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On Double Natural Transform of Boehmians

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***Abstract:**In this paper we are going to define the Double Natural transform of Bounded support and we develop this theory with the help of Zemanian technique. Here we are going to define the Integrable Boehmian and the main focus or objective of this paper is to define Double Natural transform for Integrable Boehmian, and this theory is developed and defined with the help of Mikusinski's theory. D.Nemzer, P. Mikusinski and other discovered several applications of Boehmians. Also we establish some properties of Double Natural transform with other integral transforms. The Double Natural transform have been extended to generalized function and some of its distributional properties are obtained. Further we extend this class to Integrable Boehmians.*

***Key Words: -** Double Natural Transform, Convolution Theorem, Integrable Boehmian.*

***AMS Subject Classification: -** Primary 14E20, 54C40, Secondary 46E25, 20C20.*

I. INTRODUCTION

Most of the integral transforms are applied to solve the differential equation. For example Kamal transform [1,2,11] Mahgoub transform[3] is used for solving the problems of population growth and decay, Linear voltera integral equation, coupled system of nonlinear partial differential equations. Elzaki transform [14] is used for solving Abel integral equation. In classical theory lot of application are given for solving these problems using Laplace, Fourier, Hankel, Mellin Hilbert transforms. The Double Natural transform [13] is an extension of single Natural transform. Z.H. Khan [12] has introduced and discussed the Natural transform its properties and applications for one variable. Belgacem and Silambarasan [7,8,9] discussed the application of Sumudu transform to Maxwell's equations, advances in Natural transform and theory of Natural transform in detail. Adem Kilicman and Maryam Omran[13] first extended the single Natural transform to Double Natural transform.

S.K.Q. Al-omari[5] discussed the generalized function for Double Natural transform. Also H. Eltayeb and A. Kilicman[10] discussed Double Natural transform and Double Laplace transform.

S.K.Q. Al-omari [4] discussed the application of Sumudu transform to generalized function and Boehmians. Also S.K.Q. Al-omari[6] discussed the Fourier sin(cosine) transform for ultra distribution and there extension to tempered and ultra Boehmian space.

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In this paper we study the Double Natural transform to space of distributions of bounded support and established some theorems. Also we have discussed Integral Boehmians for Double Natural transform using [15].

The Double Natural transformation of a function f(x,y) $x, y \in \mathbb{R}_+$ of two variables is defined by [13] as,

$$N_+^2\{f(x,y)\} = R_+^2\{(s,p);(u,v)\}$$

$$= \int_0^\infty \int_0^\infty e^{-(sx+py)} f(ux, vy)dxdy$$

or it can be written as

$$N_+^2\{f(x,y)\} = \frac{1}{uv}\int_0^\infty \int_0^\infty e^{-(\frac{sx}{u}+\frac{py}{v})} f(x,y)dxdy$$

provided the integral exists and f(x,y) can be expressed as convergent series.

Let f, g be integrable function of two variables then by using [5] the Double Convolution of f and g is given by

$$(f ** g) = \int_0^\infty \int_0^\infty f(\xi,\eta)g(x-\xi, y-\eta)d\xi d\eta$$

II. DOUBLE NATURAL TRANSFORM OF BOUNDED SUPPORT

Let J be compact subset I= (0, ∞). We denote the space of all complex valued infinitely smooth testing functions spaces on $I \times I$ by $\epsilon(I \times I)$such that

$$\sup_{(x,y) \in J \times J} \left|\frac{\partial^{k+m}}{\partial x^k \partial y^m}\phi(x,y)\right| < \infty$$

for k, m ≥ 0

$\epsilon(I \times I)$ is linear space under the usual definitions Moreover $D(I \times I) \subset \epsilon(I \times I)$.

A sequence $\{\phi_v\}_{v=1}^\infty$ converges in $\epsilon(I \times I)$ to ϕ if and only if all ϕ_v and ϕ in $\epsilon(I \times I)$ and for each non-negative integer k, m $(\frac{\partial^{k+m}}{\partial x^k \partial y^m}\phi_v)$ converges to $\frac{\partial^{k+m}}{\partial x^k \partial y^m}\phi$ uniformly on every compact subset $I \times I$.

