

# CFOA-Based New Structure of Fractional Order Inverse Filters



Manoj Kumar, D. R. Bhaskar, Pragati Kumar

**Abstract**— This paper presents a CFOA-based fractional order inverse filter (FOIF) structure. Proposed structure utilizes two current feedback operational amplifiers (CFOAs), two fractional capacitors (FCs) and two resistors to design FOIFs to realize fractional order inverse low pass (FOILP), fractional order inverse high pass (FOIHP) and fractional order inverse band pass (FOIBP) responses. The proposed configuration of FOIFs is simulated in PSPICE using AD844 type CFOAs. Stability analysis and MATLAB simulations are also carried out to authenticate the theoretical propositions.

**Index Terms**— Fractional order circuits, Fractional capacitor, Fractional order inverse filter, CFOA.

## I. INTRODUCTION

During last two decades, fractional order analog signal processing has achieved a lot of popularities by the research works in new emerging field of fractional order analog circuits [1-10]. Consequently, this has become an interesting research area, due to its inter-disciplinary applications in the field of science and engineering such as in biomedical, control systems, analog filters [1-9], analog inverse filters [8-9] and oscillators. Due to its design tractability and tunability (because of added degree of freedom delivered by the fractional parameter ( $\alpha$ )) [7], the fractional order filter (FOF) has become an important part of the analog signal processing.

FOFs are about one decade old topic, which commence with introduction of first order [1] and second order [2] filters in fractional domain in 2008 and 2009 respectively by Elwakil et al. Afterward many research articles are proposed in the field of FOF employing various analog active building blocks like Op-amps [2-5], CCIIs[6], CFOAs[7] and other active devices.

The fractional order inverse filter is a new born research area of fractional order analog circuits, which is originated with research article proposed by authors of [8] in 2018. In this paper, authors have introduced two configurations of FOIFs using an op-amp to realize FOILP, FOIBP and FOIHP responses. In [9], authors have proposed FOIFs using two CFOAs to realize FOILP, FOIBP and FOIHP employing 2/3/4 FCs respectively.

Therefore, with this article, a new structure of FOIF using CFOAs has been introduced that can realize FOILP, FOIHP and FOIBP responses from the same configuration using two fractional capacitors, two resistors and two CFOAs. To justify the workability of proposed structure of FOIFs, we have provided the SPICE simulation results using AD844 type CFOAs as well as MATLAB simulation results.

## II. THE PROPOSED MULTIFUNCTION FOIF STRUCTURE

A new generalized configuration of multifunction FOIFs is shown in Fig. 1. The circuit analysis of proposed FOIF assuming ideal CFOA ( $I_y = 0, V_x = V_y, I_z = I_x,$  and  $V_w = V_z$ ) yields a transfer function as:

$$\frac{V_o}{V_{in}} = \frac{y_4(y_1 + y_2 + y_3) + y_2(y_1 + y_3)}{y_2 y_3} \quad (1)$$

where  $y_i, i = 1-5$  are the admittances of circuit.

The fractional order transfer functions of FOIF have been obtained with proper selection of branch admittance(s) as follows:

(i) FOILP filter: by choosing admittances as

$$y_1 = s^\alpha C_1, y_4 = s^\alpha C_4, y_2 = \frac{1}{R_2} \text{ and } y_3 = \frac{1}{R_3}, \text{ the}$$

resulting transfer function becomes

$$\frac{V_o(s)}{V_{in}(s)} = \frac{1}{\frac{1}{C_1 C_4 R_2 R_3} (s^{2\alpha} + s^\alpha \{ \frac{1}{R_2 C_1} + \frac{1}{R_3 C_1} + \frac{1}{R_2 C_4} \} + \frac{1}{C_1 C_4 R_2 R_3})} \quad (2)$$

