OPEN ACCESS
International Journal of Recent Technology and Engineering (IJRTE)
ISSN: 2277-3878, Volume-8 Issue-5, January 2020

Pythagorean Nano Topological Space

D. Ajay, J. Joseline Charisma

Abstract: ***Our main objective is to introduce the concept of Pythagorean nano fuzzy topological space by motivating from the notion of fuzzy topological space and nano topological space and properties of Pythagorean nano topology are also examined.***

Keywords: ***Pythagorean fuzzy nano topology, Pythagorean nano topological space, Pythagorean nano open sets, Pythagorean nano closure, Pythagorean nano interior.***

I. INTRODUCTION

L.A. Zadeh [1] generalized the usual set by using fuzzy set in which every detail is described with a degree of membership function $\mu_A: X \to [0,1]$. The fuzzy set has many programs in economic system, decision making, facts mining, commercial enterprise and many others. Fuzzy set has been generalized to greater non-standard fuzzy subsets. As Intuitionistic fuzzy subset become introduced with the aid of Atanassov [2], in which every element had the degree of membership and the degree of non- membership. Yager [3,4] presented the perception of Pythagorean fuzzy subset that is atypical fuzzy subsets and which has many powerful applications in natural and social sciences. Pythagorean fuzzy subsets can be used appropriately on every instant where intuitionistic fuzzy subsets cannot be used.

Standard topology has been stepped forward by means of taking its motivation from classical analysis and applied on several sections of research inclusive of system getting to know, statistics evaluation, facts mining. Farther the scrutiny of topology refers the relationship between spatial gadgets and features and it may be used to explain some sure spatial functions and to conceive statistics units which have higher great control and extra statistics integrity. In 1968, Chang [5] described the theory of fuzzy topological space and generalized some fundamental idea of topology inclusive of open set, closed set, continuity and compactness. Following this observation, Lowen gave a different explanation of a fuzzy topological space by way of converting a primary property of topology [6]. In 1995, Coker delivered the notion of intuitionistic fuzzy topological space and studied some equivalent variations of some standards of classical topology together with continuity and compactness [7]. Furthermore, some authors studied the concept of fuzzy soft topological space and its packages in choice-making environment.

The concept of nano topological spaces turned into introduced via L. Thivagar [8]. Ramachandran & Stephan coined Intuitionistic nano topology [9] in 2017. The concept of nano topology thru neutrosophic units [10] become additionally coined through L. Thivagar et.al.

The fundamental cause of this paper is to introduce the whim of Pythagorean fuzzy nano topological space. After specifying Pythagorean fuzzy nano topological space, we inspect the simple properties of nano topological space.

II. PRELIMINARIES

Definition 2.1:

Let W be a non-empty fixed set & $O \subseteq W$ then O is stated to be a fuzzy subset and is of the form $O = \{\langle x, \rho_O(x)\rangle, x \in W\}$ where in $\rho_O: W \to [0,1]$ is the degree of membership function.

Definition 2.2:

Let D be a non-empty fixed set & $B \subseteq D$ then B is said to be an Intuitionistic fuzzy subset of the shape $B = \{\langle x, \vartheta(x), \mu(x)\rangle, x \in D\}$ where in $\vartheta: D \to [0,1], \mu: D \to [0,1]$ is the degree of membership function and non-membership function gratifying $\vartheta(x) + \mu(x) \leq 1$.

Definition 2.3:

Let $\mathcal{L}$ be a non-empty set & $I \subseteq \mathcal{L}$ then I is stated to be Pythagorean fuzzy subset & expressed as $I = \{\langle x, \vartheta(x), \mu(x)\rangle, x \in \mathcal{L}\}$ in which $\vartheta: \mathcal{L} \to [0,1]$, $\mu: \mathcal{L} \to [0,1]$ is the degree of membership and non-membership function fulfilling $\vartheta^2(x) + \mu^2(x) \leq 1$.

Definition 2.4:

Let $\mathcal{H}$ be a non-empty finite set of items known as the universe and $\mathfrak{T}$ be the equivalence relation on $\mathcal{H}$. The pair $(\mathcal{H}, \mathfrak{T})$ is stated to be approximation space. Let $K \subseteq \mathcal{H}$.

