Pythagorean Nano Topological Space

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Abstract: Our main objective is to introduce the concept of Pythagorean nano fuzzy topological space by motivating from the notion of fuzzy topological space and nano topological space and properties of Pythagorean nano topology are also examined.

Keywords: Pythagorean fuzzy nano topology, Pythagorean nano topological space, Pythagorean nano open sets, Pythagorean nano closure, Pythagorean nano interior.

I. INTRODUCTION

L.A. Zadeh [1] generalized the usual set in which every detail is described with a degree of membership function \( \mu: X \rightarrow [0,1] \). The fuzzy set has many programs in economic system, decision making, facts mining, commercial enterprise and many others. Fuzzy set has been generalized to greater non-standard fuzzy subsets. As Intuitionistic fuzzy subset become introduced with the aid of Atanassov [2], in which every element had the degree of membership and the degree of non-membership. Yager [3,4] presented the perception of Pythagorean fuzzy subset that is atypical fuzzy subsets and which has many powerful applications in natural and social sciences. Pythagorean fuzzy subsets can be used appropriately on every instant where intuitionistic fuzzy subsets cannot be used.

Standard topology has been stepped forward by means of taking its motivation from classical analysis and applied on several sections of research inclusive of system getting to know, statistics evaluation, facts mining. Farther the scrutiny of topology refers the relationship between spatial gadgets and features and it may be used to explain some sure spatial functions and to conceive statistics units which have higher great control and extra statistics integrity. In 1968, Chang [5] described the theory of fuzzy topological space and generalized some fundamental idea of topology inclusive of open set, closed set, continuity and compactness. Following this observation, Lowen gave a different explanation of a fuzzy topological space by way of converting a primary property of topology [6]. In 1995, Coker delivered the notion of intuitionistic fuzzy topological space and studied some equivalent variations of some standards of classical topology together with continuity and compactness [7]. Furthermore, some authors studied the concept of fuzzy soft topological space and its packages in choice-making environment.


The fundamental cause of this paper is to introduce the whim of Pythagorean fuzzy nano topological space. After specifying Pythagorean fuzzy nano topological space, we inspect the simple properties of nano topological space.

II. PRELIMINARIES

Definition 2.1: Let \( W \) be a non-empty fixed set & \( \emptyset \subseteq W \) then \( \emptyset \) is stated to be a fuzzy subset and is of the form \( O = \{(x, \rho_2(x)), x \in W\} \) where in \( \rho_2: W \to [0,1] \) is the degree of membership function.

Definition 2.2: Let \( D \) be a non-empty fixed set & \( B \subseteq D \) then \( B \) is said to be an Intuitionistic fuzzy subset of the shape \( B = \{(x, \theta(x), \mu_1(x)), x \in D\} \) where in \( \theta: D \to [0,1], \mu: D \to [0,1] \) is the degree of membership function and non-membership function gratifying \( \theta(x) + \mu(x) \leq 1 \).

Definition 2.3: Let \( L \) be a non-empty fixed set & \( I \subseteq L \) then \( I \) is stated to be Pythagorean fuzzy subset & expressed as \( I = \{(x, \phi(x), \mu_1(x)), x \in L\} \) in which \( \phi: L \to [0,1], \mu: L \to [0,1] \) is the degree of membership and non-membership function fulfilling \( \phi^2(x) + \mu^2(x) \leq 1 \).

Definition 2.4: Let \( \mathcal{H} \) be a non-empty finite set of items known as the universe and \( \mathcal{X} \) be the equivalence relation on \( \mathcal{H} \). The pair \( (\mathcal{H}, \mathcal{X}) \) is stated to be approximation space. Let \( K \subseteq \mathcal{H} \).

The smaller approximation of \( K \) with admire to \( \mathcal{X} \) is the set of items, which can be for certain categorised as \( K \) with recognize to \( \mathcal{X} \) and it is denoted by using \( L_2(x) \).

Let \( \mathcal{H} \) be an universe, \( \mathcal{X} \) be an equivalence relation on \( \mathcal{H} \) and \( \tau_2(K) = \{(x, U, L_2(x), B_2(x), U_2(x)) \} \) in which \( K \subseteq \mathcal{H} \).

