \frac{1}{9} \times \frac{\begin{matrix} 0.1y_{11} + 0.4y_{12} + 0.15y_{13} + \\ 0.25y_{21} + 0.3y_{22} + 0.25y_{23} + \\ 0.35y_{31} + 0.15y_{32} + 0.05y_{33} + \alpha \end{matrix}}{\begin{matrix} 0.1y_{11} + 0.3y_{12} + 0.25y_{13} + \\ 0.25y_{21} + 0.25y_{22} + 0.15y_{23} + \\ 0.35y_{31} + 0.25y_{32} + 0.1y_{33} + \beta \end{matrix}}$$

subject to

$$\begin{aligned} &y_{11} + y_{12} + y_{13} \ge 0.45 \\ &y_{21} + y_{22} + y_{23} \ge 1.05 \\ &y_{31} + y_{32} + y_{33} \ge 0.45 \\ &y_{11} + y_{21} + y_{31} \ge 1.05 \\ &y_{12} + y_{22} + y_{32} \ge 0.45 \\ &y_{13} + y_{23} + y_{33} \ge 0.15 \\ &y_{ij} \ge 0. \end{aligned}$$

This is the primal problem we have taken and by solving we get $\begin{matrix} y_{11} = 0.6, y_{12} = 0.45, y_{22} = 13, \\ y_{23} = 3386.6, y_{31} = 0.45 \end{matrix}$ is an optimal value and its objective function is $Z_2 = 0.185$

Again, the dual problem is

$$Minimize\ Z_2 = \frac{1}{9} \times \frac{\begin{matrix} 0.1y_{11} + 0.4y_{12} + 0.15y_{13} + \\ 0.25y_{21} + 0.3y_{22} + 0.25y_{23} + \\ 0.35y_{31} + 0.15y_{32} + 0.05y_{33} + \alpha \end{matrix}}{\begin{matrix} 0.1y_{11} + 0.3y_{12} + 0.25y_{13} + \\ 0.25y_{21} + 0.25y_{22} + 0.15y_{23} + \\ 0.35y_{31} + 0.25y_{32} + 0.1y_{33} + \beta \end{matrix}}$$

subject to

$$\begin{aligned} &-p_1 + q_1 + 0.01u_{12} - 0.01u_{13} + 0.005u_{22} \\ &-0.01u_{23} - 0.01u_{32} - 0.005u_{33} \le 0 \\ &-p_1 + q_2 - 0.01u_{11} - 0.055u_{13} - 0.025u_{21} - 0.01u_{22} \\ &-0.04u_{23} + 0.035u_{31} - 0.055u_{32} - 0.025u_{33} \le 0 \\ &-p_1 + q_3 + 0.01u_{11} + 0.055u_{12} + 0.025u_{21} + 0.032u_{22} \\ &+0.04u_{23} + 0.035u_{31} - 0.003u_{33} \le 0 \\ &-p_2 + q_1 + 0.025u_{12} - 0.024u_{13} + 0.013u_{22} + 0.0245u_{23} \\ &-0.025u_{32} - 0.013u_{33} \le 0 \\ &-p_2 + q_2 - 0.005u_{11} + 0.01u_{12} - 0.037u_{13} - 0.0125u_{21} \\ &+0.017u_{23} - 0.018u_{31} - 0.038u_{32} - 0.018u_{33} \le 0 \\ &-p_2 + q_3 - 0.015u_{11} - 0.015u_{12} + 0.013u_{13} - 0.024u_{21} \\ &-0.017u_{23} + 0.029u_{31} - 0.039u_{32} - 0.012u_{33} \le 0 \\ &-p_3 + q_1 + 0.035u_{12} + 0.08u_{13} + 0.018u_{22} + 0.035u_{23} \\ &-0.034u_{32} - 0.017u_{33} \le 0 \\ &-p_3 + q_2 + 0.01u_{11} + 0.05u_{12} + 0.025u_{21} + 0.037u_{22} \\ &+0.04u_{23} + 0.035u_{31} - 0.003u_{33} \le 0 \\ &-p_3 + q_3 - 0.005u_{11} + 0.02u_{12} + 0.002u_{23} - 0.012u_{21} \\ &-0.017u_{22} + 0.017u_{23} + 0.017u_{31} + 0.002u_{32} \le 0 \\ &-0.45p_1 - 1.05p_2 - 0.45p_3 - 1.05q_1 \\ &-0.45q_2 - 0.15q_3 \le 0 \end{aligned}$$