The space $\epsilon(I \times I)$ is complete and $\epsilon'(I \times I)$ i.e dual of $\epsilon(I \times I)$ consists of distribution of compact support. By the definition of Double Natural transform it follows that the Kernel $e^{-(\frac{sx}{u}+\frac{py}{v})}/uv$ is a smooth and satisfies

$$\sup_{(x,y) \in J \times J} \left|\frac{\partial^{k+m}}{\partial x^k \partial y^m} e^{-(\frac{sx}{u}+\frac{py}{v})}/uv\right| < \infty \quad (2.1)$$

for all positive real no u and v. Hence for every $f \in \epsilon'(I \times I)$ we define the distributional Double Natural transform of f of compact support with the help of Zemanian [17,18] as,

$\widehat{N}((s,p);(u,v)) \cong N_+^2\{f(x,y)\}$ (2.2)

$\cong < f(x,y).e^{-(\frac{sx}{u}+\frac{py}{v})}/uv >$ for (u, v) $\in I \times I$

Theorem 2.1 The distributional Double Natural transform $\widehat{N}$ is linear.

Proof. For any constants α, β we have

$$\widehat{N}(\alpha f(x,y) + \beta g(x,y))$$

$$\cong < \alpha f(x,y) + \beta g(x,y).\frac{e^{-(\frac{sx}{u}+\frac{py}{v})}}{uv} >$$

$$\cong < \alpha f(x,y).\frac{e^{-(\frac{sx}{u}+\frac{py}{v})}}{uv} > + < \beta g(x,y).\frac{e^{-(\frac{sx}{u}+\frac{py}{v})}}{uv} >$$

$$\cong \alpha < f(x,y).\frac{e^{-(\frac{sx}{u}+\frac{py}{v})}}{uv} > + \beta < g(x,y).\frac{e^{-(\frac{sx}{u}+\frac{py}{v})}}{uv} >$$

$$\cong \alpha\widehat{N}(f) + \beta\widehat{N}(g)$$

Theorem 2.2 Let h be distribution in $\epsilon'(I \times I)$ and let g be defined by

$$g(x,y) = \begin{cases} h(x-\tau_1, y-\tau_2) & x \geq \tau_1, y \geq \tau_2 \\ 0 & x < \tau_1, y < \tau_2 \end{cases}$$

Then $\widehat{N}(g) = e^{-(\frac{s\tau_1}{u}+\frac{p\tau_2}{v})}\widehat{N}(h)$

Proof. Here g is member of $\epsilon'(I \times I)$ by the translation property

$$\widehat{N}(g) = < h(x-\tau_1, y-\tau_2), \frac{e^{-(\frac{sx}{u}+\frac{py}{v})}}{uv} >$$

$$= < h(x,y), \frac{e^{-(\frac{s(x-\tau_1)}{u}+\frac{p(y-\tau_2)}{v})}}{uv} >$$

$$= < h(x,y), \frac{e^{\frac{-s\tau_1}{u}} e^{\frac{-sx}{u}} e^{\frac{-p\tau_2}{v}} e^{\frac{-py}{v}}}{uv} >$$

$$= e^{-(\frac{s\tau_1}{u}+\frac{p\tau_2}{v})} < h(x,y), \frac{e^{-(\frac{sx}{u}+\frac{py}{v})}}{uv} >$$

$$= e^{-(\frac{s\tau_1}{u}+\frac{p\tau_2}{v})}\widehat{N}(h)$$

Theorem 2.3 Let the $f \in \epsilon'(I \times I)$ and $\widehat{N}(f)$ be the distributional Double Natural transform of f then

$$\frac{\partial^k}{\partial u^k}\widehat{N}(f) = < f(x,y), \frac{\partial^k}{\partial u^k}\frac{e^{-(\frac{sx}{u}+\frac{py}{v})}}{uv} >$$

$$\frac{\partial^n}{\partial v^n}\widehat{N}(f) = < f(x,y), \frac{\partial^n}{\partial v^n}\frac{e^{-(\frac{sx}{u}+\frac{py}{v})}}{uv} >$$

where $\frac{\partial^k}{\partial u^k}$ and $\frac{\partial^n}{\partial v^n}$ are kth and nth derivatives with respect to u and v.

