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On 2-Uniform Ideals and Direct Summands in N-Groups

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***Abstract*: In this paper, we consider N-groups where N is a zero-symmetric right nearring. We define the notion 2-uniform ideal in N-groups and prove some important fundamental results. Further, we obtain some results interconnecting with i- uniform (i = 0, 1) ideals. Finally, we prove that in a special class of nearrings an ideal is 2-uniform if and only if it is 1-uniform.**

***Keywords*: H-essential ideal, strictly essential ideal, 0-uniform ideal, 1-uniform ideal.**

I. INTRODUCTION

A right nearring is a set N together with two binary operations '+' and '.' such that

(i) (N, +) is a group (not necessarily abelian),

(ii) $a \cdot (b \cdot c) = (a \cdot b) \cdot c$

(iii) $(a + b) \cdot c = a \cdot c + b \cdot c$ for all a, b, c $\in$ N.

In view of (iii), N satisfies the right distributive law, and so it is called a right nearring. It is evident that $0{\cdot}n = 0$ for all n $\in$N. However, $n{\cdot}0$ need not be equal to 0, in general. We denote $N_0 = \{n \in N \mid n \cdot 0 = 0\}$ the zero-symmetric part of the right nearring. If $N = N_0$ then we say that the nearring N is zero-symmetric.

Let (G, +) be a group. By an N-group, we mean a mapping $N \times G \to G$ (the image of (n, g) $\in N \times G$ is denoted by ng), satisfying the following two conditions

(i) $(n + n^1)g = ng + n^1g$ and

(ii) $(nn^1)g = n(n^1g)$ for all g $\in$G and n, $n^1 \in$ N.

We will denote this N-group by $_NG$ or simply by G.

For preliminary definitions and results on nearrings and N-groups we refer Pilz [8] and Satyanarayana - Syam Prasad [31].

An ideal I of G is said to be essential (strictly essential, resp.) in an ideal J of G if it satisfies the condition: K is an ideal (N-subgroup of G, resp.) of G, I $\cap$ K= (0), K $\subseteq$ J imply K = (0). Oswald [6] defined an N-subgroup U of N is strictly uniform if whenever A, B are N-subgroups of N contained in U with A $\cap$ B = {0} then either A = {0} or B = {0}.

Satyanarayana et. al. [28] (i) Let H be an ideal of G. H is said to be 0-uniform if for any ideals K and B with K $\subseteq$ H, B $\subseteq$ H and K $\cap$ B = (0) implies B = (0).

(ii) H is said to be 1-uniform if for any ideal K of G and an N-subgroup B of H with K $\subseteq$ H, B $\subseteq$ H and K $\cap$ B = (0) implies B = (0).

It was mentioned in Satyanarayana et. al. [28] that every 1-uniform ideal is a 0-uniform ideal (in case of zero symmetric near-rings) and also an example is provided for a 0-uniform ideal which is not 1-uniform ideal.

For the notions uniform ideals and essential ideals of modules over associative rings, we refer to authors Anderson and Fuller [1], Camillo and Zelmanowitz [2]. Fleury [4] studied the dualization of these notions. Reddy and Satyanarayana [11] introduced the concept of uniform ideals in N-groups and obtain fundamental results involving finite Goldie dimension. For comprehensive literature on uniform ideals and essential ideals in N-groups, we Satayanaranarana [12, 13, 14, 18], Satyanarayana and Syam Prasad [22, 23, 24, 31].

1.1 Note: Let G be an N-group. Let I and J be two ideals of an N-group G with I + J = G and I $\cap$ J = (0). Then a + b = b + a for a $\in$I and b $\in$J. (This result follows from proposition 2.6(a), Pilz [8]).

For completeness, we include the proof for the following known trivial result. However, the proof for ideals of nearring can be found in Theorem 2.12 of Pilz [8].

1.2 Result: Let G be an N-group, H an ideal of G which is a direct summand (that is, there exists an ideal J of G such that H + J = G and H $\cap$ J = (0)). Then we have the following

(i) n(h + j) = nh + nj for n $\in$N, h $\in$H and j $\in$J.

(ii) If K is an ideal of H, then K is an ideal of G.

Proof: (i) Let n $\in$N, h $\in$ H and j$\in$J. Since H is an ideal of G , n(h + j) – nj$\in$H.