The dual fractional programming has the form

$$Maximize\ Z_2(U) = -0.45p_1 - 1.05p_2 - 0.45p_3 - 1.05q_1 - 0.45q_2 - 0.15q_3$$

subject to

$$\begin{aligned}
&-p_1 + q_1 \le -34\\
&-p_1 + q_2 \le -135\\
&-p_1 + q_{31} \le -135\\
&-p_2 + q_1 \le -83\\
&-p_2 + q_2 \le -58\\
&-p_2 + q_3 \le 0.02\\
&-p_3 + q_1 \le -118\\
&-p_3 + q_2 \le -135\\
&-p_3 + q_3 \le -59\\
&p_1, p_2, p_3, q_1, q_2, q_3 \ge 0
\end{aligned}$$

The optimal solution of the dual fractional programming problem is given below

$p_1 = 135, p_2 = 83, p_3 = 135, q_1 = 0, q_2 = 2, q_3 = 0$

$u_{11} = 0.6, u_{12} = 0.45, u_{22} = 13, u_{23} = 3386.6, u_{31} = 0.45$ is also the optimal solution of the dual problem and the optimal value of the objective function is $Z_2(U) = 208.65$ This is a basic feasible solution for model-II and therefore, when we solve the dual problem we may not get the same solution. Thus this method is really good to find basic feasible solution in the case of duality for Model-II fractional transportation problems.

Next in this goal programming approach is used for solving duality fuzzy fractional transportation problem using a special type of hyperbolic membership function. For this we are considering he right and center values of the objective function. Positive deviational variables are only taken into consideration and negative deviational variables are taken as zero. This is solved by using LINGO [14] software and the optimal compromise solution is presented as follows

$Z_2(U) = 208.65, D_q^+ = [0.4, 0.6], D_q^- = 0$

and $\phi = 0.1$.

Model-III

$$Maximize\ Z_3 = \frac{1}{9} \times \frac{\begin{matrix}0.6z_{11} + 0.6z_{12} + 0.7z_{13} +\\ 0.9z_{21} + 0.75z_{22} + 0.6z_{23} +\\ 0.4z_{31} + 0.7z_{32} + 0.7z_{33} + \alpha\end{matrix}}{\begin{matrix}0.5z_{11} + 0.75z_{12} + 0.6z_{13} +\\ 0.9z_{21} + 0.7z_{22} + 0.7z_{23} +\\ 0.7z_{31} + 0.6z_{32} + 0.75z_{33} + \beta\end{matrix}}$$

subject to

$$\begin{aligned}
&z_{11} + z_{12} + z_{13} \ge 2.4\\
&z_{21} + z_{22} + z_{23} \ge 2.1\\
&z_{31} + z_{32} + z_{33} \ge 01.8\\
&z_{11} + z_{21} + z_{31} \ge 2.1\\
&z_{12} + z_{22} + z_{32} \ge 2.4\\
&z_{13} + z_{23} + z_{33} \ge 1.53\\
&z_{ij} \ge 0.
\end{aligned}$$

This is the primal problem we have taken and by solving we get $z_{11} = 1915.8, z_{13} = 1.53, z_{22} = 2.1,$ $z_{32} = 1.8$

is an optimal value and its objective function is $Z_3 = 0.133$.

Again, the dual problem is

$$Maximize\ Z_3 = \frac{1}{9} \times \frac{\begin{matrix}0.6u_{11} + 0.6u_{12} + 0.7u_{13} +\\ 0.9u_{21} + 0.75u_{22} + 0.6u_{23} +\\ 0.4u_{31} + 0.7u_{32} + 0.7u_{33} + \alpha\end{matrix}}{\begin{matrix}0.5u_{11} + 0.75u_{12} + 0.6u_{13} +\\ 0.9u_{21} + 0.7u_{22} + 0.7u_{23} +\\ 0.7u_{31} + 0.6u_{32} + 0.75u_{33} + \beta\end{matrix}}$$

subject to

$$\begin{aligned}
&-p_1 + q_1 - 0.15u_{12} - 0.04u_{13} - 0.09u_{21}\\
&-0.04u_{22} - 0.08u_{23} - 0.2u_{31} - 0.01u_{32} - 0.1u_{33} \le 0\\
&-p_1 + q_2 - 0.015u_{11} + 0.13u_{13} - 0.13u_{21} + 0.14u_{22}\\
&+0.03u_{23} + 0.12u_{31} + 0.16u_{32} + 0.07u_{33} \le 0\\
&-p_1 + q_3 + 0.01u_{11} - 0.16u_{12} - 0.09u_{21}\\
&-0.106u_{23} - 0.25u_{31} - 0.105u_{33} \le 0\\
&-p_2 + q_1 + 0.09u_{11} - 0.135u_{12} + 0.09u_{13} + 0.115u_{22}\\
&-0.54u_{23} - 0.2u_{31} + 0.09u_{32} - 0.04u_{33} \le 0\\
&-p_2 + q_2 + 0.05u_{11} - 0.14u_{12} + 0.04u_{13} - 0.045u_{21}\\
&-0.07u_{23} - 0.24u_{31} + 0.04u_{32} - 0.035u_{33} \le 0\\
&-p_2 + q_3 + 0.12u_{11} - 0.03u_{12} + 0.1u_{13} - 0.09u_{21}\\
&+0.105u_{22} + 0.14u_{31} + 0.13u_{32} + 0.04u_{33} \le 0
\end{aligned}$$

