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On Divisor 3-Equitable Labeling of Wheel Graphs

K. Tina Jebi Nivathitha, N. Srinivasan, A. Parthiban, Sangeeta

Abstract: ***A graph G on n vertices is said to admit a divisor 3-equitable labeling if there exists a bijection $d: V(G) \to \{1, 2, \ldots, n\}$ defined by $d(e = xy) = \begin{cases} 1, \text{ if } d(x)|d(y) \text{ or } d(y)|d(x) \\ 2, \text{ if } \frac{d(x)}{d(y)} = 2 \text{ or } \frac{d(y)}{d(x)} = 2 \\ 0, \text{ otherwise} \end{cases}$ and $|e_d(i) - e_d(j)| \leq 1$ for all $0 \leq i, j \leq 2$, where $e_d(i)$ denotes the number of edges labelled with "i". A graph which permits a divisor 3-equitable labeling is called a divisor 3-equitable graph. A wheel graph W_n is defined as $W_n = C_{n-1} \wedge K_1$, where C_{n-1} is a cycle on $n-1$ vertices and K_1 is a complete graph on a single vertex. In this paper, we prove the non-existence of a divisor 3-equitable labeling of the wheel graph W_n for $n \geq 7$.***

Keywords: 3-equitable labeling, Divisor 3-equitable labeling, Wheel graph.

I. INTRODUCTION

All graphs considered in this present investigation are "finite, undirected, simple, and connected". By $G(V, E)$, or simply G, we mean a graph G with "vertex set V and edge set E." We also denote the "number of vertices in G" by $|V(G)|$ and the "number of edges in G" by $|E(G)|$. "An assignment of integers" to the vertices of G is called a "vertex labeling" of G. Graph labeling is a "strong communication" between the structure of graphs and number theory. Labeled graphs have a variety of applications in coding theory such as "missile guidance codes, design of good radar type codes, convolution codes with optimal auto correlation properties" etc. Labeled graphs also play a vital role in the field of X-ray crystallography, in determining optimal circuit layouts, and communication networks [4]. We recall a few relevant definitions and results relevant to the study undertaken.

Definition 1. [2]
Let a and b be any two integers. If "a divides b", then that there is k, a positive integer such that $b = ka$ and denoted by $a \mid b$. If "a does not divide b", then it is denoted by $a \nmid b$.

Theorem 1. [2] (**The Division Algorithm**)
If a, b are integers with $b > 0$, then there exist "unique" integers q, r such that $a = q \cdot b + r$ with $0 \leq r < b$ where q is called "the quotient and r is called the remainder".

I. Cahit introduced the notion of "cordial labeling" in 1987. For a detailed study on cordial graphs and 3-equitable labeling, one can refer to [1, 2].

Definition 2. [3]
"A labeling (vertex) $f: V \to \{0, 1\}$ induces another labeling (edge) $f*: E \to \{0, 1\}$ defined by $f*(xy) = f(x) - f(y)|$. For $i \in \{0, 1\}$, let $v_f(i)$ and $e_f(i)$ be the number of vertices v and edges e with $f(v) = i$ and $f*(e) = i$, respectively. A graph G is cordial if there exists a labeling (vertex) f such that $v_f(0) - v_f(1) \leq 1$ and $e_f(0) - e_f(1) \leq 1$".

By merging the "divisibility concept in number theory and cordial labeling concept in graph labeling", Varatharajan et al. [6] introduced a new notion called "divisor cordial labeling" and proved various results. The definition of a divisor cordial labeling is given below.

Definition 3. [6]
Let $G = (V, E)$ be the given graph and $f: V(G) \to \{1, 2, \ldots, |V|\}$ be a bijective function. For each edge uv, assign the label "1" if either $f(u) \mid f(v)$ or $f(v) \mid f(u)$ and the label "0" otherwise. Then f is called a "divisor cordial labeling" if $|e_f(0) - e_f(1)| \leq 1$.

Further, in 2019, Sweta Srivastav and Sangeeta Gupta [5] introduced a new variant of graph labeling called a "divisor 3-equitable labeling" which is defined as follows.