Theorem 2.4 Let the $f \in \epsilon'(I \times I)$ and $\widehat{N}(f)$ be the distributional Double Natural transform then

$$\widehat{N}(e^{-ax-by}f(x,y))[(s,p);(u,v)] = \widehat{N}(f(x,y);(s+au, p+bv)$$

Proof

$$\widehat{N}(f(x,y))[(s,p);(u,v)] = < f(x,y), \frac{e^{-(\frac{sx}{u}+\frac{py}{v})}}{uv} >$$

$$\widehat{N}(e^{-ax-by}f(x,y))[(s,p);(u,v)] = < f(x,y), \frac{e^{-(\frac{sx}{u}+\frac{py}{v})}}{uv} >$$

$$\widehat{N}(e^{-ax-by}f(x,y))[(s,p);(u,v)] = < e^{-ax-by}f(x,y), \frac{e^{-(\frac{sx}{u}+\frac{py}{v})}}{uv} >$$

$$= < f(x,y), \frac{e^{-(\frac{sx}{u}+\frac{py}{v})}}{uv} e^{-ax-by} >$$

$$= < f(x,y), \frac{e^{\frac{-sx}{u}-ax-\frac{py}{v}-by}}{uv} >$$

$$= < f(x,y), e^{\frac{-(s+au)x}{u}} e^{\frac{-(p+bv)y}{v}} >$$

$$= \widehat{N}(f(x,y);(s+au, p+bv)$$

Theorem 2.5 Let the $f \in \epsilon'(I \times I)$ and $\widehat{N}(f)(x,y)$ is the distributional Double Natural transform of f then

$$\widehat{N}(f(ax,by)) = \frac{1}{ab}\widehat{N}\left(f(x,y)\left(\frac{s}{a};\frac{p}{b}\right)(u,v)\right)$$

Where a, b>0

Proof

$$\widehat{N}(f(ax,by)) = < f(ax,by), \frac{e^{-(\frac{sx}{u}+\frac{py}{v})}}{uv} >$$

$$= < f(x,y), \frac{1}{a}\frac{1}{b}\frac{e^{-(\frac{sx}{u}+\frac{py}{v})}}{uv} >$$

$$= \frac{1}{ab}\widehat{N}\left(f(x,y)\left(\frac{s}{a};\frac{p}{b}\right)(u,v)\right)$$

Theorem 2.6 Let the $f \in \epsilon'(I \times I)$ and $\widehat{N}(f)(x,y)$ is the distributional Double Natural transform of f then

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1) $\hat{N}(sx\,f(x,y))(u,v) = u^2 \frac{\partial}{\partial u}\hat{N}(f) + u\hat{N}(f(x,y))(u,v)$

2) $\hat{N}(py\,f(x,y))(u,v) = v^2 \frac{\partial}{\partial v}\hat{N}(f) + v\hat{N}(f(x,y))(u,v)$

Proof. We prove the first part
Let $f \in \epsilon'(I \times I)$ then by theorem (2.3) and

by equation (2.2) we have

$$\frac{\partial}{\partial u}\hat{N}(f(x,y)) = \frac{\partial}{\partial u} < f(x,y), \frac{e^{-\left(\frac{sx}{u}+\frac{py}{v}\right)}}{uv} >$$

$$=< f(x,y), \frac{\partial}{\partial u}\frac{e^{-\left(\frac{sx}{u}+\frac{py}{v}\right)}}{uv} >$$

By partial differentiation we get

$$=< f(x,y), \frac{e^{-\left(\frac{sx}{u}+\frac{py}{v}\right)}}{uv}\left(\frac{sx}{u^2}\right) - \frac{e^{-\left(\frac{sx}{u}+\frac{py}{v}\right)}}{u^2 v} >$$

$$\frac{\partial}{\partial u}\hat{N}(f(x,y)) = \frac{1}{u^2} < sxf(x,y), \frac{e^{-\left(\frac{sx}{u}+\frac{py}{v}\right)}}{uv} >$$

$$-\frac{1}{u} < f(x,y), \frac{e^{-\left(\frac{sx}{u}+\frac{py}{v}\right)}}{uv} >$$

Hence

$$\hat{N}(sx\,f(x,y))(u,v) = u^2 \frac{\partial}{\partial u}\hat{N}(f) + u\hat{N}(f(x,y))(u,v)$$

Similarly we can prove second part.