Let \( \mathcal{H} \) be an universe, \( \mathcal{X} \) be an equivalence relation on \( \mathcal{H} \) and \( \tau_2(K) = \{(x, U, L_2(x), B_2(x), U_2(x)) \} \) in which \( K \subseteq \mathcal{H} \).

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Definition 2.5:

Let \( J \neq \emptyset \) be a set and allow \( \rho \) be a family of Pythagorean fuzzy subsets of \( J \). If \( 1_J, 0_J \in \rho \) for any \( S_i, S_j \in \rho \), we've \( S_i \cap S_j \in \rho \). For any \( \{ S_i \}_{i \in \tau} \subseteq \tau \), are satisfied for all \( i, \cup S_i \in \tau \) where \( I \) is an arbitrary index set, then \( \tau \) is called a Pythagorean fuzzy topology on \( J \). The couple \((J, \rho)\) is told to be a Pythagorean fuzzy topological space.

III. PYTHAGOREAN NANO TOPOLOGICAL SPACE

In this portion, the idea of Pythagorean nano topology through nano Pythagorean approximations specifically Pythagorean nano lower, Pythagorean nano upper and Pythagorean nano boundary is added with its simple properties discussed.

Definition 3.1:

Let \( \mathcal{V} \) be a non-void set and \( \mathcal{R} \) an equivalence relation on \( \mathcal{V} \). Let \( D \) be a Pythagorean set in \( \mathcal{V} \) with membership \( \theta_{D} \), non-membership \( \omega_{D} \). The Pythagorean nano minor approximation, Pythagorean nano major approximation and Pythagorean nano border of \( D \) in the approximation space \((\mathcal{V}, \mathcal{R})\) denoted by \( PNL_{\mathcal{R}}(D), PNU_{\mathcal{R}}(D), PNB_{\mathcal{R}}(D) \) respectively defined as:

\[
PNL_{\mathcal{R}}(D) = \{ (x, \theta_{LD}(x), \omega_{LD}(x)) | x \in [x]_{\mathcal{R}}, x \in \mathcal{V} \}
\]

\[
PNU_{\mathcal{R}}(D) = \{ (x, \theta_{RD}(x), \omega_{RD}(x)) | x \in [x]_{\mathcal{R}}, x \in \mathcal{V} \}
\]

\[
PNB_{\mathcal{R}}(D) = PNU_{\mathcal{R}}(D) - PNL_{\mathcal{R}}(D)
\]

where \( \theta_{LD}(x) = \lambda_{y \in [x]_{\mathcal{R}}} \theta_{2}(y), \omega_{LD}(x) = \lambda_{y \in [x]_{\mathcal{R}}} \omega_{D}(y) \) and

\[
\theta_{RD}(x) = \nu_{y \in [x]_{\mathcal{R}}} \theta_{2}(y), \omega_{RD}(x) = \lambda_{y \in [x]_{\mathcal{R}}} \omega_{D}(y) \]

Definition 3.2:

Let the Universe be \( \mathcal{V} \), equivalence relation on \( \mathcal{V} \) be \( \mathcal{R} \) and if

\[
\tau_{\mathcal{R}}(Y) = \{ \emptyset_{\mathcal{V}}, \emptyset_{\mathcal{V}}, PNL_{\mathcal{R}}(Y), PNU_{\mathcal{R}}(Y), PNB_{\mathcal{R}}(Y) \}
\]

where \( Y \subseteq \mathcal{V} \). \( \tau_{\mathcal{R}}(Y) \) satisfies the following axioms:

1. If \( A_{1}, A_{2} \in \tau_{\mathcal{R}}(Y) \) for \( i = 1, 2, ... \) then \( \bigcup_{i=1}^{\infty} A_{i} \in \tau_{\mathcal{R}}(Y) \).
2. If \( A_{1}, A_{2} \in \tau_{\mathcal{R}}(Y) \) for \( i = 1, 2, ... \) then \( \bigcap_{i=1}^{\infty} A_{i} \in \tau_{\mathcal{R}}(Y) \).

