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Geodetic Global Domination in the Join of Two Graphs

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Abstract: *A set S of vertices in a connected graph $G = (V, E)$ is called a geodetic set if every vertex not in S lies on a shortest path between two vertices from S. A set D of vertices in G is called a dominating set of G if every vertex not in D has at least one neighbor in D. A set S is called a geodetic global dominating set of G if S is both geodetic and global dominating set of G. The geodetic global dominating number is the minimum cardinality of a geodetic global dominating set in G. In this paper we determine the geodetic global domination number of the join of two graphs.*

Keywords : *Geodetic set, Dominating set, Geodetic Domination, Geodetic Global Domination.*

I. INTRODUCTION

By a graph *G=(V,E)* we mean a finite, connected, undirected graph with neither loops nor multiple edges. The *order* $|V|$ and *size* $|E|$ of G and denoted by p and q respectively. For graph theoretic terminology we refer to west[9].The *open neighborhood* of any vertex v in G is $N(v) = \{x: xv \in E(G)\}$ and *closed neighborhood* of a vertex v in G is $N[v] = N(v) \cup \{v\}$. The *degree* of a vertex in the graph G is denoted by *deg(v)* and the maximum degree (minimum degree) in the graph G is denoted by $\Delta(G)(\delta(G))$. For a set $S \subseteq V(G)$ the open (closed) neighborhood $N(S)(N[S])$ in G is defined as $N(S) = \cup_{v\in S} N(v)(N[S] = \cup_{v\in S} N[v])$.

If G is a connected graph the *distance* $d(x, y)$ is the length of a shortest *x-y* path in G. The *diameter* is defined by $diam(G) = max_{x,y\in V(G)} d(x, y)$. Two vertics u and v are said to be *antipodal vertices* if $d(u, v) = diam(G)$. If $e = \{u, v\}$ is an edge of a graph G with $deg(u) = 1$ and $deg(v) > 1$, then we call e a pendant edge, u a pendant vertex and v a support vertex. A vertex v of G is said to be an *extreme vertex* of the subgraph induced by its neighborhood is complete. The set of all extreme vertices is denoted by $Ext(G)$. A vertex v is said to be *full vertex* if v is adjacent to all other vertices in G, that is, *deg(v)= p-1*. The set of all full vertices is denoted by $Fx(G)$. The *girth* of a graph G is the length of a shortest cycle contained in G and is denoted by $c(G)$. An acyclic connected graph is called a *tree*. An *x-y* path of length $d(x, y)$ is called *geodesic*. A vertex v is said to lie on an geodesic P if v is an internal vertex of P. The closed interval consists of x, y and all vertices lying on some *x-y* geodesic of G and for a non empty set $S \subseteq V(G), I[S] = \cup_{x,y\in S} I[x, y]$.

A set $S \subseteq V(G)$ in a connected graph is a *geodetic set* of G if $I[S] = V(G)$.The *geodetic number* of G, denoted by $g(G)$, is the minimum cardinality of a geodetic set of G. The geodetic number of a disconnected graph is the sum of the geodetic number of its components. A geodetic set of cardinality $g(G)$ is called $g(G)$ -set. Various concepts inspired by geodetic sets are introduced in [1, 2].

A set $S \subseteq V(G)$ in a graph G is a *dominating set* of G if for every vertex v in V-S, there exists a vertex $u \in S$ such that v is adjacent to u. The *domination number* of G, denoted by $\gamma(G)$, is the minimum cardinality of a dominating set of G[5].

The complements $\bar{G} = (\bar{V}, \bar{E})$ of $G = (V, E)$ is the graph with the vertex set $\bar{V} = \{\bar{v}: v \in V\}$ and $\bar{E} = \{(\bar{u}, \bar{v}): \bar{u}, \bar{v} \in \bar{V}, u \neq v$ and $(u, v) \notin E\}$. The domination number of $\bar{G}$ is denoted by $\gamma(\bar{G})$.

A set $S \subseteq V(G)$ is called a *global dominating set* of $G = (V, E)$ if it is a dominating set of both G and $\bar{G}$8. The *global domination number* of G, denoted by $\bar{\gamma}(G)$, is the minimum cardinality of a global dominating set of G.