Theorem 2.7 Let the $f \in \epsilon'(I \times I)$ and $\hat{N}(f)(x,y)$ is the distributional Double Natural transform of f then

$$\hat{N}\{f(x-\eta, y-\theta).H(x-\eta, y-\theta)\} = e^{-\left(\frac{s\eta}{u}+\frac{p\theta}{v}\right)}\hat{N}\{f(x-\eta, y-\theta)\}$$

where H(x, y) is Heaviside unit step function defined by

$$H(x-\eta, y-\theta) = 1 \quad x > \eta, y > \theta$$

Hence

$$H(x-\eta, y-\theta) = 0 \quad x < \eta, y < \theta$$

Proof. By definition of distributional Double Natural transform we have

$$\hat{N}\{f(x-\eta, y-\theta).H(x-\eta, y-\theta)\}$$

$$=< f(x-\eta, y-\theta).H(x-\eta, y-\theta), \frac{e^{-\left(\frac{sx}{u}+\frac{py}{v}\right)}}{uv} >$$

For $x > \eta, y > \theta$

$$=< f(x-\eta, y-\theta) \frac{e^{-\left(\frac{s(x+\eta)}{u}+\frac{p(y+\theta)}{v}\right)}}{uv} >$$

$$= e^{-\left(\frac{s\eta}{u}+\frac{p\theta}{v}\right)} < f(x-\eta, y-\theta), \frac{e^{-\left(\frac{sx}{u}+\frac{py}{v}\right)}}{uv} >$$

$$= e^{-\left(\frac{s\eta}{u}+\frac{p\theta}{v}\right)}\hat{N}\{f(x-\eta, y-\theta)\}$$

III. CONVOLUTION OF DOUBLE NATURAL TRANSFORM

Let f, g be distributions of compact support

in $\epsilon'(I \times I)$ we define the convolution of f and g by

$$< (f ** g)(t,x), \phi(t,x) >$$
$$=< f(x,y), < g(J_1,J_2), \phi(t+J_1, x+J_2) \gg \quad (3.1)$$

where $\phi \in \epsilon(I \times I)$ this has meaning if
$< g(J_1,J_2), \phi(t+J_1, x+J_2) > \in \epsilon(I \times I)$

Theorem 3.1 Let $f \in \epsilon'(I \times I)$ and $\psi \in \epsilon(I \times I)$

If $\phi(t,x) =< g(J_1,J_2), \frac{\partial^{k+m}}{\partial t^k \partial x^m}\psi(t+J_1, x+J_2) >$

Where k, m = 1,2,3,…

Proof This theorem can be proved by using ([16],PP. 130)

Theorem 3.2 (The Convolution Theorem)

If $\hat{N}(f)((s,p);(u,v))$ and $\hat{N}(g)((s,p);(u,v))$ are the distributional transforms of f and g respectively then

$$\hat{N}(f ** g)((s,p);(u,v))$$

$$= uv\,\hat{N}(f)((s,p);(u,v)).\hat{N}(g)((s,p);(u,v))$$

Proof

$$\hat{N}(f ** g)((s,p);(u,v))$$

$$=< (f ** g)(x,y), \frac{e^{-\left(\frac{sx}{u}+\frac{py}{v}\right)}}{uv} >$$

$$=< f(x,y), g < (J_1,J_2), \frac{e^{-\left(\frac{s(x+J_1)}{u}+\frac{p(y+J_2)}{v}\right)}}{uv} \gg$$

$$= \frac{1}{uv} < f(x,y), g < (J_1,J_2), e^{\frac{-sx}{u}-\frac{sJ_1}{u}}.e^{\frac{-py}{v}-\frac{pJ_2}{v}} \gg$$

$$= \frac{1}{uv} < f(x,y), < g(J_1,J_2), e^{\frac{-sx}{u}-\frac{py}{v}}.e^{-\frac{sJ_1}{u}-\frac{pJ_2}{v}} \gg$$

$$= uv < f(x,y), \frac{e^{\frac{-sx}{u}-\frac{-py}{v}}}{uv} >< g(J_1,J_2), \frac{e^{-\frac{sJ_1}{u}-\frac{pJ_2}{v}}}{uv} >$$

$$= uv\,\hat{N}(f)((s,p);(u,v)).\hat{N}(g)((s,p);(u,v))$$

IV. INTEGRABLE BOEHMIANS FOR DOUBLE NATURAL TRANSFORM

Let L^1 be the space of Lebesgue integrable functions on the I_+ where I_+ is the set of positive real numbers then by P. Mikusinski [15] a sequence $(\delta_n)_{n=1}^{\infty}$ of continuous real functions over $I \times I$ is called a delta sequence if and only if

1) $\int_0^\infty \int_0^\infty \delta_n(x,y) = 1$
2) $\int_0^\infty \int_0^\infty |\delta_n| < K$ for some positive $K \in \mathbb{R}$ and all $n \in \mathbb{N}$
3) $\int_{|(x,y)|>\rho} |\delta_n(x,y)|dxdy \;\to 0$ as $n \to \infty$

