International Journal of Recent Technology and Engineering (IJRTE)
ISSN: 2277-3878, Volume-8 Issue-5, January 2020

A modified Particle Swarm Optimization Algorithm to solve Time Minimization Transportation Problem

Gurwinder Singh, Amarinder Singh

***Abstract**: When the supply of items need urgent/earliest delivery to the destinations, the Time Minimization Transportation Problems (TMTPs) are indispensable. Traditionally these problems have been solved using the exact techniques, however, the (meta) heuristic techniques have provided a great breakthrough in search space exploration. Particle Swarm Optimization is one such meta-heuristic technique that has been applied on a wide variety of continuous optimization problems. For discrete problems, either the mathematical model of problem or the solution procedure has been changed. In this paper, the PSO has been modified to incorporate the discrete nature of variables and the non-linearity of the objective function. The proposed PSO is tested on the problems available in the literature and the optimal solutions are obtained efficiently. The exhaustive search capability of proposed PSO is established by obtaining alternate optimal solutions and the combinations of the allocated cells that are beyond* $(m+n-1)$ *in number. This proposed solution technique, therefore, provides an effective alternate to the analytical techniques for decision making in the logistic systems.*

***Keywords**: Transportation Problem, Time Minimization Transportation Problem, Particle Swarm Optimization, Optimal Solution.*

I. INTRODUCTION

Transportation Problem (TP) is primarily studied to minimize cost when goods are being transported from their sources to destinations. But, in variety of real-world problems, the time of transportation is of greater importance than cost, for instance, the dispatch of armaments from military bases to war fronts, the delivery of perishable goods for daily needs and the supply of services requested during emergency hours and so on. Under these circumstances, where delay in transportation may result in much larger loss than any other gain, the optimization of time plays a vital role to squeeze the time limit of the transportation.

Manuscript published on January 30, 2020.
* Correspondence Author
Gurwinder Singh*, Research Scholar, IKG Punjab Technical University, Jalandhar, Punjab, India. Email: gurwinder.singh@bbsbec.ac.in
Amarinder Singh, Assistant Professor, Department of Applied Sciences, BBSB Engineering College, Fatehgarh Sahib, Punjab, India. Email: amarinder77@gmail.com

So, a time minimization transportation problem (TMTP), also known as a Bottleneck Transportation Problem (BTP), is being formulated and solved to serve such market needs. The main objective of TMTP is to minimize the maximum time to transport all the goods to the destinations, satisfying certain conditions in respect of availability at sources and requirement at the destinations. Mathematically the problem is formulated as given below.

$$\text{Minimize } T(X) = Max(t_{ij} : x_{ij} > 0)$$

Subject to conditions

$$\sum_{j=1}^{n} x_{ij} = a_i \;\; (i = 1, 2, \cdots, m)$$

$$\sum_{i=1}^{m} x_{ij} = b_j \;\; (j = 1, 2, \cdots, n)$$

$$x_{ij} \geq 0 \text{ and integers}$$

where m sources $(S_1, S_2, \cdots, S_m)$ and n destinations $(D_1, D_2, \cdots, D_n)$ have capacities $a_1, a_2, \cdots, a_m$ units and demands $b_1, b_2, \cdots, b_n$ units respectively. The t_{ij} is the time of transporting certain units of the product from source S_i to destination D_j. Thus the problem is to determine x_{ij}, the amount of the product transported, that minimize the maximum transportation time. The transportation is assumed to start simultaneously from all sources at once with the carriers having sufficient capacity to transport in a single trip and the time remains independent of quantity of the commodity being transported.

The TMTP has evolved through a variety of solution techniques. For instance, the labeling techniques by Hammer[1], Schwarz[2], Gafinkel and Rao[3], the use of cost operator theory by Srinivasan[4], a lexicographic primal code technique by Issermann[5] have gained a wide attention. Whereas Bhatia et al.[6], Ramakrishnan[7] and Sharma & Swarup[8] have worked on computational aspects to make some significant improvements. Parkash[9] has incorporated branch and bound method to solve the problem. Nikolic[10] has focused on dynamic transportation routes by optimizing multiple solutions in a lexicographic manner whereas Natarajan and Pandian[11] have considered the blocking zero point method.