In [7] , authors studied geodetic global domination in graphs which is defined as follows:

Definition 1.1

Let $G = (V, E)$ be a connected graph. A subset $S \subseteq V(G)$ is called *geodetic global dominating set* of G if S is both geodetic and global dominating set of G. The *geodetic global domination number* denoted by $\bar{\gamma}_g(G)$ is the minimum cardinality of a geodetic global dominating set of G and the geodetic global dominating set with cardinality $\bar{\gamma}_g(G)$ is called the $\bar{\gamma}_g$-set of G or $\bar{\gamma}_g(G)$-set.

In this paper we mainly consider the geodetic global domination number in the join of two graphs. Let A and B be sets which are not necessarily disjoint. The *disjoint union* of A and B, denoted by $A \cup B$, is the set obtained by taking the union A and B treating each element in A as dijoint from each element in B. The *union* $G_1 \cup G_2$ of graphs G_1 and G_2 with disjoint vertices $V(G_1)$ and $V(G_2)$, respectively.

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II. SOME USEFUL KNOWN RESULTS

Theorem 2.1 [7] Let G be a connected graph. Then

(i) $Ext(G) \subseteq S$ for all geodetic global dominating sets S in G.

(ii) $Fx(G) \subseteq S$ for all geodetic global dominating sets S in G.

Theorem 2.2[7] For the complete graph $K_p, (p \geq 2), \bar{\gamma}_g(K_p) = p$.

Theorem 2.3[7] Let G be a connected graph of order p. Then, $\bar{\gamma}_g(G) = 2$ if and only if $G = K_2$ or there exists a geodetic set $S = \{u, v\}$ such that $d(u, v) = 3$.

Theorem 2.4[7]Let G be a connected graph of order p. Then, $\bar{\gamma}_g(G) = p$ if and only if G contains only the extreme and full vertices.

III. JOIN OF GRAPHS

The *join* $G + H$ of two graphs G and H is the graph with vertex set $V(G + H) = V(G) \cup V(H)$ and the edge set $E(G + H) = E(G) \cup E(H) \cup \{uv : u \in V(G), v \in V(H)\}$.

Theorem 3.1

If G_1 and G_2 are the complete graphs of order p_1 and p_2, respectively. Then $\bar{\gamma}_g(G_1 + G_2) = p_1 + p_2 = \bar{\gamma}_g(G_1) + \bar{\gamma}_g(G_2)$.

Proof. Let G_1 and G_2 be the complete graphs of order p_1 and p_2 respectively. By, Theorem 2.2, $\bar{\gamma}_g(G_1) = p_1$ and $\bar{\gamma}_g(G_2) = p_2$. Since the join of two complete graphs is again a complete graph, $G_1 + G_2$ is of order $p_1 + p_2$ in which every vertices of $G_1 + G_2$ are extreme. Hence by, Theorem 2.4,$\bar{\gamma}_g(G_1 + G_2) = p_1 + p_2 = \bar{\gamma}_g(G_1) + \bar{\gamma}_g(G_2)$.

Theorem 3.2

For non-empty graphs G and H, $\bar{\gamma}_g(G + H) = 2$ if and only if $G \simeq K_1$ and $H \simeq K_1$.

Proof. Let G and H be non-empty graphs. Assume $\bar{\gamma}_g(G + H) = 2$. Then, by Theorem 2.3, $G + H \simeq K_2$ or there exists a geodetic set $S = \{u, v\}$ in $G + H$ such that $d_{G+H}(u, v) = 3$. If $d_{G+H}(u, v) = 3$, then either G or H must be empty. Therefore, the only possibility is $G + H \simeq K_2$, implies that $G \simeq K_1$ and $H \simeq K_1$. The converse part is obvious.

Theorem 3.3

For non-empty graphs G and H,$\bar{\gamma}_g(G + H) = 3$ if and only if $G + H \simeq K_3$ or $G + H \simeq \bar{K}_2 + H_1$, where $\delta(H_1) = 0$.

Proof. If $G + H \simeq K_3$. Then by Theorem 2.2, $\bar{\gamma}_g(G + H) = 3$. Suppose $G + H \simeq \bar{K}_2 + H_1$, where $\delta(H_1) = 0$. Let $x, y \in \bar{K}_2$ and let $v \in V(H_1)$ such that $\delta(v) = 0$. Then, in $G + H_1$ every vertex in $V(H_1)$ lies in a xy geodesic and also dominated by x and y. Since, $\delta(v) = 0, \{v\}$ is a dominating set in $\bar{H}_1$. This mean that $S = \{x, y, w\}$ is a geodetic global dominating set in $\bar{K}_2 + H_1$ and so $\bar{\gamma}_g(\bar{K}_2 + H_1) \leq |S| = 3$. Since $\bar{K}_2 + H_1$ contains at least three vertices, by Theorem 3.2, $\bar{\gamma}_g(\bar{K}_2 + H_1) \geq 3$. It follows that $\bar{\gamma}_g(G + H) = \bar{\gamma}_g(\bar{K}_2 + H_1) = 3$.