(ii) FOIBP filter: by choosing admittances as

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$y_3 = s^\alpha C_3, y_4 = s^\alpha C_4, y_1 = \frac{1}{R_1}$  and  $y_2 = \frac{1}{R_2}$ , the resulting transfer function becomes

$$\frac{V_0(s)}{V_{in}(s)} = \frac{1}{\frac{(s^\alpha)}{R_2 C_4}} \frac{1}{s^{2\alpha} + s^\alpha \left\{ \frac{1}{R_1 C_3} + \frac{1}{R_2 C_3} + \frac{1}{R_2 C_4} \right\} + \frac{1}{C_3 C_4 R_1 R_2}} \quad (3)$$

(iii) FOIHP filter: by choosing admittances as

$y_2 = s^\alpha C_2, y_3 = s^\alpha C_3, y_1 = \frac{1}{R_1}$  and  $y_4 = \frac{1}{R_4}$ , the resulting transfer function becomes

$$\frac{V_0(s)}{V_{in}(s)} = \frac{1}{s^{2\alpha}} \frac{1}{s^{2\alpha} + s^\alpha \left\{ \frac{1}{R_4 C_3} + \frac{1}{R_4 C_2} + \frac{1}{R_1 C_3} \right\} + \frac{1}{C_2 C_3 R_1 R_4}} \quad (4)$$

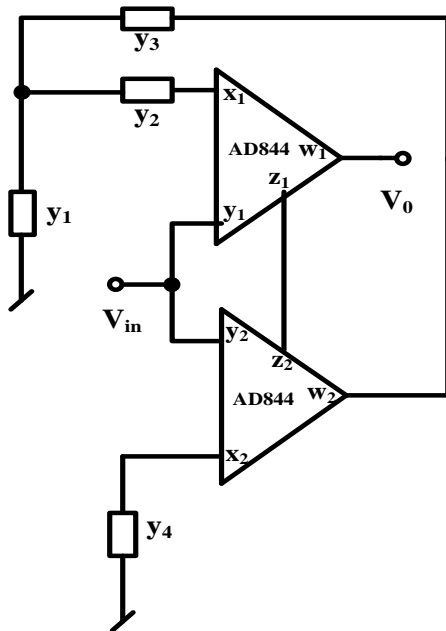


Fig. 1. The proposed structure of FOIFs

III. STABILITY ANALYSIS OF FOIFs:

The stability of FOIFs can be analyzed using method suggested in paper [8, 10]. Since, FOIF is non-integer order system, for stability analysis, system is transformed into w-domain from s-domain assuming  $S = W^m$  and  $\alpha = p/m$  ('p' and 'm' be integers) to make integer order system in w-domain. Now s-plane ( $\theta_s = \pm\pi$ ) is mapped as  $\theta_w \leq \left| \frac{\pi}{10} \right|$  in w-plane (m = 10), which is shown in Fig. 2.

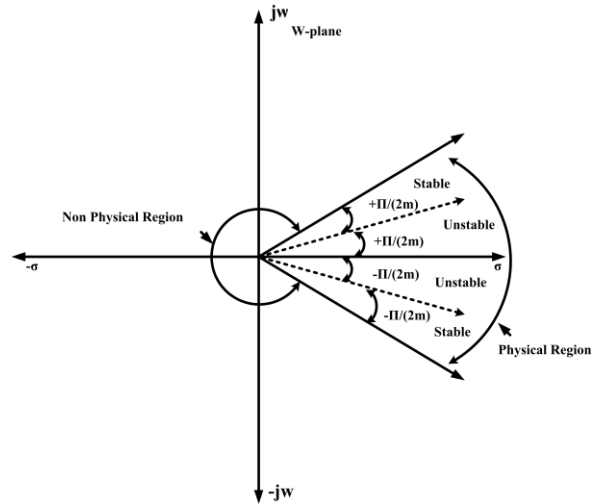


Fig.2 w-plane

With this transformation, the characteristic equation (CE) of FOIF represented by following equation in s-domain

$$s^{2\alpha} + bs^\alpha + c = 0, \quad 0 < \alpha < 1 \quad (5)$$

can be represented in w-domain as

$$w^{2p} + bw^p + c = 0 \quad (6)$$

From equation (6), roots of CE may be calculated in w-plane and if all roots of CE lies in stable region of w-plane (as shown in Fig.2), then system be stable otherwise unstable. The stability of proposed FOIFs is checked and it be stable. For exemplary, we have shown the plot of roots of CE of proposed FOIBP for  $\alpha = 0.9$  and  $m = 10$  in Fig.3. It is observed that minimum root angles are  $\pm 20.00^\circ$  (be greater than  $\pm 9^\circ$ ) for FOIBP.