Several variants of TMTP are given in literature such as the bi-level time minimization transportation problem by Khandewal and Puri[12], a capacitated two-stage time minimization transportation problem by Sharma et al.[13], two-level hierarchical time minimization transportation problem, incorporating hierarchy in levels by Sonia and Puri[14] and later an iterative method was developed by Sharma and Verma[15].

Now recently a priority-based time minimization transportation problem is discussed by Kaushal and Arora[16] wherein the destinations are prioritized into primary and secondary groups and served by assigning weights.

The methods (either exact or heuristic) have been used to solve the small size TMTPs and are restricted by limitation on computer time and becomes difficult to deal as the problem size grows exponentially. Therefore, there has been an increasing interest in meta-heuristics or evolutionary techniques to solve such complex problem. The meta-heuristics are well known global search techniques that have gained significant attention due to their adaptive applicability to wide range of optimization problems. To name a few, Vignaux and Michalewicz[17] were the first to use genetic algorithm (GA) to solve linear & non-linear TPs. Later, Gen and Li[18], Gen et al.[19] and Syarif and Gen[20] have developed spanning-tree based GAs for TP. Also Tabu Search (TS) has been applied by Sun and Mckeown[21],[22] to solve a variant of TP. Likewise Huang[23] and Elsherbiny et al.[24] have used PSO to solve TP.

So in this motivation, a modified particle swarm optimization algorithm (discussed in section III) is proposed to solve the TMTP. The proposed PSO is able to override the rigid conditions of conventional methods such as Modified Distribution Method (MODI) and Stepping Stone Method (SSM), to solve the TP. The conditions such as the number of allocations $(m+n-1)$ and the independent positions of allocations, are required to be followed on each iteration. But, the proposed algorithm does not depend on such conditions to iterate towards optimal solution. Therefore, it provides a good alternative to existing techniques and will provide the necessary decision support to decision-makers to handle time oriented logistic problems.

II. PARTICLE SWARM OPTIMIZATION

PSO is a population based stochastic algorithm perceived by Kennedy and Eberhart[25] whereby the collective behavior of birds/insects has been designed to solve different optimization problems. This behavior is, basically employed when a group of randomly initialized particles (swarm) is constructed with the initial position & velocity and then updated iteratively towards a global optimum. The path of the particles, in search space, is coordinated by the interaction of the individual best (Pbest) and the global best (Gbest) position respectively. Mathematically, this mechanism results in following equations:

$$V_i(t+1) = \omega . V_i(t) + c_1 r_1 (P_{i,best} - X_i) + c_2 r_2 (P_{g,best} - X_i) \quad \textbf{(1)}$$

$$X\_i(t+1) = X\_i(t) + V\_i(t+1) \quad \textbf{(2)}$$

where the ith particle of the swarm is represented by $X_i = (x_{i1}, x_{i2}, \cdots, x_{in})$ and the velocity of the particle is denoted by $V_i = (v_{i1}, v_{i2}, \cdots, v_{in})$. The best visited position of Xi and the global best position of the swarm may be considered as $P_{i,best} = (p_{i1}, p_{i2}, \cdots, p_{in})$ and $P_{g,best} = (p_{g1}, p_{g2}, \cdots, p_{gn})$. And the parameter ω, the inertia weight, c_1, c_2 acceleration coefficients and r_1, r_2 random variables uniformly distributed within [0, 1], play a vital role to retain the accelerated rate of particles towards their pbest and gbest locations.

Since its inception, the PSO algorithm has been improved, as discussed by Wang et al.[26], in terms of a) the theoretical analysis: to analyse its working mechanism, b) the parameter selection & tuning: to improve its performance, c) the neighbourhood topology: to configure particles' mutual-interaction, and d) the convergence & stability analysis: to overcome the premature-convergence & oscillating behaviour. These improvements have enabled PSO to be applicable for a wide variety of problems.

The Pseudo-code of basic PSO is devised as follows :

Algorithm 1: Basic PSO

```
Initialize required parameters such as ω,
  c1,c2, Popsize, Maxiters, Maxrun.
Initialize particle positions
Initialize particle velocities.
do
    for each particle do
        Calculate the objective value by
          using the defined objective function
        Update particle's personal best
          position(PBest) if required
        Update particle's global best
          position(GBest) if required.
    Update the inertia weight ω.
    for each particle do
        Update particle's velocity using
          equation (1)
        Update particle's position using
          equation (2)
while the end condition is not arrived
return the GBest solution
```

In particular, PSO has been used to solve the discrete problems by incorporating encoding/decoding techniques that discretize the solution space wherein the candidate solution is confined to the discrete/integer values only. Jordehi and Jasni[27] have reviewed these encoding/decoding techniques which keep the solution particles in the feasible solution space. Recently, Singh & Singh [28] has hybridized PSO to solve TP and resolved this issue by incorporating additional modules into the basic PSO. Thus, in this paper, a similar approach has been adopted to solve the time-minimization transportation problem.