Also since N is zero symmetric, we have nh$\in$H. Therefore n(h + j) – nj – nh$\in$H.

Similarly, we have n(h + j) – nh – nj$\in$J. Since nj + nh = nh + nj, we have n(h + j) – nj – nh$\in$H $\cap$ J = (0). Therefore n(h + j) = nh + nj.

(ii) Suppose K is an ideal of H. We show that K is an ideal of G.

Clearly K is a subgroup of G. Let k $\in$K, g $\in$G. Assume g = h + j for some h$\in$H and j $\in$ J.

Now g + k – g = (h + j) + k – (h + j) = h + j + k – j – h = j + h + k –h – j (by 2.6(a) of Pilz [7]) $\in$ K (since K $\subseteq$ H). Therefore, K is normal in G. Take n $\in$N, g $\in$ G and k $\in$K. Now by Note 1.1, we get

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$n(g + k) – ng = n(h + j + k) – n(h + j) = n (h + k) + nj – nh – nj$, is in K.

1.3 Note: A straight forward verification shows that, if H and K are ideals of G such that $K \subseteq H$ then K is an ideal of H.

II. UNIFORM IDEALS OF N-GROUPS

We introduce the notion of 2-uniform ideals in N-groups.

2.1 Definition: Let G be an N-group. H an ideal of G. H is said to be 2-uniform if for any ideals $(0) \neq K \subseteq H$, $B \subseteq H$ with $K \cap B = (0)$ implies $B = (0)$.

2.2 Note: If we consider G as an ideal of G, then G is 0-uniform if and only if it is 2-uniform.

2.3 Remark: $(Z_6, +)$ is the additive group of integers modulo 6. We define multiplication on Z_6.
as follows.

$$a \cdot b = \begin{cases} 0 & if\ b \in \{0, 2, 3, 4, 5\} \\ a & if\ b = 1 \end{cases}.$$

Then we have the following

(i) $(Z_6, +, .)$ is a nearring but not a ring [because $2(1+1) = 2.2 = 0 \neq 4 = 2+2 = 2.1+2.1$].

(ii) (0), {0, 2, 4}, {0, 3}, Z_6 are subgroups of the additive group $(Z_6, +)$.

(iii) {0}, Z_6 are only ideals of the nearring Z_6.

(iv) {0}, {0,2,4}, {0,3}, Z_6 are N-subgroups of N-group $(Z_6,+)$.

(v) As there are no non-trivial ideals in Z_6 we get that Z_6 is 2-uniform,1-uniform and also 0-uniform.

(vi) As H = {0, 2, 4}, J = {0, 3} are N-subgroups, $H \cap J = (0)$ but $H \neq 0$ and $J \neq 0$ we get that Z_6 cannot be strictly uniform (in the sense of Oswald [6]).

2.4 Proposition: Let I and U be two ideals of G such that I is a direct summand in U. If U is 2-uniform, then I is also 2-uniform.

Proof: Let U be the 2-uniform ideal of G and A be an ideal of G such that $A \subseteq U$. We show that A is 2-uniform. Let K and B be ideals of A such that $K \cap B = (0)$. Since A is a direct summand in U, the ideals K and B of A are also ideals of U. Since U is 2-uniform, we have that B = (0). Therefore, A is 2-uniform.

2.5 Lemma: Let H be an ideal of G. If H is 2-uniform, then H is 0-uniform.

Proof : To show that H is 0-uniform, let K be an ideal of G such that $K \subseteq H$. We have to prove that K is essential in H. To prove this, take B an ideal of G such that $B \subseteq H$ and $K \cap B = (0)$. Since K and B are two ideals of G with $K \subseteq H$, $B \subseteq H$, by the Note 1.3, we get that K and B are ideals of H. Since H is 2-uniform, we have that B = (0). The proof is complete.

2.6 Theorem: Let H be an ideal such that $H \oplus J = G$ for some ideal J of G. Then H is 0-uniform if and only if H is 2-uniform.

Proof: By Lemma, we know that “H is 2-uniform implies H is 0-uniform”, is true.
We have to show the converse, that is “H is 0-uniform imp lies H is 2-uniform”
Take K an ideal of H. To show that $(0) \neq K$ is essential in H, let B be an ideal of H such that $K \cap B = (0)$. We have to prove that B = (0). Since K and B are ideals of H, and H is a direct summand, by Result 1.2 (ii), we have that K and B are also ideals of G. Now since H is 0-uniform, we get that B = (0). The Proof is complete.