For every $\rho > 0$
The space of all integrable Boehmians is denoted by $B_{L^1 \times L^1}$ which is a Convolution algebra

1) $\mu\left[\frac{f_n}{\delta_n}\right] = [\mu f_n/\delta_n]$
2) $[f_n/\delta_n] + [g_n/\psi_n] = (f_n ** \psi_n) + (g_n ** \delta_n)/(\delta_n ** \psi_n)$
3) $\left[\frac{f_n}{\delta_n}\right] ** \left[\frac{g_n}{\psi_n}\right] = [f_n ** g_n / \delta_n ** \psi_n]$

Lemma 4.1 If $[f_n/\delta_n] \in B_{L^1 \times L^1}$ then the sequence

$$N(f_n(x,y))(s,p;u,v) = \int_0^\infty \int_0^\infty \frac{1}{uv} e^{-(\frac{sx}{u}+\frac{py}{v})} f_n(x,y)dxdy$$

Converges uniformly an each compact set $J \times J$ in $I \times I$

Proof We know that if (δ_n) is sequence then its Double Natural transform $N(\delta_n) = \tilde{\delta}_n$ converges uniformly to the function $\frac{1}{uv}$. Hence for each $J \times J$ $\tilde{\delta}_n$ is positive on $J \times J$ *and*

$$N(f_n(x,y))(s,p;u,v) = N(f_n(s,p;u,v)\frac{\tilde{\delta}_k}{\tilde{\delta}_k}) \tag{4.1}$$

$$= \frac{Nf_k}{\tilde{\delta}_k}\tilde{\delta}_n \; on\; J \times J$$

i.e $Nf_n \to \frac{Nf_k}{uv\tilde{\delta}_k}$ $as\ n \to \infty$ $on\ J \times J$

$(\tilde{\delta} \to \frac{1}{uv}\ as\ n \to \infty)$

Now we define the Double Natural transform of an integrable Boehmians depending upon the above result as

$$\mathcal{P}[f_n/\delta_n] = \lim_{x\to\infty} \widehat{N} f_n \qquad (4.2)$$

where the limit ranges over compact subsets of $I \times I$. Thus we get a continuous function of a Double Natural transform of an integrable Boehmians.

Theorem 4.2 The Double Natural transformation of an integrable Boehmians is well defined.

Proof Let $\gamma_1 = [f_n/\phi_n], \gamma_2 = [g_n/\psi_n]$ are in $B_{L^1 \times L^1}$ *such that* $\gamma_1 = \gamma_2$ then

$$[f_n/\phi_n] = [g_n/\psi_n]$$

$$\Longrightarrow f_n ** \psi_m = g_m ** \phi_n \qquad m, n\epsilon N$$

Applying Double Natural Transform

$$\widehat{N}(f_n ** \psi_m) = \widehat{N}(g_m ** \phi_n) = \widehat{N}(g_n ** \phi_m)$$

From equation (4.2) and theorem (3.2) we have

$$\lim_{x\to\infty} \widehat{N} f_n = \lim_{x\to\infty} \widehat{N} g_n$$

i.e.

$$\mathcal{P}[f_n/\phi_n] = \mathcal{P}[g_n/\psi_n]$$

Theorem 4.3
The Double natural transform $\mathcal{P}$ *is linear*.
Proof Let $\beta_1, \beta_2 \in B_{L^1 \times L^1}$ be such that

$$\beta_1 = [f_n/\phi_n],\ \beta_2 = [g_n/\psi_n]$$

$$\beta_1 + \beta_2 = [(f_n ** \psi_n) + g_n ** \phi_n/(\phi_n ** \psi_n)]$$

$$\mathcal{P}(\beta_1 + \beta_2) = \lim_{x\to\infty} \widehat{N}(f_n ** \psi_n) \lim_{x\to\infty} \widehat{N}(g_n ** \phi_n)$$

$$= \lim_{x\to\infty} \widehat{N} f_n + \lim_{x\to\infty} \widehat{N} g_n$$

$$= \mathcal{P}(\beta_1) + \mathcal{P}(\beta_2) \text{ on compact sets.}$$

Also $\mathcal{P}(\alpha\beta) = \mathcal{P}[\alpha f_n/\delta_n] = \alpha \lim f_n = \alpha\, \mathcal{P}(\beta)$