III. PROPOSED PSO FOR TMTP

The proposed PSO incorporates three modules into the Algorithm 1, which are explained as follow:

First Module (Algorithm 2) is of Initialization wherein the particles are initialized randomly, satisfying the demand and supply constraints, to get the Initial Basic Feasible Solution (IBFS) of the time-minimization transportation problem.

Retrieval Number: E6606018520/2020©BEIESP
DOI:10.35940/ijrte.E6606.018520
Journal Website: www.ijrte.org

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Second Module (Algorithm 3) is to amend the negative allocations in the candidate solutions. This situation arises due to the deviation of the current candidate position from its personal best position and the global best position as mentioned in the velocity equation of PSO.

The negative allocations are repaired in this module and then move to the next module to repair the fractional allocations in the candidate solutions.

Third Module (Algorithm 4) is to amend the fractional allocations arising because of various parameters in the velocity equation of PSO. These are repaired as in the algorithm given below so that the candidate solutions are brought back into the feasible solution space. Then the proposed-PSO is examined on different test problems (taken from various research papers) and the convergence towards the respective optimal solution is also recorded.

Algorithm 2: Initialization

Input: $TMTP$, $Supply$, $Demand$

Output: Initialized Matrix X

Set $[m, n] = size(TMTP)$ and $s = Supply$ and $d = Demand$

Set a matrix RX of random numbers of dimension $m \times n$

Initialize the solution matrix X as $X = zeros(m, n)$

Set $maxitrs = m * n$ for maximum number of iterations

for $iter = 1$ **to** $maxitrs$ **do**

  Find maximum random number by using $maxr = max(RX)$

  Locate (i, j) coordinates of $maxr$ in matrix RX

  Assign available amount to x_{ij} as $x_{ij} = min(s_i, d_j)$

  Update supply and demand as

  $s_i = s_i - x_{ij}$ and $d_j = d_j - x_{ij}$

  Drop that location by $maxr = 0$

return X

Algorithm 3: Amend Negatives

Input: Infeasible Matrix X

Output: Negatives-Repaired Matrix X

do

  Locate the k^{th} row and j^{th} column of most negative element x_{kj} in given matrix X

  Set $x_0 = x_{kj}$

  Take the most positive element x_{hj} of j^{th} column of X

  Update $x_{hj} = x_{hj} - |x_0|$

  and $x_{kj} = 0$

  Change elements in row k into

$$x_{kj} = \begin{cases} x_{kj} & x_{kj} = 0 \\ x_{kj} - \frac{|x_0|}{u} & x_{kj} > 0 \end{cases}$$

  (u is the number of times when the $x_{kj} > 0$ (positive))

  Change elements in row h into

$$x_{hj} = \begin{cases} x_{hj} & x_{kj} = 0 \\ x_{hj} + \frac{|x_0|}{u} & x_{kj} > 0 \end{cases}$$

while $(x_{ij} < 0;\ i \in \{1, 2, ..., m\}, j \in \{1, 2, ..., n\})$

return X

Algorithm 4: Amend Fractions

Input: Negative-Repaired Matrix X

Output: Fractions-Repaired Matrix Y

Count the crossed entries (if available) of given Matrix X and assign it to $countcros$

Assign row wise total of x_{ij} to s_i and column-wise total to d_j

Take a matrix rX of random numbers with dimension $m \times n$

Initialize Y as zero matrix of dimension $m \times n$

Set $mn = m * n$

for $i = 1$ **to** $mn - countcros$ **do**

  Choose location (i, j) of maximum random number $maxr$ from rX

  Assign value to y_{ij} as: $y_{ij} = min(s_i, d_j)$

  Update supply and demand as:

  $s_i = s_i - y_{ij}$

  $d_j = d_j - y_{ij}$ and Set $maxr = 0$

return Y

III.I Merits of Proposed PSO

Some vantage points of proposed PSO in comparison to the analytic techniques are described as follows:

III.I.I Pre-conditions of Analytic optimal technique

The known analytic optimal techniques have two pre-conditions viz. a) the number of allocations should be equal to $(m + n - 1)$, b) these allocations should be in independent positions. Moreover, during each iteration, the non-degenerate solution has reduced to degenerate one. But as the dimension of problem increases, it becomes very difficult to identify the cells for virtual allocation(s) and, those too, in independent positions. The proposed PSO does not depend on the number of allocations to move towards optimal solution. This is particularly advantageous in case of the higher dimension of the problem as the iterations are performed irrespective of the number of allocations and their locations.