Conversely, assume $\bar{\gamma}_g(G + H) = 3$. Let $S = \{x, y, z\}$ be a minimum geodetic global dominating set in $G + H$. If $G + H \simeq K_3$, then we are done. So, suppose $G + H \not\simeq K_3$. We show that $G + H \simeq \bar{K}_2 + H_1$, where H_1 contain atleast one isolated vertex. Since $|S| = 3$ and $G + H \not\simeq K_3$, S have atleast two non-adjacent vertices in $G + H$, say x, y. By the definition of join of G and H, x, y are either in G or in H. Without loss of generality, assume that x, y in $V(G)$. It is easy to see that $d(x, y) = 2$ in $G + H$. Therefore, every vertex in $V(G + H) - \{x, y\}$ lies in a $x - y$ geodesic of length 2 in $G + H$, Necessarily, every vertex in $V(G) - \{x, y\}$ lies in a $x - y$ geodesic of length 2 in G. If $V(G) \neq \{x, y\}$, then every vertex in $V(G) - \{x, y\}$ is adjacent to both x and y. This implies that S is not a geodetic global dominating set in $G + H$, which is a contradiction. Hence, $V(G) = \{x, y\}$ and so $G \simeq \bar{K}_2$. By the definition of S, w is adjacent to every vertices in $\bar{H}_1$ implies, $\delta_{H_1}(w) = 0$. Therefore, $H \simeq H_1$, where $\delta(H_1) = 0$. It shows that $G + H \simeq \bar{K}_2 + H_1$.

Theorem 3.4

For non-trivial connected graphs G and H, $\bar{\gamma}_g(G + H) \geq 4$.

Proof. Let G and H be non-trivial connected graphs. To prove $\bar{\gamma}_g(G + H) \geq 4$. That is, to see every subset S of $V(G + H)$ having atmost 3 vertices is not a geodetic global dominating set in $G + H$. Suppose on the contrary that S is a geodetic global dominating set of cardinality less than or equal to 3. That is $|S| \leq 3$. Since S is a geodetic set, $|S| = 2$ or 3.

case(i) $|S| = 2$. In other words, $S = \{u, v\}$ be a minimum geodetic global dominating set in $G + H$. By Theorem 2.3, $G + H \simeq K_2$ or $g(G + H) = 2$ and $d_{G+H}(u, v) = 3$. Since, $diam(G + H) \leq 2$, the only possibility is $G + H \simeq K_2$, implies $G \simeq K_1$ and $H \simeq K_1$, which is a contradiction.

case(ii) $|S| = 3$. In other words $S = \{u, v, w\}$ be a geodetic global dominating set in $G + H$. Consider the following sub cases.

sub case(i) Suppose $S \subseteq V(G)$ or $S \subseteq V(H)$. Assume that $S \subseteq V(G)$. Clearly, S is a geodetic set in $G + H$. Since, $diam(G) \leq 2$, S is a dominating set in $G + H$. By the definition of $G + H$, every vertex in G is adjacent to every vertex in H. Therefore in $\overline{G + H}$, G and H are disconnected components, implies that S is not a geodetic global dominating set in $G + H$, contrary to an assumption about S. Similarly S is not a geodetic global dominating set in $G + H$ whenever $S \subseteq V(H)$.

sub case(ii) Suppose $|V(G) \cap S| = 2$ or $|V(H) \cap S| = 2$. Assume that $|V(G) \cap S| = 2$, say $u, v \in V(G)$. Then $w \in V(H)$. Let $S_1 = \{u, v\}$ and so that $S = S_1 \cup \{w\}$. If u and v are adjacent in G, then the only possibility is $G \simeq K_2$ and $H \simeq K_1$. This is contrary to our hypothesis. Therefore, u and v are non-adjacent in G. since w is adjacent to both u and v, w lies on every $u - v$ geodetic and S_1 is a geodetic set in $G + H$. Since $diam(G + H) = 2$, S_1 is a dominating set in $G + H$. Moreover, H is non-trivial connected, there exists atleast one more vertex $z \neq w$ in H, which is adjacent to w in $G + H$. But in $\overline{G + H}$, z is non-adjacent to u, v and w

implies that S is not a minimum geodetic global dominating set in $G+H$ and so $\overline{\gamma}_g(G+H) > |S|$, a contrary to our assumption about S. Using a similar argument, it can be shown that $\overline{\gamma}_g(G+H) > |S|$ whenever $|V(H) \cap S| = 2$. Therefore, $\overline{\gamma}_g(G+H) \geq 4$.