Then \( \tau_{\mathcal{R}}(Y) \) is termed as Pythagorean Nano Topology on \( \mathcal{V} \) w.r.t \( \mathcal{R} \). Whereas \( \emptyset_{\mathcal{V}}, \emptyset_{\mathcal{V}} \) are called Pythagorean Nano Topological space. The elements of \( \tau_{\mathcal{R}}(Y) \) are called Pythagorean Nano Open (PNO) Sets.

Example 3.3:

Let \( \mathcal{V} = \{ p, q, r \} \) be the Universe, \( \mathcal{V}_{/\mathcal{R}} = \{ [p, q], [r] \} \) be the equivalence relation on \( \mathcal{V} \). Let

\[
A = \{ (q|(.4,4)), (q|(.6,6)), (r|(.5,5)) \}
\]

Then \( PNL_{\mathcal{R}}(A) = \{ (q|(.4,4)), (q|(.6,6)) \} \) and

\[
PNU_{\mathcal{R}}(A) = \{ (q|(.4,4)), (q|(.6,6)) \}
\]

\[
PNB_{\mathcal{R}}(A) = \{ (q|(.4,4)), (q|(.6,6)), (r|(.5,5)) \}
\]

Thus

\[
\tau_{\mathcal{R}}(A) = \{ (q|(.4,4)), (q|(.6,6)), (r|(.5,5)) \}
\]

is a PNT.

Note: Pythagorean nano topology, Pythagorean Nano Open, Pythagorean Nano closed sets and with respect to will be denoted as PNT, PNOs, PNCs and w.r.t in the following theorems and sections.

Proposition 3.4:

Let \( \mathcal{V} \) be a non-void finite universe and \( Y \subseteq \mathcal{V} \).

a) If \( PNL_{\mathcal{R}}(Y) = \emptyset_{\mathcal{V}} \) and \( PNU_{\mathcal{R}}(Y) = \emptyset_{\mathcal{V}} \), then \( \tau_{\mathcal{R}}(Y) = \{ \emptyset_{\mathcal{V}}, \emptyset_{\mathcal{V}} \} \), the indiscrete Pythagorean nano topology on \( \mathcal{V} \).

b) If \( PNL_{\mathcal{R}}(Y) = PNU_{\mathcal{R}}(Y) = Y \), then the Pythagorean nano topology is

\[
\tau_{\mathcal{R}}(Y) = \{ \emptyset_{\mathcal{V}}, \emptyset_{\mathcal{V}}, PNL_{\mathcal{R}}(Y) \}
\]

c) If \( PNL_{\mathcal{R}}(Y) = \emptyset_{\mathcal{V}} \) & \( PNU_{\mathcal{R}}(Y) \neq \emptyset_{\mathcal{V}} \) then \( \tau_{\mathcal{R}}(Y) = \{ \emptyset_{\mathcal{V}}, \emptyset_{\mathcal{V}}, PNL_{\mathcal{R}}(Y) \} \) is the Pythagorean nano topology.

d) If \( PNL_{\mathcal{R}}(Y) \neq \emptyset_{\mathcal{V}} \), \( PNU_{\mathcal{R}}(Y) = \emptyset_{\mathcal{V}} \) then \( \tau_{\mathcal{R}}(Y) = \{ \emptyset_{\mathcal{V}}, \emptyset_{\mathcal{V}}, PNL_{\mathcal{R}}(Y), PNB_{\mathcal{R}}(Y) \} \) is the Pythagorean nano topology.

Theorem 3.5:

Let \( \mathcal{V} \) be a non-void, finite universe and \( Y \subseteq \mathcal{V} \). Let \( \tau_{\mathcal{R}}(Y) \) be the PNT on \( \mathcal{V} \) w.r.t \( \mathcal{R} \). Then \( [\gamma_{\mathcal{R}}(Y)]^{c} \) whose elements are \( A^{c} \) for \( A \in \tau_{\mathcal{R}}(Y) \), is a PNT on \( \mathcal{V} \).