III.I.II Alternate optimal solution

The analytical techniques have identified the existing of alternate optimal solution of the transportation problem if the cell evaluation $(z_{ij} - c_{ij})$ of some unallocated cell is equal to zero. The alternate optimal solution is obtained by carrying out an iteration with respect to unallocated cell. However, the proposed PSO implicitly deals with the alternate optimal solutions, if they exist. As it has an exhaustive search capability to explore all the combinations of solution and identify the alternative solution corresponding to the same value of the objective function, without underlying any particular conditions of cell-evaluation.

Time_Matrix				S_i
10	7	4	25	4
3	9	13	12	16
8	8	15	5	20
14	9	15	8	11
D_j 16	5	18	12	

Particle	InitialSolution				OptimalSolution			
Particle_1	0	1	2	1	0	0	4	×
	0	0	16	0	0	1	14	1
	16	4	0	0	16	3	×	1
	0	0	0	11	0	1	×	10
Particle_2	0	0	0	4	0	2	2	×
	16	0	0	0	0	0	16	0
	0	5	15	0	16	3	×	1
	0	0	3	8	0	0	×	11
Particle_3	4	0	0	0	0	2	2	×
	0	4	0	12	0	0	16	0
	12	1	7	0	16	2	×	2
	0	0	11	0	0	1	×	10
Particle_4	0	3	0	1	0	2	2	×
	16	0	0	0	0	0	16	0
	0	2	18	0	16	3	×	1
	0	0	0	11	0	0	×	11
Particle_5	4	0	0	0	0	0	4	×
	0	4	0	12	0	1	14	1
	12	1	7	0	16	4	×	0
	0	0	11	0	0	0	×	11

Figure 1: Time Matrix and Initial Solution Particles with its Optimal Solution.

III.I.III Number of allocations

It is generally observed that the number of allocated cells in the optimal solution is less than or equal to $(m+n-1)$, however, the proposed PSO explores all the combinations of variables and produces even those optimal solutions where the number of allocated cells is even more than $(m+n-1)$ as brought out in the numerical example given in Figure 1. This establish the existing of alternate optimal solution and the different number of allocated cells.

IV. NUMERICAL ILLUSTRATION

To elaborate the procedure of proposed PSO, one test problem is illustrated with the parameters as mentioned in Table 1:

Table 1: Parameters used for proposed PSO

Parameter	Value	Description
c_1	2	Accleration Coefficient (Cognitive)
c_2	2	Accleration Coefficient (Social)
r_1	$rand()$	Random Variable between (0,1)
r_2	$1-r_1$	Random Variable
ω	0.9 to 0.4	Time Decreasing
popsize	5	Population or Swarm Size
maxitr	100	Maximum iterations

1. A test problem P1, as given in Table 2, is considered wherein each entry in the cell depicts transportation time between sources and destinations along with the availability at each source and the demand at each destination. As it is balanced transportation problem, so there is no need to add a dummy

2. A swarm, collection of particles, is initialized using the Algorithm 2 and the Initial Basic feasible solution (IBFS) are determined. One of such particle is given in Table 2 andrepresented by basic $x_{14}, x_{21}, x_{32}, x_{33}, x_{43}, x_{44}$ with maximum time 25 units. This solution is marked as the Personal best of the respective solution, i.e. the particle. Although it is a degenerate solution, i.e. the number of basic variables is less than $(m+n-1)$, but the proposed algorithm may still be applied. Then, more such particles are raised using the Algorithm 2 and their personal bests are recorded. The Global best of the swarm, among all the raised particles, is the best value of the objective function, i.e. the minimum value of the time taken to complete the transportation by all the particles.