The smaller approximation of K with admire to $\mathfrak{T}$ is the set of items, which can be for certain categorised as K with recognize to $\mathfrak{T}$ and it is denoted by using $L_{\mathfrak{T}}(x)$. $L_{\mathfrak{T}}(x) = \bigcup_{x \in \mathcal{H}} \mathfrak{T}(x): \mathfrak{T}(x) \subseteq X$, $\mathfrak{T}$ (x) is the equivalence relation determined by way of x. The greater approximation is denoted as $U_{\mathfrak{T}}(x)$ and described as $U_{\mathfrak{T}}(x) = \bigcup_{x \in \mathcal{H}} \mathfrak{T}(x): \mathfrak{T}(x) \cap K \neq \emptyset$. The boundary vicinity of K, $B_{\mathfrak{T}}(x) = U_{\mathfrak{T}}(x) - L_{\mathfrak{T}}(x)$.

Let $\mathcal{H}$ be an universe, $\mathfrak{T}$ be an equivalence relation on $\mathcal{H}$ and

Manuscript published on January 30, 2020.
* Correspondence Author
D. Ajay*, P.G and Research Department of Mathematics, Sacred Heart College, Tirupattur – 635 601. Email: dajaypravin@gmail.com
J. Joseline Charisma, P.G and Research Department of Mathematics, Sacred Heart College, Tirupattur-635 601. Email: joecharish@gmail.com

$\tau_{\mathfrak{X}}(K) = \{\emptyset, U, L_{\mathfrak{X}}(x), B_{\mathfrak{X}}(x), U_{\mathfrak{X}}(x)\}$ in which $K \subseteq \mathcal{H}$. $\tau_{\mathfrak{X}}(K)$ satisfies the subsequent axioms: $\mathcal{H}$ and $\phi \in \tau_{\mathfrak{X}}(K)$, union of the factors of any sub-series of $\tau_{\mathfrak{X}}(K)$ is in $\mathfrak{T}(K)$, intersection of the factors of any finite sub-collection of $\tau_{\mathfrak{X}}(K)$ is in $\tau_{\mathfrak{X}}(K)$. Then $(\mathcal{H}, \tau_{\mathfrak{X}}(K))$ is named as the Nano topological space.

Definition 2.5:

Let $J \neq \emptyset$ be a set and allow ρ be a family of Pythagorean fuzzy subsets of J. If $1_X, 0_X \in \rho$, for any $S_1, S_2 \in \rho$, we've $S_1 \cap S_2 \in \rho$, for any $\{S_i\}_{i\in I} \subset \tau$, are satisfied for all i, $\cup S_i \in \tau$ where I is an arbitrary index set, then τ is called a Pythagorean fuzzy topology on J. The couple (J, ρ) is told to be a Pythagorean fuzzy topological space.

III. PYTHAGOREAN NANO TOPOLOGICAL SPACE

In this portion, the idea of Pythagorean nano topology through nano Pythagorean approximations specifically Pythagorean nano lower, Pythagorean nano upper and Pythagorean nano boundary is added with its simple properties discussed.

Definition 3.1:

Let $\mathcal{V}$ be a non-void set and $\mathfrak{R}$ an equivalence relation on $\mathcal{V}$. Let D be a Pythagorean set in $\mathcal{V}$ with membership θ_D, non-membership ω_D . The Pythagorean nano minor approximation, Pythagorean nano major approximation and Pythagorean nano border of D in the approximation space $(\mathcal{V}, \mathfrak{R})$ denoted by $PNL_{\mathfrak{R}}(D), PNU_{\mathfrak{R}}(D), PNB_{\mathfrak{R}}(D)$ are respectively defined as:

$PNL_{\mathfrak{R}}(D) = \{\langle x, \theta_{LD}(x), \omega_{LD}(x) | z \in [x]_{\mathfrak{R}}, x \in \mathcal{V}\rangle\}$

$PNU_{\mathfrak{R}}(D) = \{\langle x, \theta_{RD}(x), \omega_{RD}(x) | z \in [x]_{\mathfrak{R}}, x \in \mathcal{V}\rangle\}$