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* Correspondence Author
K. Tina Jebi Nivathitha*, Research Scholar, Department of Mathematics, St. Peter's Institute of Higher Education and Research, Avadi, Chennai-600 054, Tamil Nadu, India. Email: niveditha1992@yahoo.com
Dr. N. Srinivasan, Professor and Head, Department of Mathematics, St. Peter's Institute of Higher Education and Research, Avadi, Chennai-600 054, Tamil Nadu, India. Email: sri24455@yahoo.com
Dr. A. Parthiban*, Assistant Professor, Department of Mathematics, School of Chemical Engineering and Physical Sciences, Lovely Professional University, Phagwara-144 411, Punjab, India. Email: mathparthi@gmail.com
Mrs. Sangeeta, Research Scholar, Department of Mathematics, School of Chemical Engineering and Physical Sciences, Lovely Professional University, Phagwara-144 411, Punjab, India. Email: sangeeta1028@gmail.com

Definition 4. [5]

A divisor 3-equitable labeling is a "bijection" $d : V(G) \to \{1, 2, \ldots, n\}$ defined by $d\,(e = xy) = \begin{cases} 1, if\, d(x)|d(y) \text{ or } d(y)|d(x) \\ 2, \quad if\ \frac{d(x)}{d(y)} = 2 \text{ or } \frac{d(y)}{d(x)} = 2 \\ 0, \qquad \text{otherwise} \end{cases}$ such that $|e_d(i) - e_d\,(j)| \leq 1$ for all $0 \leq i, j \leq 2$, where $e_d(i)$ denotes the number edges with label *"i"* under d. A graph which permits a divisor 3-equitable labeling is called a "divisor 3-equitable graph".

In this paper, we prove that the wheel graph W_n, $n \geq 7$ does not admit a divisor 3-equitable labeling.

II. MAIN RESULTS

In this section, first we recall a few important definitions and results concerning a divisor 3-equitable labeling of graphs which are relevant to the study undertaken. We also prove the non-existence of a divisor 3-equitable labeling of wheel graphs.

Theorem 2. [5]

The path P_n is a divisor 3-equitable graph.

Theorem 3. [5]

The cycle C_n is a divisor 3- equitable graph.

Definition 5. [7]

The "complete graph" K_n is a graph in which "any two vertices are adjacent".

Definition 6. [7]

A wheel graph W_n is defined as $W_n = C_{n-1} \wedge K_1$, where C_{n-1} is a cycle on $n-1$ vertices and K_1 is a complete graph.

One can easily obtain the "divisor 3-equitable labeling of W_n, $1 \leq n \leq 6$." One such example is given in Figure 1.

Theorem 4.

A wheel graph W_n does not permit a divisor 3-equitable labeling for all $n \geq 7$.

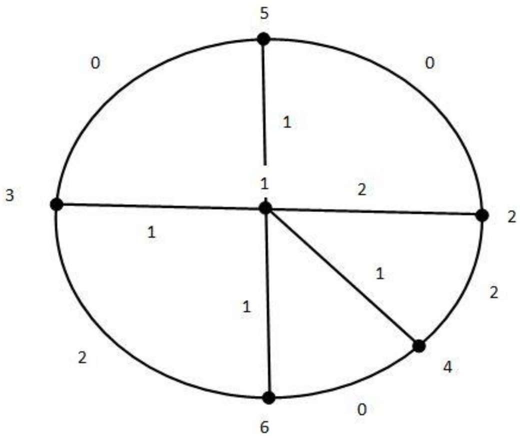

Fig. 1. A divisor 3-equitable labeling of a wheel graph W_6

Proof.

Let $W_n = \{v_0, v_1, v_2, \ldots, v_n\}$ be the given wheel graph on $n \geq 7$ vertices. For the sake of discussion, we take $n = 7$. One can clearly observe that there are $n-1$ vertices of degree 3 on the rim and a vertex v_0 of degree 6 at the centre of a wheel as the central vertex. |Now define a bijection $d : V(W_7) \to \{1, 2, \ldots, 7\}$ defined by $d\,(e = xy) = \begin{cases} 1, if\, d(x)|d(y) \text{ or } d(y)|d(x) \\ 2, \quad if\ \frac{d(x)}{d(y)} = 2 \text{ or } \frac{d(y)}{d(x)} = 2 \\ 0, \qquad otherwise \end{cases}$. We prove this theorem by a method of contradiction. Let's assume that W_7 has a divisor 3-equitable labeling d such that "$|e_d(i) - e_d\,(j)| \leq 1$" for all $0 \leq i, j \leq 2$, where $e_d(i)$ denotes the number edges with label *"i"* under d. Note that $|V(W_7)| = 7$ and $|E(W_7)| = 12$. So as per the definition of a divisor 3-equitable labeling, the number of edges labelled with label either 0 or 1 or 2 in W_7 is at most 4. In fact, the number of edges with labels 0, 1, and 2 is exactly 4. One can also see that the central vertex v_0 can take any value between 1 and 7. Now the following seven cases arise.