Table 2: Test Problem P1 and Initial Solution

P1	Time_Matrix				InitialSolution				S_i
	10	7	4	25	0	0	0	4	4
	3	9	13	12	16	0	0	0	16
	8	8	15	5	0	5	15	0	20
	14	9	15	8	0	0	3	8	11
				D_j	16	5	18	12	

3. These particles are then updated using velocity equation (1) with the parameters as given in Table 1. The velocity of the particle (Table 2) is mentioned in Table 3. The position of the particle is then updated using the equation (2) and the updated position matrix is given in Table 3. This position updation satisfies the constraints of the transportation problem but renders some of the transportation allocations infeasible as some of the allocations attain fractional and/or negative values. These allocations need to be brought back into the feasible solution space. The negative correction is carried out using the Algorithm 3 followed by the fractional corrections using Algorithm 4, to obtain a feasible solution given in Table 3. This solution is an improved solution since the value of the objective function corresponding to these transportation allocations is 15 units.
4. Similarly, the Algorithm 3 and 4 are applied on the other particles as well to update their Personal best. The Global best of the swarm is also updated to the current best particle that corresponds to the minimum of the values of the objective function.

Retrieval Number: E6606018520/2020©BEIESP
DOI:10.35940/ijrte.E6606.018520
Journal Website: www.ijrte.org

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Table 3: Particle at iteration 1

Velocity Matrix				Position Matrix				Negative Repair				Fractions Repair			
2.5679	-0.6420	-1.2839	-0.6420	2.5679	0.3580	0.7161	0.3580	2.5679	0.3580	0.7161	0.3580	2	1	1	0
0	2.5679	-10.2715	7.7036	0	2.5679	5.7285	7.7036	0	2.5679	5.7285	7.7036	1	2	6	7
-2.5679	-1.9259	4.4938	0	13.4321	2.0741	4.4938	0	13.4321	2.0741	4.4938	0	13	2	4	1
0	0	7.0616	-7.0616	0	0	7.0616	3.9384	0	0	7.0616	3.9384	0	0	7	4

Table 4: Particle at end of phase-I

Velocity Matrix				Position Matrix				Negative Repair				Fractions Repair			
-1.9506	1.3717	1.4437	-0.8646	2.0492	1.3717	1.4437	-0.8646	1.7610	1.0635	1.1555	0	2	1	1	0
-0.2101	0.9756	-4.2057	3.4403	0.7899	2.9756	1.7943	10.4403	1.1307	2.2508	2.6679	9.9506	1	3	2	10
2.7874	-0.2817	-0.0346	-2.4711	13.7874	1.7183	4.9654	-0.4711	13.1083	1.6657	5.2260	0	13	1	6	0
-0.6264	-2.0656	2.7966	-0.1045	-0.6264	-1.0656	9.7966	2.8955	0	0	8.9506	2.0494	0	0	9	2

5. Continuing in this manner, the particles are updated with velocity & position equations and then the negatives & fractions, if required, are repaired.

Thus, the obtained optimal solution is mentioned in Table 4. This completes Phase I of the solution procedure.

6. In Phase II, all the cells in the current problem wherein the transportation time is greater than or equal to 15 units are dropped from further consideration and a random initial solution is generated using Algorithm 2. One such random initial basic feasible solution of the updated problem is given in Table 5. This solution, non-degenerate one, is an improved solution from the previous one as the value of the objective function corresponding to this transportation allocation is 14 units. This particle is given a random velocity as given in Table 6.

Table 5: Updated Time Matrix & Initial Solution

P1	Time Matrix				Initial Solution				S_i
	10	7	4	×	0	0	4	×	4
	3	9	13	12	0	0	14	2	16
	8	8	×	5	15	5	×	0	20
	14	9	×	8	1	0	×	10	11
				D_j	16	5	18	12	

7. The updated position of the particle is given in Table 6. This updated position is subjected repeatedly to Algorithms 3 and 4 in order to obtain a feasible solution as given in Table 6.

8. Continuing in the same manner, the positions of all the particles are updated and the Global best is also updated to the position of the particle that corresponds to the minimum transportation time. The basic variables corresponding to this solution are $x_{13}, x_{22}, x_{23}, x_{24}, x_{31}, x_{32}, x_{34}, x_{42}, x_{44}$ as given in Table 7 and the respective value of the objective function is 13 units. This completes Phase II.