$PNB_{\mathfrak{R}}(D) = PNU_{\mathfrak{R}}(D) - PNL_{\mathfrak{R}}(D)$

where $\theta_{LD}(x) = \wedge_{y\in[x]_{\mathfrak{R}}} \theta_D(y), \omega_{LD}(x) = \vee_{y\in[x]_{\mathfrak{R}}} \omega_{LD}(y)$

and

$$\theta_{RD}(x) = \vee_{y\in[x]_{\mathfrak{R}}} \theta_D(y), \omega_{RD}(x) = \wedge_{y\in[x]_{\mathfrak{R}}} \omega_D(y).$$

Definition 3.2:

Let the Universe be $\mathcal{V}$, equivalence relation on $\mathcal{V}$ be $\mathfrak{R}$ and if

$\tau_{\mathfrak{R}}(Y) = \{ \emptyset_P, \mathcal{V}_P, PNL_{\mathfrak{R}}(Y), PNU_{\mathfrak{R}}(Y), PNB_{\mathfrak{R}}(Y)\}$ where $Y \subseteq \mathcal{V}$, $\tau_{\mathfrak{R}}(Y)$ satisfies the following axioms:

1. $\emptyset_P, \mathcal{V}_P \in \tau_{\mathfrak{R}}(Y)$

2. $If\ A_i \in \tau_{\mathfrak{R}}(Y)\ for\ i = 1,2, \ldots then \bigcup_{i=1}^{\infty} A_i \in \tau_{\mathfrak{R}}(Y)$

3. $If\ A_i \in \tau_{\mathfrak{R}}(Y)\ for\ i = 1,2, \ldots, n\ then \bigcap_{i=1}^{n} A_i \in \tau_{\mathfrak{R}}(Y)$

then $\tau_{\mathfrak{R}}(Y)$ is termed as Pythagorean Nano Topology on $\mathcal{V}$ *w.r.t* Y. Whereas

$\emptyset_P = \{\langle x, 0,1 | \forall\ x \in \mathcal{V}\rangle\}, \mathcal{V}_P = \{\langle x, 1,0 | \forall\ x \in \mathcal{V}\rangle\}$. We call $(\mathcal{V}, \tau_{\mathfrak{R}}(Y))$ as Pythagorean Nano Topological space. The elements of $\tau_{\mathfrak{R}}(Y)$ are called Pythagorean Nano Open (PNO) Sets.

Example 3.3:

Let $\mathcal{V} = \{p, q, r\}$ be the Universe,

$\mathcal{V}/_{\mathfrak{R}} = \{\{p, q\}, \{r\}\}$ be the equivalence relation on $\mathcal{V}$. Let $A = \{\langle p | (.7, .4)\rangle, \langle q | (.6, .6)\rangle, \langle r | (.5, .5)\rangle\}$ then

$PNL_{\mathfrak{R}}(A) = \{\langle p | (.6, .6)\rangle, \langle q | (.6, .6)\rangle, \langle r | (.5, .5)\rangle\}$

$PNU_{\mathfrak{R}}(A) = \{\langle p | (.7, .4)\rangle, \langle q | (.7, .4)\rangle, \langle r | (.5, .5)\rangle\}$

$PNB_{\mathfrak{R}}(A) = \{\langle p | (.4, .4)\rangle, \langle q | (.4, .4)\rangle, \langle r | (.5, .5)\rangle\}$

Thus

$$\tau_{\mathfrak{R}}(A) = \left\{\begin{array}{c} \emptyset_P, \mathcal{V}_P, \{\langle p | (.6, .6)\rangle, \langle q | (.6, .6)\rangle, \langle r | (.5, .5)\rangle\}, \\ \{\langle p | (.7, .4)\rangle, \langle q | (.7, .4)\rangle, \langle r | (.5, .5)\rangle\}, \{\langle p | (.4, .4)\rangle, \langle q | (.4, .4)\rangle, \langle r | (.5, .5)\rangle\} \end{array}\right\}$$

is a PNT.

Note: Pythagorean nano topology, Pythagorean Nano Open, Pythagorean Nano closed sets and with respect to will be denoted as PNT, PNOs, PNCs and *w.r.t* in the following theorems and sections.