Proof:

The PNT on \( \mathcal{V} \) w.r.t \( \mathcal{R} \) is given by

\[
\gamma_{\mathcal{R}}(Y) = \{ \emptyset_{\mathcal{V}}, \emptyset_{\mathcal{V}}, PNL_{\mathcal{R}}(Y), PNU_{\mathcal{R}}(Y), PNB_{\mathcal{R}}(Y) \}
\]

Therefore,

\[
[\gamma_{\mathcal{R}}(Y)]^{c} = \{ \emptyset_{\mathcal{V}}, \emptyset_{\mathcal{V}}, PNL_{\mathcal{R}}(Y), PNU_{\mathcal{R}}(Y), PNB_{\mathcal{R}}(Y) \}
\]

Consider

\[
PNU_{\mathcal{R}}(Y)^{c} \cup [PNU_{\mathcal{R}}(Y)]^{c} = [PNU_{\mathcal{R}}(Y) \cap \mathcal{V}_{/\mathcal{R}}]^{c} = [PNU_{\mathcal{R}}(Y) \cap \mathcal{V}_{/\mathcal{R}}]^{c} \in [\gamma_{\mathcal{R}}(Y)]^{c}
\]

and

\[
PNL_{\mathcal{R}}(Y)^{c} \cup [PNL_{\mathcal{R}}(Y)]^{c} = [PNL_{\mathcal{R}}(Y) \cap \mathcal{V}_{/\mathcal{R}}]^{c} = [PNL_{\mathcal{R}}(Y) \cap \mathcal{V}_{/\mathcal{R}}]^{c} \in [\gamma_{\mathcal{R}}(Y)]^{c}
\]

Consider

\[
PNU_{\mathcal{R}}(Y)^{c} \cup [PNU_{\mathcal{R}}(Y)]^{c} = [PNU_{\mathcal{R}}(Y) \cap \mathcal{V}_{/\mathcal{R}}]^{c} = [PNU_{\mathcal{R}}(Y) \cap \mathcal{V}_{/\mathcal{R}}]^{c} \in [\gamma_{\mathcal{R}}(Y)]^{c}
\]
Thus arbitrary union of elements of \([\tau_{\mathcal{V}}(Y)]^c\) is in \([\tau_{\mathcal{V}}(Y)]^c\).
\[\mathcal{P}NL_{\mathcal{V}}(Y)^c \cap \mathcal{P}NU_{\mathcal{V}}(Y)^c = \mathcal{P}NU_{\mathcal{V}}(Y)^c \in [\tau_{\mathcal{V}}(Y)]^c\]
and
\[\mathcal{P}NL_{\mathcal{V}}(Y)^c \cap \mathcal{P}NB_{\mathcal{V}}(Y)^c = \mathcal{P}NU_{\mathcal{V}}(Y)^c \in [\tau_{\mathcal{V}}(Y)]^c\]
since \(\mathcal{P}NL_{\mathcal{V}}(Y) \cup \mathcal{P}NB_{\mathcal{V}}(Y) \in \tau_{\mathcal{V}}(Y)\),
\[\mathcal{P}NB_{\mathcal{V}}(Y)^c \cap \mathcal{P}NU_{\mathcal{V}}(Y)^c = \mathcal{P}NU_{\mathcal{V}}(Y)^c \in [\tau_{\mathcal{V}}(Y)]^c\]
i.e., finite intersection of elements of \([\tau_{\mathcal{V}}(Y)]^c\) is in \([\tau_{\mathcal{V}}(Y)]^c\).
\[\therefore [\tau_{\mathcal{V}}(Y)]^c\] is a PNT on \(\mathcal{V}\).

**Remark 3.6:**
\([\tau_{\mathcal{V}}(Y)]^c\) is called the dual PNT of \(\tau_{\mathcal{V}}(Y)\). Elements of dual PNT are called as Pythagorean nano closed sets (PNCS). We have a result from the above theorem that “a subset \(A\) of \(\mathcal{V}\) is PNC in \(\tau_{\mathcal{V}}(Y)\) if and only if \(\mathcal{V} - A\) is a PNO in \(\tau_{\mathcal{V}}(Y)\)”.