Table 8. Optimal Solution

Phase – II	Optimal Solution				S_i	Time_Matrix			
	0	0	4	×	4	10	7	4	×
	0	1	14	1	16	3	9	×	12
	16	3	×	1	20	8	8	×	5
	0	1	×	10	11	×	9	×	8
D_j	16	5	18	12					

9. Further improvement in the solution provided in Table 8 is not possible, as if all the cells corresponding to the transportation time greater than equal 13 units are dropped from consideration, then the destination D_3 will not be available for transportation and hence the problem shall not be amenable to solution. Thus the solution provided in Table 8 is the optimal solution of the problem.

Table 6: Particle at start of Phase II

Velocity Matrix				Position Matrix				Negative Repair				Fractions Repair			
0	0	0	×	0	0	4.0000	×	0	0	4.0000	×	0	0	4	×
0	3.5193	0	-3.5193	0	3.5193	14.0000	-1.5193	0	2.0000	14.0000	0	0	2	14	0
1.7596	-3.5193	×	1.7596	16.7596	1.4807	×	1.7596	16.0000	1.4807	×	2.5193	16	1	×	3
-1.7596	0	×	1.7596	-0.7596	0	×	11.7596	0	1.5193	×	9.4807	0	2	×	9

Table 7: Particle at end of Phase II

Velocity Matrix				Position Matrix				Negative Repair				Fractions Repair			
0	0	0	×	0	0	4.0000	×	0	0	4.0000	×	0	0	4	×
0	0.2956	0	-0.2956	0	2.2956	14.0000	-0.2956	0	2.0000	14.0000	0	0	1	14	1
0.1478	1.3575	×	-1.5053	16.1478	3.3575	×	0.4947	16.0000	2.7044	×	1.2956	16	3	×	1
-0.1478	-1.6531	×	1.8009	-0.1478	-0.6531	×	11.8009	0	0.2956	×	10.7044	0	1	×	10

V RESULTS

In this paper, the modified PSO is evaluated on nine balanced TMTPs and one unbalanced TMTPs, taken from exiting literature, as given in Table 9.

For each test problem, the procedure is repeated with 10 runs iterated 100 times with the population size of 5 in MATLAB. The sequence of the proposed algorithm is divided into one, two or more phases, as per the need of respective problem, when converging towards its optimal solution. The obtained results are given in Table 10. It is observed that the optimal solution for problem P4 & P10 are obtained in four phases, P1 & P3 in two phases and others in one phase as given in Table 10. The obtained optimal solutions are also compared with each solution of existing methods in Table 11, wherein it can be observed that problem P6 has better optimal solution than the existing one.