Corollary 3.5

Let G and H be non-trivial connected graphs. Then, every geodetic global dominating set of $G+H$ need atleast two vertices from both G and H.

Proof. Let S be a geodetic global dominating set in $G+H$. By Theorem 3.4, S contains atleast four vertices. Since S is a global dominating set of $G+H$, G and H are disconnected in $\overline{G+H}$ implies S contains atleast two vertices from both G and H.

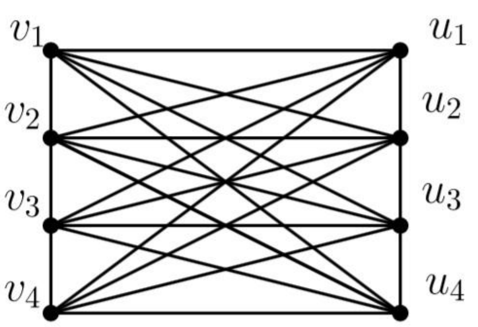
Theorem 3.6

Let S be a geodetic global dominating set in $G+H$. Then $S \cap V(G)$ and $S \cap V(H)$ are dominating sets in $\bar{G}$ and $\bar{H}$ respectively.

Proof. Let S be a geodetic global dominating set in $G+H$. Then S is a global dominating set in $G+H$ implies S is a dominating set in $G+H$ as well as $\overline{G+H}$. Since, $\bar{G}$ and $\bar{H}$ are disconnected, S is a dominating set in $\bar{G}$ and $\bar{H}$, implies $S \cap V(G)$ is a dominating set in $\bar{G}$ and $S \cap V(H)$ is a dominating set in $\bar{H}$.

Remark 3.7

The converse of Theorem 3.6 is not true. Consider the graph $G = P_4$ with vertex set $\{v_1, v_2, v_3, v_4\}$ and $H = P_4$ with vertex set $\{u_1, u_2, u_3, u_4\}$ and the graph $G+H = P_4 + P_4$ shown in Figure 3.1. Let $S = \{v_1, v_2, u_1, u_2\}$. Clearly, the sets $S_1 = S \cap V(G) = \{v_1, v_2\}$ and $S_2 = S \cap V(H) = \{u_1, u_2\}$ are dominating sets in $\bar{G}$ and $\bar{H}$, respectively. However, S is not a geodetic global dominating set in $G+H$. It is easy to see that $\overline{\gamma}_g(P_4 + P_4) = 4$.



$P_4 + P_4$
Figure 3.1

Remark 3.8

1) Every full vertices of G and H belong to every geodetic global dominating set of $G+H$.

2) Every extreme vertices of G and H need not belong to every geodetic global dominating set of $G+H$.

Theorem 3.9

Let G and H be connected graphs. Then $\overline{\gamma}_g(G+H) = 4$ if and only if $G+H \simeq K_4$ or there exist non-adjacent vertices $x, y \in V(G)$ and $u, v \in V(H)$ such that $N_G(x) \cap N_G(y) = \phi$ and $N_H(u) \cap N_H(v) = \phi$.

Proof. Let G and H be connected graphs. Suppose $G+H \simeq K_4$, then by Theorem 2.2, $\overline{\gamma}_g(G+H) = 4$. Assume that $G+H \not\simeq K_4$, and there exist non-adjacent vertices $x, y \in V(G)$ and $u, v \in V(H)$ such that $N_G(x) \cap N_G(y) = \phi$ and $N_H(u) \cap N_H(v) = \phi$. To prove $\overline{\gamma}_g(G+H) = 4$. Let $S = \{x, y, u, v\}$. First show that S is a geodetic set in $G+H$. Since, S contains vertex x and y with $xy \notin E(G)$ implies $V(H) \subseteq I_{G+H}[x, y] \subseteq I_{G+H}[S]$. Similarly S contains vertex u and v with $uv \notin E(H)$ implies $V(G) \subseteq I_{G+H}[u, v] \subseteq I_{G+H}[S]$. Therefore, $V(G+H) \subseteq I_{G+H}[S]$ follows that S is a geodetic set in $G+H$.