**Definition 3.7:**
Let the PNT space be \(\langle \mathcal{V}, \tau_{\mathcal{V}}(Y) \rangle\) w.r.t \(Y \subseteq \mathcal{V}\), then the coalition of all PNO subsets of \(\mathcal{V}\) is the Pythagorean nano interior denoted as \(\mathcal{P}NI\mathcal{nt}(A)\). Otherwise defined as the largest PNO subset of \(A\).

The intersection of all PNCs having \(A \subseteq \mathcal{V}\) is named as Pythagorean nano closure denoted by \(\mathcal{P}NC\mathcal{l}(A)\). Likewise defined as the smallest PNC set containing \(A\).

**Remark 3.8:**
Let \(\langle \mathcal{V}, \tau_{\mathcal{V}}(Y) \rangle\) be a PNT space w.r.t \(Y\), where \(Y\) is non-void subset of \(\mathcal{V}\). The PNC sets in \(\mathcal{V}\) are \(\mathcal{P}NL_{\mathcal{V}}(Y)^c\), \(\mathcal{P}NU_{\mathcal{V}}(Y)^c\), \(\mathcal{P}NB_{\mathcal{V}}(Y)^c\).
\[\mathcal{P}NL_{\mathcal{V}}(Y)^c \subseteq Y \Rightarrow [\mathcal{V}]^c \subseteq [\mathcal{P}NL_{\mathcal{V}}(Y)^c]^c\] and \([\mathcal{P}NL_{\mathcal{V}}(Y)^c]^c\) is a PNCs but it does not contain \(Y\).
Since \(Y \subseteq \mathcal{P}NU_{\mathcal{V}}(Y)^c\) which implies \([\mathcal{P}NU_{\mathcal{V}}(Y)^c]^c \subseteq [\mathcal{V}]^c\) and \([\mathcal{P}NU_{\mathcal{V}}(Y)^c]^c\) does not contain \(Y\), \([\mathcal{P}NU_{\mathcal{V}}(Y)^c]^c\) is a PNCs.
Also, \([\mathcal{P}NB_{\mathcal{V}}(Y)^c]^c\) is a PNCs, but does not contain \(Y\).
Thus, the PNC sets \([\mathcal{P}NL_{\mathcal{V}}(Y)^c]^c\), \([\mathcal{P}NU_{\mathcal{V}}(Y)^c]^c\), \([\mathcal{P}NB_{\mathcal{V}}(Y)^c]^c\) in \(\langle \mathcal{V}, \tau_{\mathcal{V}}(Y) \rangle\) but none contains \(Y\).
\[\therefore \mathcal{V}\] is the only PNC set that contains \(Y\), i.e., \(\mathcal{P}NC\mathcal{l}(Y) = \mathcal{V}\) in \(\langle \mathcal{V}, \tau_{\mathcal{V}}(Y) \rangle\).

**Theorem 3.9:**
Let \(\langle \mathcal{V}, \tau_{\mathcal{V}}(Y) \rangle\) be a PNT space w.r.t \(Y\), where \(Y \subseteq \mathcal{V}\). Let \(A, B \subseteq \mathcal{V}\). Then
a. \(A \subseteq \mathcal{P}NC\mathcal{l}(A)\)
b. if \(A \subseteq \mathcal{P}NC\mathcal{l}(A)\)
c. \(\mathcal{P}NC\mathcal{l}([\mathcal{P}P_{\mathcal{O}}(P)] = \mathcal{P}P_{\mathcal{O}}(P)\)
d. \(A \subseteq B \Rightarrow \mathcal{P}NC\mathcal{l}(A) \subseteq \mathcal{P}NC\mathcal{l}(B)\)
e. \(\mathcal{P}NC\mathcal{l}(A \cup B) = \mathcal{P}NC\mathcal{l}(A) \cup \mathcal{P}NC\mathcal{l}(B)\)
f. \(\mathcal{P}NC\mathcal{l}(A \cap B) \subseteq \mathcal{P}NC\mathcal{l}(A) \cap \mathcal{P}NC\mathcal{l}(B)\)
g. \(\mathcal{P}NC\mathcal{l}(\mathcal{P}NC\mathcal{l}(A)) = \mathcal{P}NC\mathcal{l}(A)\)