Case 1: When $d(v_0) = 1$

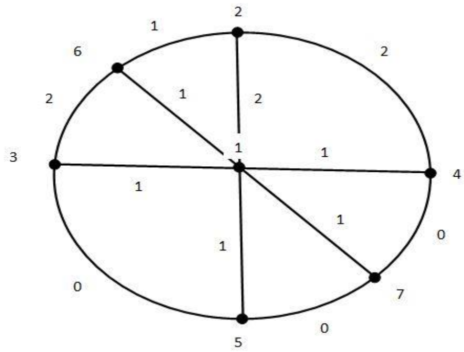

Fig. 2. The central vertex in W_7 is assigned the label "1"

As the central vertex is adjacent to all other vertices and the label 1 divides all other labels, $d(v_0 v_i) = 1$ for all $1 \leq i \leq 6$ except for the edge whose end vertex, say v_2, is labelled with 2 gives $d(v_0 v_2) = 2$. So the number of edges labelled with 1 is at least 5, a contradiction.

Case 2: When $d(v_0) = 2$

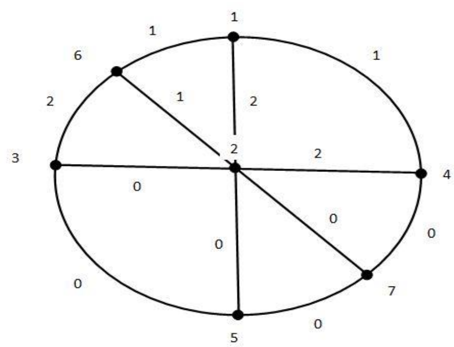

Fig. 3. The central vertex in W_7 is assigned the label "2"

One can note there are three at least five edges (with all possible assignments of numbers 1, 3, 4, 5, 6, 7 on the rim vertices) with label 0, a contradiction.

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Case 3: When $d(v_0) = 3$

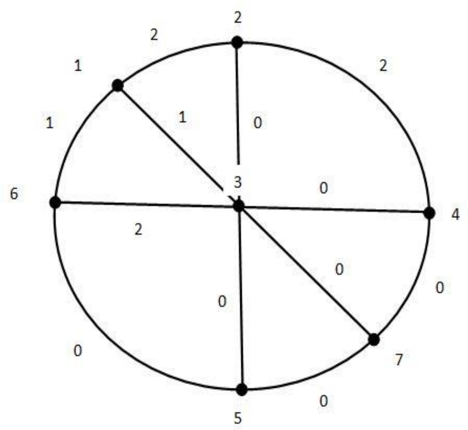

Fig. 4. The central vertex in W_7 is assigned the label "3"

Interestingly there are more than 4 edges with label 0, and just three edges with label 2, a contradiction.

Case 4: When $d(v_0) = 4$

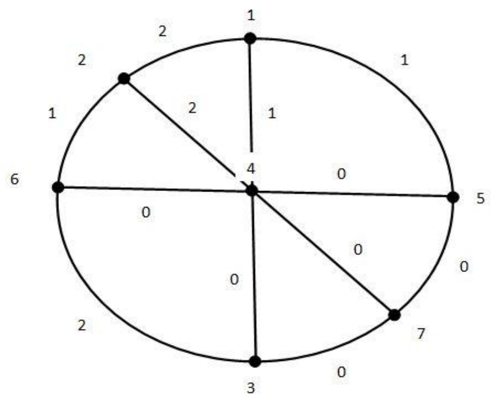

Fig. 5. The central vertex in W_7 is assigned the label "4"

The number of edges with label 2 is again 3 and the number of edges with label 0 is more than 4, a contradiction.

Case 5: When $d(v_0) = 5$

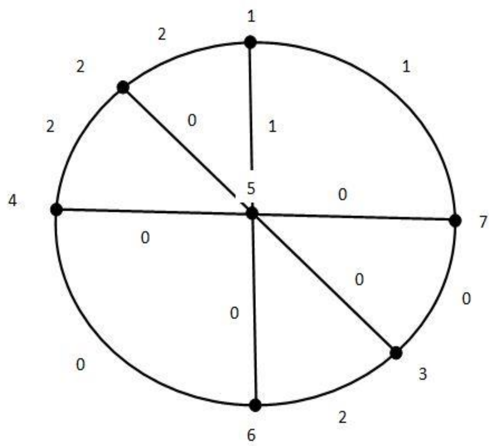

Fig. 6. The central vertex in W_7 is assigned the label "5"

Observe that the edges incident with the central vertex get the label 0 except the one edge whose end vertex is labelled with 1, a contradiction. This is because there at least 5 edges with label 0.