2.7 Example: Let $G = D_8 = \{e, r, r^2, r^3, s, sr, sr^2, sr^3\}$, the dihedral group of order 8. Consider the nearring $N = (Z_8, +, \cdot)$ where the operations are define as addition modulo and multiplication modulo 8. Then G is an N-group with the following external operation $*: N \times G \rightarrow G$, as follows.

*	e	r	r^2	r^3	s	sr	sr^2	sr^3
0	e	e	e	e	e	e	e	e
1	e	r	r^2	r^3	s	sr	sr^2	sr^3
2	e	r^2	e	r^2	e	e	e	e
3	e	r^3	r^2	r	s	sr	sr^2	sr^3
4	e	e	e	e	e	e	e	e
5	e	r	r^2	r^3	s	sr	sr^2	sr^3
6	e	r^2	e	r^2	e	e	e	e
7	e	r^3	r^2	r	s	sr	sr^2	sr^3

Then we have the following.

(i) The only non-trivial ideals and non-trivial N-subgroups of G are $I_1 = \langle s, r^2 \rangle$, $I_2 = \langle r \rangle$, $I_3 = \langle r^2 \rangle$, $I_4 = \langle s \rangle$.

(ii) I_1 is not 2-uniform, since I_3, I_4 are ideals of I_1 satisfying $I_3 \subseteq I_1$, $I_4 \subseteq I_1$ with $I_3 \cap I_4 = \{e\}$ but $I_3 \neq \{e\}$ and $I_4 \neq \{e\}$.

(iii) I_1 is not 1-uniform, since $I_3 \subseteq I_1$ as an ideal, $I_4 \subseteq I_1$ considered as an N-subgroup with $I_3 \cap I_4 = \{e\}$ but $I_3 \neq \{e\}$ and $I_4 \neq \{e\}$.

(iv) I_1 is not 0-uniform, when we consider I_3 and I_4 as ideals of G such that $I_3 \cap I_4 = \{e\}$ whereas $I_3 \neq \{e\}$ and $I_4 \neq \{e\}$.

Hence we conclude that I_1 is not 0-uniform, not 1-uniform and not 2-uniform.

III. UNIFORM IDEALS IN A SPECIAL CLASS OF NEARRINGS

In this section we consider a special class of nearrings described in Satyanarayana [13] and obtain some equivalences between i – uniform (i = 1, 2).

3.1 Result: If $H \oplus J = G$ then H is 1-uniform implies H is 2-uniform.

Proof: Let K and B are ideals of H such that $K \cap B = (0)$.
Now we have to show that B = (0).
Since K is an ideal of H and H is a direct summand, by Result 1.2, we have that K is an ideal of
G. Since B is an ideal of H and N is zero symmetric, we have that B is N-subgroup of H and so B is N-subgroup of G. Since H is 1-uniform, we get that B = (0). The Proof is complete.

3.2 Note: In the above Result 3.1 if H is not a direct summand and N is not zero-symmetric, in general, this result does not hold. For this, refer the example 3.6.

3.3 Remark (Satyanarayana [13]:

(i) There exists a class of Near-rings in which every N-subgroup of N is an ideal. Let us call this class as “class-S”.

(ii) Class- S contains the class of all Boolean near-rings (that is, every element of N is an idempotent).

(iii) Class- S contains the class of all strongly regular near-rings (that is, for every $a \in N$, there is an $x \in N$ such that $a = x.a^2$).

3.4 Theorem: Consider a nearing N which belongs to class-S. Consider G = N as the N-subgroup of N. (refer Example 1.18(a) of Pilz [8]). Let H be an ideal of G.
(i) H is 2-uniform implies H is 1-uniform
(ii) H is 1-uniform implies H is 2-uniform
(iii) H is 2-uniform $\Leftrightarrow$ H is 1-uniform