Now, we show that S is a global dominating set in $G+H$. Since $V(G) \subseteq N_{G+H}[x] \subseteq N_{G+H}[S]$ and $V(H) \subseteq N_{G+H}[u] \subseteq N_{G+H}[S]$ implies $V(G+H) \subseteq N_G[S]$. This means that S is a dominating set in $G+H$. Since, $N_G(x) \cap N_G(y) = \phi$, no vertex in $V(G)$ is adjacent to both x and y. Similarly,$N_H(u) \cap N_H(v) = \phi$, no vertex in $V(H)$ is adjacent to both u and v. Therefore, S dominates $\bar{G}$ implies that S is a geodetic global dominating set in $G+H$ and so $\overline{\gamma}_g(G+H) \leq |S| = 4$. By Theorem 3.4, we conclude $\overline{\gamma}_g(G+H) = 4$.

Conversely assume that $\overline{\gamma}_g(G+H) = 4$. Let S be a $\overline{\gamma}_g$-set of $G+H$. Let $x, y \in V(G)$ and $u, v \in V(H)$. By corollary 3.5, S contains two vertices from G and two vertices from H. Without loss of generality, take that $S = \{x, y, u, v\}$. If x and y are adjacent in G, then u and v are adjacent in H, otherwise $V(H)$Û$I_{G+H}[S]$ implies S is not a $\overline{\gamma}_g$-set of $G+H$. Hence, the only possibility is $G+H \simeq K_4$. If x and y are non adjacent in G, then u and v are also non adjacent in H, otherwise $V(G)$Û$I_{G+H}[S]$ follows that S is not a $\overline{\gamma}_g$-set of $G+H$. Now, we show that $N_G(x) \cap N_G(y) = \phi$ and $N_H(u) \cap N_H(v) = \phi$. Suppose, $N_G(x) \cap N_G(y) \neq \phi$. Let $z \in N_G(x) \cap N_G(y)$ implies z is adjacent to both x and y in $\bar{G}$. This means that $S \cap V(G)$ is not a dominating set in $\bar{G}$. By Theorem 3.6, S is not a geodetic global dominating set in $G+H$, which is a contradiction. Similarly, we show that $N_H(u) \cap N_H(v) = \phi$. Hence the Theorem.

Theorem 3.10

Let G be a non-complete connected graph. Let $S \subseteq V(G+K_p)$ is a geodetic global dominating set in $G+K_p$. Then $S \cap V(G)$ is a geodetic global dominating set in G.

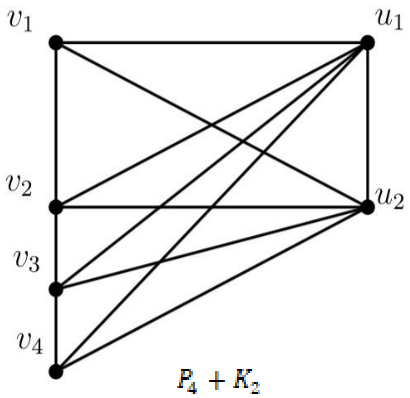
Proof. Let G be a non-complete connected graph. Let $S \subseteq V(G+K_p)$ be a geodetic global dominating set in $G+K_p$. Let $S_1 = S \cap V(G)$. To prove S_1 is a geodetic global dominating set in G. Since S is a geodetic set in $G+K_p$, $S_1 \neq \phi$. If $S_1 = V(G)$ then the proof is obvious. So assume that $S_1 \neq V(G)$. Let $x \in V(G) - S_1$. Then there exists $u, v \in S$ such that $x \in I_{G+K_p}[u, v]$. Since, $x \neq u$ and $x \neq v, d_{G+K_p}(u, v) \neq 1$ Hence, $d_{G+K_p}(u, v) = 2$. This implies that $u, v \in S_1$.

Also, any $u-v$ geodesic path P containing x cannot contain a vertex of $V(K_p)$. Since every $u-v$ geodesic P in $G+K_p$ that does not contain a vertex of K_p is an geodesic path in G, and $x \in I_{G+K_p}[u,v]$, it follows that $x \in I_G[u,v]$. Therefore, $I_G[S_1] = V(G)$. This shows that S_1 is a geodetic set in G. Since, S is a global dominating set in $G+K_p$ and every vertex of K_p is of degree $p-1$, $S - V(K_p)$ is a global dominating set in G.