Proof:

a. By definition of Pythagorean nano closure (i.e., closure of a PNO set is the minutest PNC set that contains \(A\)).

b. Consider \(A\), the PNC set, then it is the closest PNC that contains \(A\).
\[\therefore \mathcal{P}NC\mathcal{l}(A) = A\]

Conversely, let \(\mathcal{P}NC\mathcal{l}(A) = A\). By definition \(\mathcal{P}NC\mathcal{l}(A)\) is the tiniest closed set that contains \(A\). This implies \(A\) is PNC set.

c. It is a known fact that \(\mathcal{O}_{\mathcal{P}}, \mathcal{V}_{\mathcal{P}}\) are PNC sets then by (b)
\[\mathcal{P}NC\mathcal{l}(\mathcal{O}_{\mathcal{P}}) = \mathcal{O}_{\mathcal{P}}\]
\[\mathcal{P}NC\mathcal{l}(\mathcal{V}_{\mathcal{P}}) = \mathcal{V}_{\mathcal{P}}\]

d. Let \(A \subseteq B\), definition of Pythagorean nano closure implies that \(B \subseteq \mathcal{P}NC\mathcal{l}(B)\).
\[A \subseteq B \land B \subseteq \mathcal{P}NC\mathcal{l}(B) \Rightarrow A \subseteq \mathcal{P}NC\mathcal{l}(B)\]
Since \(\mathcal{P}NC\mathcal{l}(A)\) is the closest closed set containing \(A\), \(A \subseteq \mathcal{P}NC\mathcal{l}(A)\).
\[\therefore \mathcal{P}NC\mathcal{l}(A) \subseteq \mathcal{P}NC\mathcal{l}(B)\]

\[\begin{align*}
e. & A \subseteq A \cup B, B \subseteq A \cup B, \text{ by (d) we get} \\
& \mathcal{P}NC\mathcal{l}(A) \subseteq \mathcal{P}NC\mathcal{l}(A \cup B) \\
& \mathcal{P}NC\mathcal{l}(B) \subseteq \mathcal{P}NC\mathcal{l}(A \cup B)
\end{align*}\]

f. \(A \cup B \subseteq A \text{ and } A \cup B \subseteq B\) that implies
\[\mathcal{P}NC\mathcal{l}(A \cup B) \subseteq \mathcal{P}NC\mathcal{l}(A) \cup \mathcal{P}NC\mathcal{l}(B)\]

\[\begin{align*}
g. & \therefore \mathcal{P}NC\mathcal{l}(A) \subseteq \mathcal{P}NC\mathcal{l}(B) \subseteq \mathcal{P}NC\mathcal{l}(B) \\
& \therefore \mathcal{P}NC\mathcal{l}(A \cap B) \subseteq \mathcal{P}NC\mathcal{l}(A) \land \mathcal{P}NC\mathcal{l}(B)
\end{align*}\]

\[\begin{align*}
& \therefore \mathcal{P}NC\mathcal{l}(\mathcal{P}NC\mathcal{l}(A)) = \mathcal{P}NC\mathcal{l}(A)
\end{align*}\]

**Theorem 3.10:** (Kuratowski closure axiom)
The Pythagorean nano closure in a PNT space \(\langle \mathcal{V}, \tau_{\mathcal{V}}(Y) \rangle\) is the Kuratowski closure operator.

Proof:
The Kuratowski’s closure axioms are
\[\begin{align*}
\mathcal{P}NC\mathcal{l}(\mathcal{O}_{\mathcal{P}}) = \mathcal{O}_{\mathcal{P}}\land A \subseteq \\
\mathcal{P}NC\mathcal{l}(A), \mathcal{P}NC\mathcal{l}(A) \subseteq \mathcal{P}NC\mathcal{l}(A) \cup \mathcal{P}NC\mathcal{l}(B) \subseteq \\
\mathcal{P}NC\mathcal{l}(B) \land \mathcal{P}NC\mathcal{l}(\mathcal{P}NC\mathcal{l}(A)) = \\
\mathcal{P}NC\mathcal{l}(A), \text{ where } A, B \subseteq \mathcal{V}
\end{align*}\]