Proof: (i) Suppose H is 2-uniform. Let K be an ideal of G such that K $\subseteq$H and Abe an N-subgroup in H such that K $\cap$ A =(0), Now we have to show that A = (0). Since K is ideal of G and K $\subseteq$H, we have that K is ideal of H. Since A is an N-subgroup of H, we have that A is an N-subgroup of G. Since N is in class-S, we have that A is an ideal of G. Now A is an ideal of G and A $\subseteq$H, we have that A is an ideal of H. Since H is 2-uniform we get that A= 0.
The proof is complete for (i).
(ii) Suppose H is 1-uniform. To show that His 2-uniform suppose K and A are ideals of H such that K $\cap$ A =(0). We have to prove that A = (0). Since K and A are ideals of H and N is a zero symmetric nearing, we have that K and A are N-subgroups of H. Then K is also N-subgroup of G because H is an N-subgroup of G. Since N belongs to class-S, we have that K is an ideal of G.
Since ideals of G that are contained in H are also ideals of H, we conclude that K is an ideal of H.
Now we proved in the above that K is an ideal of H and A is an N-subgroup of H.
Since H is 1-uniform we get that A = (0).
The proof is complete for (ii).
(iii) follows from (i) and (ii).

3.5 Note: If N is zero-symmetric, in general, Theorem 3.4 does not hold. For this refer the following example.

3.6 Example: (Z_{12}, +), the additive group of integers modulo 12. We define multiplication on Z_{12} as follows.
a * b = a for all a, b $\in Z_{12}$. Then we have the following.

(i) N = (Z_{12}, +, .) is a nearring but not a ring. Consider ${}_NN$ (the N-group over the nearing N).
(ii) {<0>, <2>, <3>, <4>, <6>} is the set of ideals of ${}_NN$.
(iii) The only N-subgroup is Z_{12}.
(iv) The ideal H = <2> is 1-uniform in Z_{12} over itself.
(v) Since <4> and <6> are ideals of Z_{12} and <4>$\subseteq$<2>, <6>$\subseteq$<2> with <4>$\cap$<6> = {0}, and neither <4> = <0> nor <6> = (0), we have that H = <2> is not 2-uniform.

IV. CONCLUSIONS

In this paper, we have considered module over a nearring (N-group) which is a generalization of module over a ring. The concepts like essential submodule and uniform submodule of module over rings are well known. The notion essential submodule is analogue of the dense subset in a topological space. In case of N-groups, it is interesting that substructures are not just submodules but ideals, N-subgroups, etc. Hence it is possible to define and characterize various essential ideals and uniform ideals in N-groups. In the recent published papers, we have considered i – uniform (i = 0,1) and obtained fundamental results, where as in the present paper we have considered 2-uniform ideals and equivalences among other types. As an extension of this, we plan to obtain theorems on finite Goldie conditions in N-groups corresponding to i-uniform ideals (i = 0,1,2).