`This shows that S_1 is a geodetic global dominating set in G.

Remark 3.11

The converse of Theorem 3.10 is not true. Consider the graph $P_4 + K_2$ given in Figure 3.2. Let $S = \{v_1, v_4, u_1, u_2\}$. Clearly $S_1 = S \cap V(P_4) = \{v_1, v_4\}$ is a geodetic global dominating set in P_4. But, it is easy to verify that S is not a geodetic global dominating set in $P_4 + K_2$.



$P_4 + K_2$
Figure 3.2

Observation 3.12

If G is a non-complete connected graph and $S \subseteq V(G)$ is a geodetic global dominating set in G, then S is not a geodetic global dominating set in $G+H$ for any connected graph H.

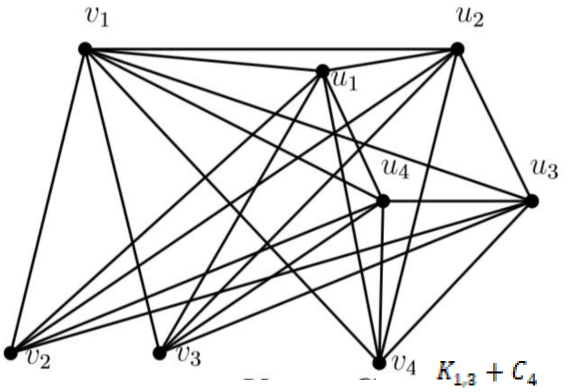
Theorem 3.13

Let G and H be connected graphs with $diam(G) = 2$. If G is non-complete and $S \subseteq V(G)$ is a geodetic global dominating set in G, then $S \cup W$ is a geodetic global dominating set in $G+H$ for all dominating set W in $\bar{H}$.

Proof. Let $S \subseteq V(G)$ be a geodetic global dominating set in G. Then $V(G) \subseteq I_{G+H}[S]$. Also G is non-complete, S contains vertices u and v with $uv \notin E(G)$. Since $diam(G) = 2$ implies, $V(H) \subseteq I_{G+H}[S]$. It follows that S is a geodetic set in $G+H$. Now, $V(H) \subseteq N_{G+H}[S]$ follows that S is a dominating set in $G+H$. Let W be a dominating set in $\bar{H}$. That $S \cup W$ is a global dominating set in $G+H$ follows from the fact that $V(\bar{G}) \subseteq N_{\overline{G+H}}[S]$ and $V(\bar{H}) \subseteq N_{\overline{G+H}}[W]$. Therefore, $V(\bar{G}) \cup V(\bar{H}) \subseteq N_{\overline{G+H}}[S \cup W]$, implies that S is a geodetic global dominating set in $G+H$.

Remark 3.14

The converse of Theorem 3.13 need not be true. Consider the graph $G = K_{1,3}$ with the vertex set $\{v_1, v_2, v_3, v_4\}$ and $H = C_4$ with the vertex set $\{u_1, u_2, u_3, u_4\}$. Then the graph $G+H = C_4 + K_{1,3}$ shown in Figure 3.3. Let $S = \{v_1, v_2, v_3\} \subseteq V(G)$ and let $W = \{u_1, u_2, u_3\} \subseteq V(H)$. It is easily verified that $S \cup W$ is a geodetic global dominating set in $G+H$, where W is a dominating set in $\bar{H}$. but it is clear that S is not a geodetic global dominating set in G.



$K_{1,3} + C_4$
Figure 3.3

Theorem 3.15

Let G and H be any graphs. If $diam(G) = 2$ and $S \subseteq V(G)$ is a geodetic global dominating set in G. Then, $S \cup \{v\}$ is a geodetic global dominating set in $G+H$ if and only if $\delta_H(v) = 0$ for some $v \in V(H)$.

Proof. Let $S \subseteq V(G)$ is a geodetic global dominating set in G. Let $diam(G) = 2$ and let $v \in V(H)$. First, assume $S \cup \{v\}$ is a geodetic global dominating set in $G+H$. By Theorem 3.13, $\{v\}$ is a dominating set in $\bar{H}$. It follows that v is an isolate vertex in H, implies $\delta_H(v) = 0$.