Table 9: Data of Test Problems

TPNo	Time Matrix	Supply Matrix	Demand Matrix	Reference
1	$[t_{ij}] = \begin{bmatrix} 10 & 7 & 4 & 25 \\ 3 & 9 & 13 & 12 \\ 8 & 8 & 15 & 5 \\ 14 & 9 & 15 & 8 \end{bmatrix}$	$[s_i] = \begin{bmatrix} 4 \\ 16 \\ 19 \\ 11 \end{bmatrix}$	$[d_j] = \begin{bmatrix} 16 & 5 & 18 & 12 \end{bmatrix}$	[2]
2	$[t_{ij}] = \begin{bmatrix} 6 & 21 & 19 & 12 & 7 \\ 9 & 13 & 10 & 14 & 15 \\ 14 & 11 & 12 & 9 & 12 \\ 12 & 16 & 8 & 20 & 19 \end{bmatrix}$	$[s_i] = \begin{bmatrix} 8 \\ 5 \\ 4 \\ 5 \end{bmatrix}$	$[d_j] = \begin{bmatrix} 2 & 6 & 4 & 7 & 3 \end{bmatrix}$	[2]
3	$[t_{ij}] = \begin{bmatrix} 7 & 9 & 6 & 4 & 2 \\ 8 & 13 & 11 & 7 & 3 \\ 4 & 6 & 9 & 2 & 8 \\ 9 & 4 & 6 & 9 & 10 \\ 1 & 3 & 4 & 9 & 8 \end{bmatrix}$	$[s_i] = \begin{bmatrix} 5 \\ 6 \\ 8 \\ 2 \\ 7 \end{bmatrix}$	$[d_j] = \begin{bmatrix} 7 & 7 & 3 & 4 & 7 \end{bmatrix}$	[3]
4	$[t_{ij}] = \begin{bmatrix} 12 & 13 & 34 & 7 & 8 & 29 & 19 \\ 7 & 18 & 36 & 40 & 38 & 6 & 10 \\ 11 & 20 & 30 & 21 & 21 & 29 & 31 \\ 27 & 12 & 39 & 31 & 5 & 36 & 12 \\ 15 & 17 & 32 & 36 & 22 & 16 & 14 \\ 17 & 38 & 16 & 33 & 23 & 30 & 29 \end{bmatrix}$	$[s_i] = \begin{bmatrix} 15 \\ 7 \\ 45 \\ 30 \\ 12 \\ 16 \end{bmatrix}$	$[d_j] = \begin{bmatrix} 20 & 13 & 11 & 27 & 9 & 5 & 40 \end{bmatrix}$	[6]
5	$[t_{ij}] = \begin{bmatrix} 25 & 30 & 20 & 40 & 45 & 37 \\ 30 & 25 & 20 & 30 & 40 & 20 \\ 40 & 20 & 40 & 35 & 45 & 22 \\ 25 & 24 & 50 & 27 & 30 & 25 \end{bmatrix}$	$[s_i] = \begin{bmatrix} 37 \\ 22 \\ 32 \\ 14 \end{bmatrix}$	$[d_j] = \begin{bmatrix} 15 & 20 & 15 & 25 & 20 & 10 \end{bmatrix}$	[8]
6	$[t_{ij}] = \begin{bmatrix} 3 & 4 & 2 & 2 & 5 \\ 4 & 1 & 2 & 4 & 2 \\ 3 & 2 & 4 & 5 & 3 \\ 2 & 5 & 1 & 3 & 4 \end{bmatrix}$	$[s_i] = \begin{bmatrix} 40 \\ 45 \\ 50 \\ 45 \end{bmatrix}$	$[d_j] = \begin{bmatrix} 40 & 50 & 35 & 30 & 25 \end{bmatrix}$	[29]
7	$[t_{ij}] = \begin{bmatrix} 11 & 3 & 10 & 2 & 5 \\ 2 & 7 & 3 & 8 & 1 \\ 12 & 2 & 4 & 5 & 7 \\ 9 & 4 & 6 & 3 & 5 \end{bmatrix}$	$[s_i] = \begin{bmatrix} 14 \\ 13 \\ 22 \\ 16 \end{bmatrix}$	$[d_j] = \begin{bmatrix} 15 & 10 & 15 & 10 & 15 \end{bmatrix}$	[10]
8	$[t_{ij}] = \begin{bmatrix} 10 & 68 & 73 & 52 \\ 66 & 95 & 30 & 21 \\ 97 & 63 & 19 & 23 \end{bmatrix}$	$[s_i] = \begin{bmatrix} 8 \\ 19 \\ 17 \end{bmatrix}$	$[d_j] = \begin{bmatrix} 11 & 3 & 14 & 16 \end{bmatrix}$	[11]
9	$[t_{ij}] = \begin{bmatrix} 10 & 2 & 20 \\ 3 & 7 & 9 \\ 12 & 14 & 16 \end{bmatrix}$	$[s_i] = \begin{bmatrix} 10 \\ 15 \\ 5 \end{bmatrix}$	$[d_j] = \begin{bmatrix} 10 & 8 & 12 \end{bmatrix}$	[30]
10	$[t_{ij}] = \begin{bmatrix} 5 & 3 & 7 & 9 & 5 & 1 & 10 & 6 & 0 \\ 13 & 4 & 6 & 12 & 12 & 10 & 9 & 3 & 0 \\ 8 & 13 & 2 & 9 & 3 & 8 & 9 & 6 & 0 \\ 4 & 1 & 4 & 4 & 9 & 6 & 13 & 13 & 0 \\ 2 & 6 & 2 & 6 & 13 & 12 & 5 & 5 & 0 \\ 9 & 10 & 4 & 8 & 7 & 6 & 4 & 4 & 0 \end{bmatrix}$	$[s_i] = \begin{bmatrix} 9 \\ 8 \\ 8 \\ 10 \\ 9 \\ 8 \end{bmatrix}$	$[d_j] = \begin{bmatrix} 5 & 8 & 6 & 2 & 6 & 3 & 2 & 3 & 17 \end{bmatrix}$	[16]