By previous theorem, Pythagorean nano closure in PNT space satisfies the above axiom. Thus the Pythagorean nano closure is a Kuratowski closure operator.
Theorem 3.11:
In a PNT space \((\mathcal{V}, \tau)\), \(x \in PNC\ell(A) \iff G \cap A \neq \phi\)
for every PNO set \(G\) containing \(x\), where \(A \subseteq \mathcal{V}\).
Proof:
First consider \(x \in PNC\ell(A)\). Let \(G\) be a PNO set containing \(x\). Since \(G\) is PNO, \(\mathcal{V} - G\) is a PNC set. If \(A \cap G = \phi\), then \(A \subseteq \mathcal{V} - \mathcal{V} \cap \mathcal{V}\), which is contradiction to the fact that \(x \in PNC\ell(A)\) but \(x \notin \mathcal{V} - G\).

Therefore \(A \cap G \neq \phi\) for every PNO set \(G\) having \(x\).

Conversely, let \(A \cap G \neq \phi\) for every PNO set \(G\) having \(x\). If \(x \notin PNC\ell(A)\), then \(x \notin \mathcal{V} - PNC\ell(A)\). \(\mathcal{V} - PNC\ell(A)\) is PNO set and thus \(\mathcal{V} - PNC\ell(A) \cap A \neq \phi\) (by assumption).

\(\mathcal{A} \subseteq PNC\ell(A) \implies PNC\ell(A)^c \subseteq \mathcal{A}^c \implies (\mathcal{V} - PNC\ell(A)) \cap A \subseteq (\mathcal{V} - \mathcal{V}^c) \cap A\)
which is contradiction.
\(: x \in PNC\ell(A)\).

Theorem 3.12:
Let \((\mathcal{V}, \tau)\) be a PNT space w.r.t \(Y\) where \(Y \subseteq \mathcal{V}\). Let \(A \subseteq \mathcal{V}\).

1. \(\mathcal{V} - PNC\ell(A) = \text{PNC}\ell(V_p - A)\)
2. \(\mathcal{V} - PNC\ell(A) = \text{PNC}\ell(V_p - A)\)

Proof:
1. Let \(x \in \mathcal{V} - PNC\ell(A) \implies x \in PNC\ell(A)^c\).

Thus, \(G \subseteq A\) for every PNO set \(G\) containing \(x\).
\(: G \subseteq A \implies G \cap (\mathcal{V} - A) \neq \phi\) for each PNO set \(G\) having \(x\).

By previous theorem, \(G \cap (\mathcal{V} - A) \neq \phi\) for every PNO set \(G\) having \(x\).
\(x \in PNC\ell(\mathcal{V} - A)\)
\(x \in PNC\ell(V_p - A)\)
\(x \in PNC\ell(V_p - A)\)
\(\text{PNC}\ell(V_p - A)\)
\(\text{PNC}\ell(V_p - A)\)

Now let \(x \in PNC\ell(V_p - A)\), then \(G \cap (\mathcal{V} - A) \neq \phi\) for each PNO set \(G\) containing \(x\).
\(\text{i.e., } G \subseteq A\) for every PNO set \(G\) containing \(x\) i.e., \(x \in PNC\ell(A)\).

Therefore,
\(x \in \mathcal{V} - PNC\ell(A) \implies x \in PNC\ell(V_p - A) \subseteq \text{PNC}\ell(V_p - A)\)
\(\mathcal{V} - PNC\ell(A) = \text{PNC}\ell(V_p - A)\)

2. Let \(x \in PNC\ell(A)\) and \(x \notin PNC\ell(A)\).

Since \(x \in PNC\ell(A)\), by previous theorem for every PNO set \(G\) containing \(x\), \(G \cap A = \phi\)
\(G \subseteq \mathcal{V} \cap A \subseteq \mathcal{V}\).
\(\implies x \in PNC\ell(V_p - A) \subseteq \text{PNC}\ell(V_p - A)\)

\(\text{PNC}\ell(V_p - A) \subseteq \text{PNC}\ell(V_p - A)\)