Conversely, assume $\delta_H(v) = 0$. This shows that v is an isolate vertex in H. Since S is a geodetic set in $G, V(H) \subseteq I_{G+H}[S]$. Further, if $diam(G) = 2$, then $V(G) \subseteq I_{G+H}[S]$. This means that S is geodetic set in $G+H$. Clearly, S is a dominating set in $G+H$. Moreover, S is a dominating set in $\bar{G}$ and v is adjacent to every vertex in $\bar{H}, S \cup \{v\}$ is a global dominating set in $G+H$. Hence, $S \cup \{v\}$ is a geodetic global dominating set in $G+H$.

Corollary 3.16

Let $G = H + \bar{K}_n$, where H is any graph and $n \geq 1$. If $diam(H) = 2$, then $\bar{\gamma}_g(G) = \bar{\gamma}_g(H) + 1$.

Proof. Let S be a geodetic global dominating set in H such that $\bar{\gamma}_g(H) = |S|$. Let $v \in \bar{K}_n$. Then $\delta(v) = 0$ in $\bar{K}_n$. Therefore, by Theorem 3.15, $S \cup \{v\}$ is a geodetic global dominating set in G. Since, S is minimum, $S \cup \{v\}$ is a minimum geodetic global dominating set in $G+H$ and so $\bar{\gamma}_g(G) = |S \cup \{v\}| = \bar{\gamma}_g(H) + 1$.

Theorem 3.17

Let G and H be non complete connected graphs. A subset S of $V(G+H)$ is a geodetic global dominating set in $G+H$ if and only if the following conditions holds:

(i) $S_1 = S \cap V(G)$ is a dominating set in $\bar{G}$.

(ii) $S_2 = S \cap V(H)$ is a dominating set in $\bar{H}$ and

(iii) There exist $x, y \in S_1$ and $u, v \in S_2$ such that $xy \notin E(G)$ or $uv \notin E(H)$ or $xy \notin E(G)$ and $uv \notin E(H)$.

Proof. Assume $S \subseteq V(G+H)$ is a geodetic global dominating set in $G+H$. Then by Theorem 3.6, S_1 and S_2 are

dominating sets in $\bar{G}$ and $\bar{H}$ respectively. Since, S is a geodetic global dominating set in $G+H$ by Corollary 3.5, S contains atleast two vertices from both G and H. Without loss of generality, let $x,y \in V(G)$ and $u,v \in V(H)$, such that $\{x,y,u,v\} \subseteq S$. This implies that $x,y \in S_1$ and $u,v \in S_2$.
If S_1 is a geodetic set in G, then $xy \notin E(G)$ and either $uv \in E(H)$ or $uv \notin E(H)$. Suppose, S_2 is a geodetic set in H then $uv \notin V(H)$ and either $xy \in E(G)$ or $xy \notin E(G)$. Both S_1 and S_2 are geodetic sets in G and H, then $xy \notin E(G)$ and $uv \notin E(H)$.

Conversely, assume conditions (i),(ii) and (iii) holds. Let $S = S_1 \cup S_2 \subseteq V(G+H)$. We show that S is a geodetic global dominating set in $G+H$. Since, G is non-complete, there exists vertices x,y in $V(G)$ with $xy \notin E(G)$. Therefore, $V(H) \subseteq I_{G+H}[x,y]$. Similarly, H is non-complete, implies there exists u,v in $V(H)$ such that $uv \notin E(H)$. Hence, $V(G) \subseteq I_{G+H}[u,v]$. By (iii), $x,y,u,v \in S$, implies $V(H) \subseteq I_{G+H}[S]$ and $V(G) \subseteq I_{G+H}[S]$. It follows that S is a geodetic set in $G+H$. Also, $V(G) \subseteq N_{G+H}[u] \subseteq N_{G+H}[S]$ and $V(H) \subseteq N_{G+H}[x] \subseteq N_{G+H}[S]$. This means that S is a dominating set in $G+H$. Moreover, by (i) and (ii) S is a dominating set in $\bar{G}$ and $\bar{H}$. Since,$\bar{G}$ and $\bar{H}$ are components of $\overline{G+H}$, S is a dominating set in $\overline{G+H}$. Therefore, S is a global dominating set in $\overline{G+H}$. This shows that S is a geodetic global dominating set in $G+H$.

IV. CONCLUSION

In this paper, we get a deep knowledge about geodetic global domination in join of graphs. It has many applications in social networking and modern technologies. For our future work, we can extend it for large families of graphs.

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