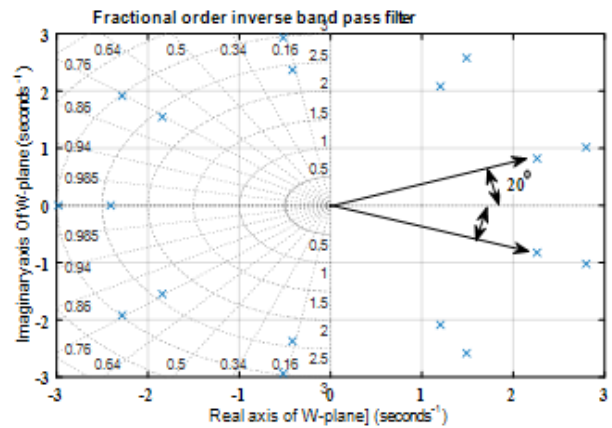


Fig. 3 Plot of roots of CE of FOIBP in w-Plane  $\alpha = 0.9$

IV. PSPICE SIMULATION AND MATLAB RESULTS OF NEW PROPOSED FOIFs:

The PSPICE simulation and MATLAB results of proposed FOIFs and their integer order inverse filters are shown in Fig. 4. Fractional capacitors of order 0.9 and 0.8 having value  $0.0995 \mu F / s^{(1-\alpha)}$  are used in simulation of FOIFs. FC is designed using VALSA method [8, 11] and its equivalent RC ladder circuit is shown in Fig. 5.

The component values of RC ladder circuit of FC of order  $\alpha = 0.9$  are provided in Table.I and component values, which are used to design FOIFs of Fig.1 are given in Table.II. The comparative results of cut off /peak frequency of derived FOIFs and conventional inverse filters are shown in Table III, while stop band attenuations are given in Table IV. PSPICE simulation and MATLAB results of presented FOIFs are almost matched, which confirm the functionality of proposed FOIFs.