Conversely, if \(x \in PNC\ell(V_p - A)\), then \(\mathcal{V} = \mathcal{V} - A\) for each PNO set \(G\) containing \(x\).
\(\text{PNC}\ell(V_p - A) \subseteq \text{PNC}\ell(V_p - A)\)

\(\text{PNC}\ell(V_p - A) \subseteq \text{PNC}\ell(V_p - A)\)

By theorem, \(x \in PNC\ell(A)\), but \(x \in \mathcal{V} \subseteq \mathcal{V}\).
\(\implies x \in PNC\ell(V_p - A) \subseteq \text{PNC}\ell(V_p - A)\)

\(\text{PNC}\ell(V_p - A) \subseteq \text{PNC}\ell(V_p - A)\)

Remark 3.13:
We get \(PNC\ell(A) = \mathcal{V} - PNC\ell(V_p - A)\) and \(\text{PNC}\ell(A) = \text{PNC}\ell(V_p - A)\) by taking complement on above theorem.

Theorem 3.14:
Let \((\mathcal{V}, \tau)\) be a PNT space w.r.t \(Y\) where \(Y \subseteq \mathcal{V}\) and \(A, B\) are subsets of \(\mathcal{V}\), then:

a. \(A\) is PNO \(\iff\) \(PNC\ell(A) = A\)

b. \(PNC\ell(\emptyset) = \emptyset\) and \(PNC\ell(\mathcal{V}) = \mathcal{V}\)

c. \(A \subseteq B \implies PNC\ell(A) \subseteq PNC\ell(B)\)

d. \(PNC\ell(A) \cup PNC\ell(B) \subseteq PNC\ell(A \cup B)\)

e. \(PNC\ell(A) \cap PNC\ell(B) \subseteq PNC\ell(A \cap B)\)

Proof:
\(\text{a.}\)
Let \(A\) be a PNO set
\(\iff 0 = \emptyset\) is a PNC in \(\mathcal{V}\)
\(\iff PNC\ell(\emptyset) = \emptyset\) (by result)
\(\iff PNC\ell(\emptyset) = A\)
\(\iff PNC\ell(\emptyset) = (\mathcal{V} - A)^c\)

\(\text{b.}\)
\(\emptyset\) and \(\mathcal{V}\) are PNO sets, thus
\(\text{PNC}\ell(\emptyset) = \emptyset\) and \(\text{PNC}\ell(\mathcal{V}) = \mathcal{V}\)

\(\text{c.}\)
\(A \subseteq B \implies \mathcal{V} - B \subseteq \mathcal{V} - A\)
\(\iff PNC\ell(\mathcal{V} - B) \subseteq PNC\ell(\mathcal{V} - A)\)

\(\text{d.}\)
\(\text{PNC}\ell(A) \cup \text{PNC}\ell(B) \subseteq \text{PNC}\ell(A \cup B)\)

\(\text{e.}\)
\(\text{PNC}\ell(A) \cap \text{PNC}\ell(B) \subseteq \text{PNC}\ell(A \cap B)\)
\[ \text{d. } A \subseteq U, B \subseteq U \Rightarrow \overline{P \text{int}(A \cup B)} \subseteq \overline{P \text{int}(A)} \cup \overline{P \text{int}(B)} \]

Thus
\[ \overline{P \text{int}(A \cup B)} \subseteq \overline{P \text{int}(A \cup B)} \]

\[ \text{e. } A \cap B \subseteq C \subseteq A \cup B \Rightarrow \overline{P \text{int}(C)} \subseteq \overline{P \text{int}(A)} \cap \overline{P \text{int}(B)} \]

\[ \overline{P \text{int}(A \cap B)} \subseteq \overline{P \text{int}(A \cap B)} \]

\[ \overline{P \text{int}(A \cap B)} = \overline{P \text{int}(A \cap B)} \]

f. \( \overline{P \text{int}(P \text{int}(A))} \) is a PNO set, thus
\[ \overline{P \text{int}(P \text{int}(A))} \]

IV. CONCLUSION

Herein the idea of Pythagorean nano topological space has been established. Some of its properties have been discussed. Further, continuity of Pythagorean nano topological spaces, applications of Pythagorean nano topological space may be studied.

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