Case 6: When $d(v_0) = 6$

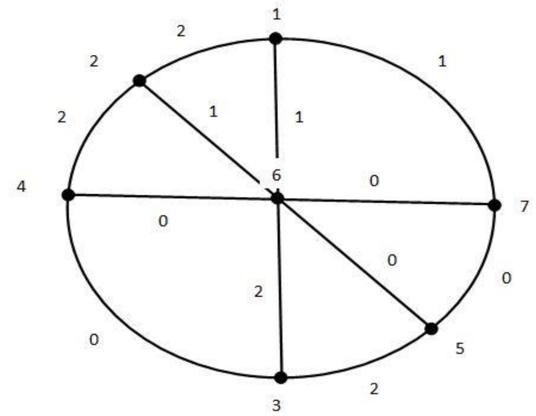

Fig. 7. The central vertex in W_7 is assigned the label "6"

Now again there are only three edges with label 2 and more than four edges with label 0, a contradiction.

Case 7: When $d(v_0) = 7$

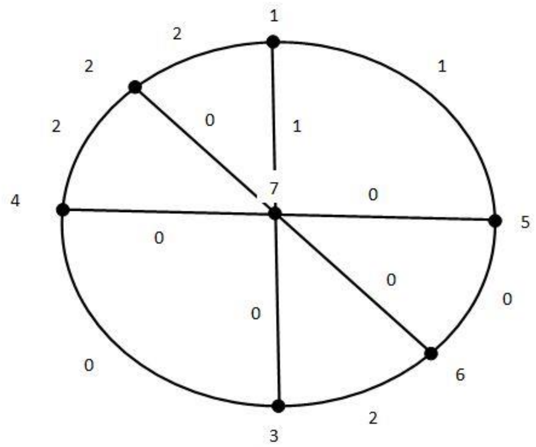

Fig. 8. The central vertex in W_7 is assigned the label "7"

This case is also rejected in a similar fashion that is of case 5.

One can also explore all other possible cases and subcases of assignments of numbers (1 to 7) to the vertices of W_7 in any possible ways (permutation and combination). These cases and subcases are treated and rejected in a similar fashion. Hence W_n, when $n = 7$ does not admit a divisor 3-equitable labeling. A similar argument holds good for any $n \geq 8$.

III. CONCLUSION

The existence and non-existence of a divisor 3-equitable labeling of wheel graphs are established. Investigating divisor 3-equitable labeling of other classes of graphs is still open and this is for future research. One can also explore the exclusive applications of divisor 3-equitable labeling in real life situations.

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AUTHORS PROFILE

Dr. A. Parthiban, born on June 03, 1988. He is working as an Assistant Professor in the Department of Mathematics, School of Chemical Engineering and Physical Sciences, Lovely Professional University, Punjab, India. He did his Under graduation, Post graduation, and Master of Philosophy at Loyola College, Chennai. He pursued his Doctor of Philosophy (Ph.D) at Madras Christian College, Chennai under the supervision of Prof. N. Gnanamalar David. He has also qualified SET. He is the awardee of UGC-SRF/JRF (RGNF) during 2015-2018. He has worked as JRF in a NBHM research project at Velammal Engineering College, Chennai during 2011-2013. His area of specialization is graph theory and his research area includes graph labeling, graph coloring, equitable power domination, algebraic graph theory etc. He has more than 25 publications in reputed journals/conference proceedings (SCIE/Scopus/UGC-CARE/Peer-reviewed). He has attended various international and national conferences/workshops across the world. He has supervised 2 PG and 3 UG students and currently supervising 4 Ph.D. scholars and 2 PG students.

K.Tina Jebi Nivathitha, born on Jan 22nd 1992. She is a Ph.D. research scholar at the Department of Mathematics, St. Peter's Institute of Higher Education and Research, Chennai. She did her M.Phil in St. Peter's Institute of Higher Education and Research, Chennai and her Post Graduation in Jaya College of Arts & Science, Chennai. She completed her under graduation in Guru Nanak College, Chennai. Her area of research is graph labeling.

Prof. N. Srinivasan, is the head of the Department of Mathematics, St. Peter's Institute of Higher Education and Research, Chennai. His area of research includes discrete mathematics, graph theory, etc. He has a very good number of research publications in reputed journals and conference proceedings.

Mrs. Sangeeta has received M.Sc. Degree in Mathematics in 2002. She is currently working as Assistant Professor in Department of Computer Science, Govt. Ranbir College, Sangrur, Punjab, India. She is having 12 years of experience in teaching. Her area of interest is Graph Theory.