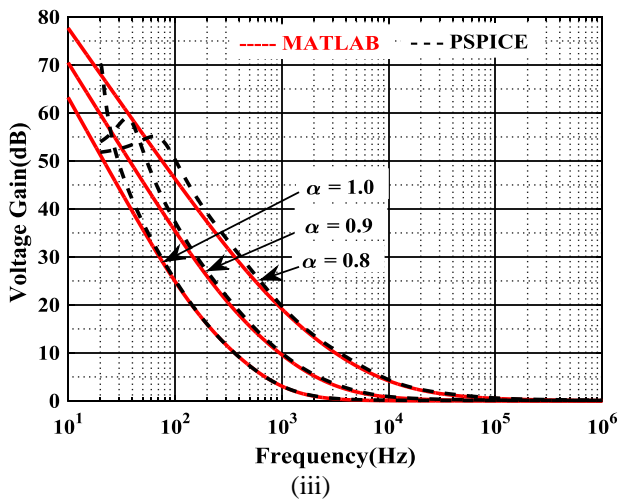
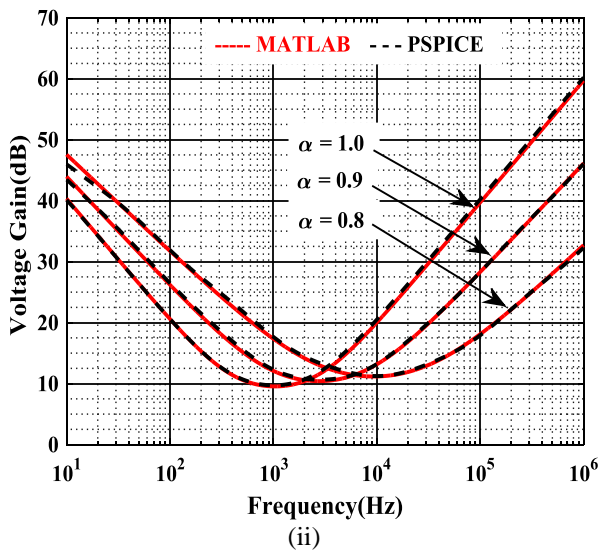
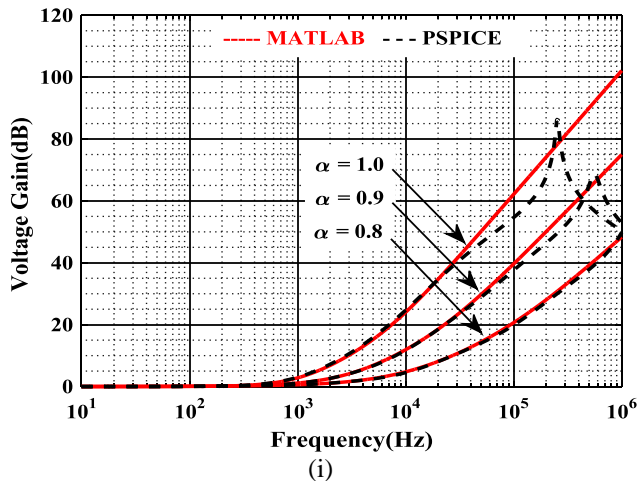


Fig. 4. Frequency response of FOIFs (i) FOILP (ii) FOIBP (iii) FOIHP

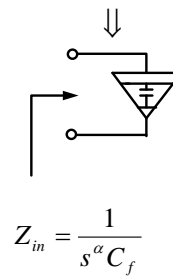
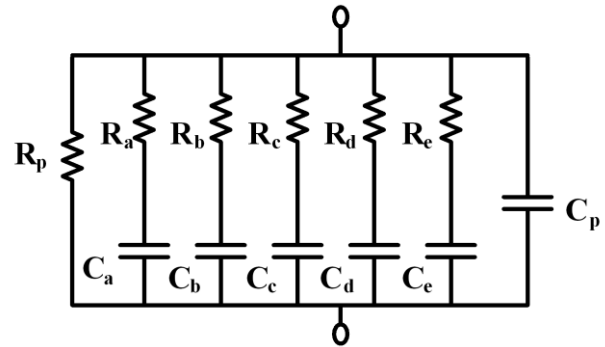


Fig.. 5 R-C Ladder circuits of FC

TABLE I. Element values to realize FC ( $\alpha = 0.9$ )

$C_f = 0.0955 \mu\text{F/s}^{(1-\alpha)}$	
$R_a = 676.342 \text{ k}\Omega$	$C_a = 14.79 \text{ nF}$
$R_b = 69.65 \text{ k}\Omega$	$C_b = 11.49 \text{ nF}$
$R_c = 7.17 \text{ k}\Omega$	$C_c = 8.92 \text{ nF}$
$R_d = 0.74 \text{ k}\Omega$	$C_d = 6.93 \text{ nF}$
$R_e = 0.08 \text{ k}\Omega$	$C_e = 5.38 \text{ nF}$
$R_p = 5890.9 \text{ k}\Omega$	$C_p = 18.736 \text{ nF}$

TABLE II. Component values used in design of FOIFs.