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REFERENCES

1. Anderson F.W. and Fuller K.R. “Rings and Categories of Modules”, Springer–Verlag, New York, 1974.
2. Camillo V. and Zelmanowitz J. “On the dimension of a sum of modules”, Communications in Algebra, 6:4(1978), 345-352.
3. Camillo V. and Zelmanowitz J. “Dimension modules”, Pacific Jr. Math., 91:2(1980), 249- 261.
4. Fleury P. "A Note on Dualizing Goldie Dimension", Canadian. Math. Bull. 17(4) 1974.
5. Hamsa Nayak, Syam Prasad K. and B. S. Kedukodi, θΓ-N-Group, MATEMATICKIVESNIK, 70, 1 , 64-78. (1). 2018.
6. Meldrum J. D. P. “Near-rings and their links with groups”, Pitman Advanced Publishing Program, Boston-London-Melbourne 1985.
7. Oswald A. “Near-rings in which every N-group is Principal”, Proc. London Math. Soc. (3) 28 (1974), 67-88.
8. Pilz G., Nearrings, North Holland, 1983.
9. Reddy Y.V., and Satyanarayana Bh. "The f-prime Radical in Near-rings", Indian J. Pure & Appl. Math. 17 (1986) 327-330.
10. Reddy Y.V., and Satyanarayana Bh. "A Note on Modules”, Proc. Japan Academy 63, Series 6(1987) 208-211.
11. Reddy Y.V., and Satyanarayana Bh. "A Note on N-groups", Indian J. Pure & Appl. Math. 19 (1988) 842- 845.
12. Satyanarayana Bh. "Tertiary Decomposition in Noetherian N-groups", Communications in Algebra, 10 (1982) 1951 – 1963.
13. Satyanarayana Bh. "Primary Decomposition in Noetherian Nearrings", Indian J. Pure & Appl. Math. 15 (1984) 127-130.
14. Satyanarayana Bh. “Contributions to Nearring Theory”, Doctoral Dissertation, Acharya Nagarjuna University, 1984.
15. Satyanarayana Bh. "A Note on E-direct and S-inverse systems", Proc. of the Japan Academy, 64-A (1988) 292-295.
16. Satyanarayana Bh. "The Injective Hull of a Module with FGD", Indian J. Pure & Appl. Math. 20 (1989) 874-883.
17. Satyanarayana Bh. “Modules with Finite Spanning Dimension”, J. Austral, Math. Society. (Series A) 57 (1994) 170-178.
18. Satyanarayana Bh. "On Modules with Finite Goldie Dimension" J. Ramanujan Math. Society. 5 (1990) 61-75.
19. Satyanarayana Bh. "On Essential E-irreducible submodules", Proc. 4th Ramanujan symposium on Algebra and its Applications, University of Madras Feb 1-3 (1995), pp 127-129.
20. Satyanarayana Bh. “Contributions to Near-ring Theory”, VDM Verlag Dr Muller, Germany, 2010, (ISBN 978-3-639-22417-7).
21. Satyanarayana Bh. and Koteswara Rao G. "On a Class of Modules and N-groups", Journal of Indian Math. Society. 59 (1993) 39 – 44.
22. Satyanarayana Bh. and Syam Prasad K. "A Result on E-direct systems in N-groups", Indian J. Pure & Appl. Math. 29 (1998) 285 – 287.
23. Satyanarayana Bh. and Syam Prasad K. "On Direct and Inverse Systems in N-groups", Indian J. Math. (BN Prasad Birth Commemoration Volume) 42 (2000) 183-192.
24. Satyanarayana Bh. and Syam Prasad K. "Linearly Independent Elements in N-groups with Finite Goldie Dimension", Bulletin of the Korean Mathematical Society, 42 (2005), No. 3, pp 433-441.
25. Satyanarayana Bh. and Syam Prasad K. "On Finite Goldie Dimension of M_n(N)-group N^n" in the book ‘Nearrings and Nearfields (Editors: Hubert Kiechle, Alexander Kreuzer and MommeJohs Thomsen) (2005), (Proc. 18th International Conference on Nearrings and Nearfields, UniversitatBundeswar, Hamburg, Germany July 27-Aug 03, 2003) Springer Verlag, Netherlands, 2005, pp 301-310.
26. Satyanarayana Bh., Syam Prasad K. “Discrete Mathematics and Graph Theory”, Prentice Hall India Learning Ltd. (2014) ISBN 978-81-203-4948-3 (2nd Edition).
27. Satyanarayana Bh., Syam Prasad K. and Nagaraju D. “A Theorem on Modules with finite Goldie dimension” Soochow Journal of Mathematics, 32 (2) 311-315 (2006).
28. SatyanarayanaBh., Syam Prasad K., and Venugopala Rao P., and Mallikarjuna B. ‘A Note dimensions in N-Groups’, Italian Journal of Pure and Applied Math., accepted, 2019.

29. Satyanarayana Bh., Syam Prasad K., and Venugopala Rao P., and Mallikarjuna B. '1-Uniform ideals in N-Groups', Bulletin of Pure and Applied Sciences Section E-Math & Stat. Vol. 38E (2) 586-591, 2019.
30. Syam Prasad K. and Satyanarayana Bh. "Finite Dimension in N-Groups and Fuzzy Ideals of Gamma Nearrings", VDM Verlag, Germany, 2011. ISBN: 978-3-639-36838-3.
31. Satyanarayana Bh. and Syam Prasad K., Near Rings, Fuzzy Ideals, and Graph Theory, Chapman and Hall, (2013), Taylor and Francis Group (London, New York), ISBN 13: 9781439873106.
32. Syam Prasad K. "Contributions to Nearring Theory II", Doctoral Dissertation, Acharya Nagarjuna University, India, 2000.
33. Syam Prasad K., Srinivas K.B., Harikrishnan P.K., and Satyanarayana Bh., Nearrings, Nearfields and Related Topics, World Scientific (Singapore), (2017) ISBN: 978-981-3207-35-6.

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