$$-p_3+q_1+0.22u_{11}+0.12u_{12}+0.25u_{13}+0.27u_{21}$$
$$+0.245u_{22}+0.14u_{23}+0.25u_{32}+0.19u_{33}\le 0$$
$$-p_3+q_2-0.01u_{11}-0.165u_{12}-0.09u_{21}-0.04u_{22}$$
$$-0.10u_{23}+0.25u_{31}-0.105u_{33}\le 0$$
$$-p_3+q_3+0.14u_{11}-0.075u_{12}+0.105u_{13}+0.045u_{21}$$
$$+0.07u_{22}-0.04u_{23}+0.19u_{31}-0.105u_{32}\le 0$$
$$-2.4p_1-2.1p_2-1.8p_3-2.1q_1-2.4q_2-1.53q_3\le 0$$

The dual fractional programming has the form

$$Minimize\ Z_3(U)=-2.4p_1-2.1p_2-1.8p_3-2.1q_1-2.4q_2-1.53q_3$$

$$subject\ to$$
$$-p_1+q_1\le 0.17$$
$$-p_1+q_2\le -288$$
$$-p_1+q_{31}\le 19$$
$$-p_2+q_1\le -173$$
$$-p_2+q_2\le -96$$
$$-p_2+q_3\le -230$$
$$-p_3+q_1\le -423$$
$$-p_3+q_2\le 19$$
$$-p_3+q_3\le -269$$
$$p_1,p_2,p_3,q_1,q_2,q_3\ge 0$$

The optimal solution of the dual fractional programming problem is given below

$$p_1=0,p_2=230,p_3=423,q_1=0,q_2=2,q_3=0$$
$$u_{11}=1915.8,u_{13}=1.53,u_{22}=2.1,u_{32}=1.8$$

is also the optimal solution of the dual problem and the optimal value of the objective function $Z_3(U)=1244$. This is a basic feasible solution for model-III and therefore, when we solve the dual problem we may not get the same solution. Thus this method is really good to find basic feasible solution in the case of duality for Model-III fractional transportation problems. Next in this goal programming approach is used for solving duality fuzzy fractional transportation problem using a special type of hyperbolic membership function. For this we are considering he right and center values of the objective function. Positive deviational variables are only taken into consideration and negative deviational variables are taken as zero. The problem is solved by LINGO [14] software and the optimal compromise solution is presented as follows

$$Z_3(U)=1244,D_q^+=[0.366,0.605],D_q^-=0$$

and $\phi=0.1$.

VII. RESULT AND DISCUSSION

This paper is motivated by [17, 21] it will be very useful in solving decision making problems. There are no optimization models in literature based on this topic. So, this motivated the authors to arise up with duality fuzzy fractional transportation of type-2 fuzzy. Here all the variables and constraints are taken type-2 fuzzy nature. In this firstly the problem splits into three different models by using type-2 fuzzy fractional transportation problem. Duality method is applied for all the three models and a special type of hyperbolic membership function is used to find out the goals separately for all the three models. Finally combining all the three optimal solution will give the optimal solution of original dual fuzzy fractional transportation problem of type-2 for triangular fuzzy numbers and it is given as

$$Z^*(U)=(Z_1,Z_2,Z_3)(U)=(3696,208.65,1244).$$

Thus, we can conclude our result by saying that our proposed algorithm is a new way to handle duality fractional transportation using type-2 fuzzy transportation problem.

VIII. CONCLUSION

The transportation problem with fractional objectives are universally used in our day to day situations. Goal programming is such a technique to handle decision theory and allow us to find solutions which fulfill our goals. In our research we have attempted for the first time an effective goal programming technique is adopted to discover the optimal solution of duality fuzzy fractional transportation problem using type-2 fuzzy transportation problem and a special type non-linear(hyperbolic)membership function where the triangular interval valued fuzzy numbers are taken into consideration. This method can be used in several areas like fuzzy environment, intuitionistic fuzzy environment etc. The proposed method gives the basic feasible solution and this approach achieves its goal speedily and accurately.

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