Where $\alpha \in \mathbb{C}\ \beta = [f_n/\phi_n]$

Theorem 4.4 Convolution theorem
Let $[f_n/\phi_n], [g_n/\psi_n] \in B_{L^1 \times L^1}$ then
$\mathcal{P}([f_n/\phi_n] ** [g_n/\psi_n]) = uv\mathcal{P}([f_n/\phi_n])\mathcal{P}([g_n/\psi_n])$

Proof

$$\mathcal{P}([f_n/\phi_n] ** [g_n/\psi_n]) = \mathcal{P}[(f_n ** g_n)/(\phi_n ** \psi_n)]$$

$$= \lim_{x\to\infty} \widehat{N}\,(f_n ** g_n)$$

$$= \lim_{x\to\infty} uv\, \widehat{N}\,(f_n)\, \widehat{N}\,(g_n)$$

$$= uv \lim_{x\to\infty} \widehat{N}\,(f) \lim_{x\to\infty} \widehat{N}\,(g)$$

$$= uv\, \mathcal{P}[f_n/\phi_n]\mathcal{P}[g_n/\psi_n]$$

Theorem 4.5 If $\beta = [h_n/\delta_n]$ in $B_{L^1 \times L^1}$ and $\mathcal{P}(\beta) = 0\ in \mathbb{C}$ then $\beta = 0$ in $B_{L^1 \times L^1}$

Proof. Let $\beta = [h_n/\delta_n]$ and $\mathcal{P}(\beta) = 0$ then by definition of $\mathcal{P}$ we get $\lim_{x\to\infty} \widehat{N}\, h_n = 0$ on compact sets by definition of Natural transform $h_n \to 0$ almost everywhere in $B_{L^1 \times L^1}$ and h_n/δ_n is a zero quotient of functions. Equivalently $\beta = [h_n/\delta_n]$ is zero equivalence class in $B_{L^1 \times L^1}$

Theorem 4.6 The generalized Double Natural transform $\mathcal{P}$ is 1-1 mapping from $B_{L^1 \times L^1}$ into the space $\mathbb{C}$ of continuous function.

Proof Let $\mathcal{P}(\beta_1) = \mathcal{P}(\beta_2)$ $\quad \beta_1, \beta_2 \in B_{L^1 \times L^1}$

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$\mathcal{P}(\beta_1 - \beta_2) = 0\ in\ \mathbb{C}$ by (Theorem 4.5) we get

$$\beta_1 - \beta_2 = 0$$

$$\beta_1 = \beta_2$$

Theorem 4.7 The Double Natural transform $\mathcal{P}: B_{L^1 \times L^1} \to \mathbb{C}$ is continuous w.r.t. $\triangle$ convergence.

Proof Let $\beta_n \overset{\triangle}{\to} \beta$ as $n \to \infty$ in $B_{L^1 \times L^1}$ then the $\triangle$ convergence implies that

$$(\beta_n - \beta) ** \delta_n = [f_n ** \delta_n / \delta_n] \to 0$$

For some

$f_n \in B_{L^1 \times L^1}$ $\delta_n \in \triangle$ and $f_n \to 0\ as\ n \to \infty$

$$\mathcal{P}((\beta_n - \beta) ** \delta_n) = \mathcal{P}[f_n ** \delta_n / \delta_n]$$

$$= \lim_{n \to \infty} \widehat{N}\,(f_n ** \delta_n)$$

$= \lim_{n \to \infty} \widehat{N}\, f_n \to 0\ as\ n \to \infty$ In $\mathbb{C}$.

Since $f_n \to 0\ \mathcal{P}(\beta_n) - \mathcal{P}(\beta) \to 0\ asn \to \infty$

$\Longrightarrow \mathcal{P}(\beta_n) \longrightarrow \mathcal{P}(\beta)$ as $n \to \infty$

V. RESULT

1. We obtained the Double Natural transform of bounded support.
2. Some properties like convolution, linearity, in distributional sense as well as in bohemian space are obtained.
3. Also $\triangle$ convergence and δ-convergence are defined for Double Natural transform.

VI. CONCLUSION

In this paper the Double Natural transform is extended to distributional sense and some properties are obtained. Double Natural transform of bounded support is also obtained. Also the convolution of Double Natural transform of Boehmians is obtained. Lastly we have made an attempt to extend the Double Natural Transform to Integrable Boehmians.

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