FOIFs of Fig. 1		
Components		
FOILP	FOIHP	FOIBP
$R_2 = 570\Omega$	$R_1 = 4200\Omega$	$R_1 = 1540\Omega$
$R_3 = 570 \Omega$	$R_4 = 4200 \Omega$	$R_2 = 1540 \Omega$
$C_1 = 0.0995 \mu\text{F}$	$C_2 = 0.0995 \mu\text{F}$	$C_3 = 0.0995 \mu\text{F}$
$C_4 = 0.0995 \mu\text{F}$	$C_3 = 0.0995 \mu\text{F}$	$C_4 = 0.0995 \mu\text{F}$

TABLE III. Cut off / Peak frequency (Hz) of FOIFs

Order	2.0	1.8	1.6
Type of filter			
FOILPF (MATLAB)	1043	2245	5898
FOILPF (PSPICE)	1000	2239	6310
FOIBPF (MATLAB)	1038	2697	9903
FOIBPF (PSPICE)	1000	2630	9550
FOIHPF (MATLAB)	1017	3342	14560
FOIHPF (PSPICE)	1026	3639	15170

TABLE IV. Pass attenuation (dB/decade) of proposed FOIFs

Order Type of filter	2.0	1.8	1.6
FOILPF (MATLAB)	38.07	35.37	28.21
FOILPF (PSPICE)	30.00	26.85	15.04
FOIBPF (MATLAB)	19.70	17.82	14.59
FOIBPF (PSPICE)	19.71	17.75	14.64
FOIHPF (MATLAB)	38.44	35.08	31.44
FOIHPF (PSPICE)	40.10	32.29	30.25

V. CONCLUSIONS

A new FOIF structure is presented which can realize FOILP, FOIBP and FOIHP responses from the same topology through proper selection of admittances. Through MATLAB and PSPICE simulations, workability of proposed structure has been tested. The stability analysis is also performed for FOIFs

REFERENCES

1. A. G. Radwan, A. M. Soliman, and A. S. Elwakil, "First-order filters generalized to the fractional domain", *Journal of Circuits, Systems, and Computers*, 2008, 17(01), pp. 55–66.
2. A. G. Radwan, A. S. Elwakil, and A. M. Soliman, "On the generalization of second-order filters to the fractional-order domain," *Journal of Circuits, Systems and Computers*, 2009, 18(2), pp. 361–386.
3. M. C. Tripathy, K. Biswas, and S. Sen, "A design example of a fractional-order Kerwin-Huelsman-Newcomb biquad filter with two fractional capacitors of different order," *Circuits, Systems and Signal Processing*, 2013, 32(4), pp. 1523-1536.
4. A. Soltan, A. G. Radwan, and A. M. Soliman, "Fractional order filters with two fractional elements of dependent orders," *Microelectronics Journal*, 2012, 43(11), pp. 818–827.
5. T. J. Freeborn, B. Maundy, and A. S. Elwakil, "Fractional-step Tow-Thomas biquad filters," *Nonlinear Theory and Its Applications, IEICE*, 2012, 3(3), pp. 357–374.
6. A. Soltan, A. G. Radwan, and A. M. Soliman, "CCII based fractional filters of different orders," *Journal of advanced research*, 2014, 5(2), pp. 157–164.
7. L. A. Said, A. G. Radwan, A. H. Madian and A. M. Soliman, "Fractional-order inverting and non-inverting filters based on CFOA," *In Telecommunications and Signal Processing (TSP), IEEE 39<sup>th</sup> International Conference 2016*, pp. 599-602.
8. D. R. Bhaskar, M. Kumar, and P. Kumar, "Fractional order inverse filters using operational amplifier," *Analog Integrated Circuits Signal Process*, 2018, 97(1), pp. 149-158.
9. E. M. Hamed, L. A. Said, H. M. Ahmed and A. G. Radwan, "On the Approximations of CFOA-Based Fractional-Order Inverse Filters," *Circuits, Systems and Signal Processing*, 2019, pp. 1-28.
10. A. G. Radwan, A. M. Soliman, A. S. Elwakil and A. Sedeek, "On the stability of linear systems with fractional-order elements," *Chaos, solutions fractals*, 2009, 40(5), pp. 2317-2328.
11. J. Valsa, P. Dvorak, and M. Friedl, "Network model of the CPE," *Radioengineering*, 2011, 20(3), pp. 619–626.

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