Table 10: Optimal Solution for P1 to P10

Phases	P1	P2	P3	P4	P5	P6	P7	P8	P9	P10
Phase-I	**15**	**13**	7	31	**40**	**2**	**9**	**66**	**12**	7
Phase-II	13	-	**6**	30	-	-	-	-	-	6
Phase-III	-	-	-	27	-	-	-	-	-	5
Phase-IV	-	-	-	**21**	-	-	-	-	-	**4**

Table 11: Comparison of Proposed PSO with existing optimal solutions

Problem	Dimension	Author's Solution	Proposed PSO
P1	4*4	13	13
P2	4*5	13	13
P3	5*5	6	6
P4	6*7	21	21
P5	4*6	40	40
P6	4*5	4	**2**
P7	4*5	9	9
P8	3*4	66	66
P9	3*3	12	12
P10	6*9	4	4

The quality of solution (Gbest) has been measured and compared in terms of the Relative Percentage Deviation (RPD), calculated as follows:

$$RPD = \left(\frac{solution - optimal\ solution}{optimal\ solution} \right) * 100$$

Retrieval Number: E6606018520/2020©BEIESP
DOI:10.35940/ijrte.E6606.018520
Journal Website: www.ijrte.org

Published By:
Blue Eyes Intelligence Engineering
& Sciences Publication

For each test problem, the value of average RPD is calculated for the proposed PSO (phases-wise) and plotted as given in Figure 2. It is pertinent to note from these average RPD bars P5, P7 & P9 are minimum as compared to the problems. The optimal transportation time indicates the promising attainment & potential level of proposed algorithm.

V. CONCLUSION

To ensure the efficient movement and in-time availability of goods and services, the transportation time plays a crucial role to make better economic decisions. In this paper, a modified particle swarm optimization algorithm has been presented to solve time minimization transportation problem. The balance of exploration-exploitation capability of PSO is utilized to attain quality of solution. The proposed algorithm has been evaluated on 10 test problems and the results have been compared with exiting exact/heuristic methods. These simulation results have revealed that the proposed algorithm is quite effective in terms of its minimum possible iterations, quality of solution and the identification/attainment of the optimal solution even if the initial or the intermediate solutions are degenerate. Hence the procedure of this paper can be extended to other types of transportation problems like the bulk or fixed charge transportation problems with single or multi-objective goals.

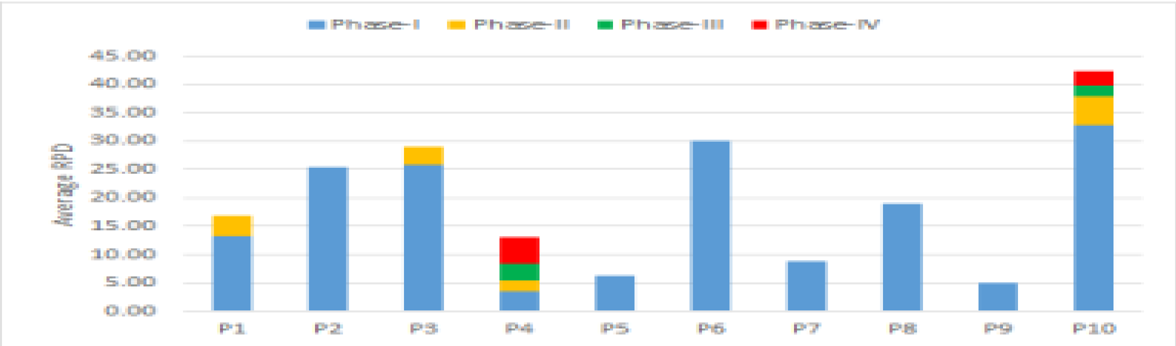

Figure 2: Plot of Average Relative Percentage Deviation (RPD) for problem P1 to P10

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AUTHORS PROFILE

Gurwinder Singh, is a Research Scholar at IKG Punjab Technical University, Punjab, India. His research interest includes optimization techniques especially heuristic /meta-heuristic algorithms. He has focused on the application of these techniques to variants of Transportation Problem and other combinatorial optimization problems.

Amarinder Singh, received his PhD in Mathematics at Thapar Institute of Engineering & Technology, Patiala. He is Assistant Professor at the Department of Applied Sciences, Baba Banda Singh Bahadur Engineering College, Fatehgarh Sahib. His research interests are related to the optimization techniques and applications of heuristic techniques on the facility location problems, transportation problems, routing problems, etc. He has published papers in national and international journals and conference proceedings.