Proposition 3.4:

Let $\mathcal{V}$ be a non-void finite universe and $Y \subseteq \mathcal{V}$.

a) If $PNL_{\mathfrak{R}}(Y) = \emptyset_P$ and $PNU_{\mathfrak{R}}(Y) = \mathcal{V}_P$, then $\tau_{\mathfrak{R}}(Y) = \{ \emptyset_P, \mathcal{V}_P\}$, the indiscrete Pythagorean nano topology on $\mathcal{V}$.

b) If $PNL_{\mathfrak{R}}(Y) = PNU_{\mathfrak{R}}(Y) = Y$, then the Pythagorean nano topology is $\tau_{\mathfrak{R}}(Y) = \{ \emptyset_P, \mathcal{V}_P, PNL_{\mathfrak{R}}(Y)\}$.

c) If $PNL_{\mathfrak{R}}(Y) = \emptyset_P$ & $PNU_{\mathfrak{R}}(Y) \neq \mathcal{V}_P$ then $\tau_{\mathfrak{R}}(Y) = \{ \emptyset_P, \mathcal{V}_P, PNU_{\mathfrak{R}}(Y)\}$ is the Pythagorean nano topology.

d) If $PNL_{\mathfrak{R}}(Y) \neq \emptyset_P$, and $PNU_{\mathfrak{R}}(Y) = \mathcal{V}_P$, then $\tau_{\mathfrak{R}}(Y) = \{ \emptyset_P, \mathcal{V}_P, PNL_{\mathfrak{R}}(Y), PNB_{\mathfrak{R}}(Y)\}$ is the Pythagorean nano topology.

$\tau_{\mathfrak{R}}(Y) = \{ \emptyset_P, \mathcal{V}_P, PNL_{\mathfrak{R}}(Y), PNU_{\mathfrak{R}}(Y), PNB_{\mathfrak{R}}(Y)\}$ is the discrete Pythagorean nano topology on $\mathcal{V}$.

Theorem 3.5:

Let $\mathcal{V}$ be a non-void, finite universe and $Y \subseteq \mathcal{V}$. Let $\tau_{\mathfrak{R}}(Y)$ be the PNT on $\mathcal{V}$ *w.r.t* Y. Then $[\tau_{\mathfrak{R}}(Y)]^c$ whose elements are A^c for $A \in \tau_{\mathfrak{R}}(Y)$, is a PNT on $\mathcal{V}$.

Proof:

The PNT on $\mathcal{V}$ *w.r.t* Y is given by

$\tau_{\mathfrak{R}}(Y) = \{ \emptyset_P, \mathcal{V}_P, PNL_{\mathfrak{R}}(Y), PNU_{\mathfrak{R}}(Y), PNB_{\mathfrak{R}}(Y)\}$.

Therefore,

$[\tau_{\mathfrak{R}}(Y)]^c =$

$\{ \emptyset_P, \mathcal{V}_P, [PNL_{\mathfrak{R}}(Y)]^c, [PNU_{\mathfrak{R}}(Y)]^c, [PNB_{\mathfrak{R}}(Y)]^c\}$.

Consider

$[PNL_{\mathfrak{R}}(Y)]^c \cup [PNU_{\mathfrak{R}}(Y)]^c = [PNL_{\mathfrak{R}}(Y) \cap PNU_{\mathfrak{R}}(Y)]^c = [PNL_{\mathfrak{R}}(Y)]^c \in [\tau_{\mathfrak{R}}(Y)]^c$

and

$[PNL_{\mathfrak{R}}(Y)]^c \cup [PNB_{\mathfrak{R}}(Y)]^c = [PNL_{\mathfrak{R}}(Y) \cap PNB_{\mathfrak{R}}(Y)]^c = [\emptyset_P]^c = \mathcal{V}_P \in [\tau_{\mathfrak{R}}(Y)]^c$
and
$[PNB_{\mathfrak{R}}(Y)]^c \cup [PNU_{\mathfrak{R}}(Y)]^c = [PNB_{\mathfrak{R}}(Y) \cap PNU_{\mathfrak{R}}(Y)]^c = [PNB_{\mathfrak{R}}(Y)]^c \in [\tau_{\mathfrak{R}}(Y)]^c$

Thus arbitrary union of elements of $[\tau_{\mathfrak{R}}(Y)]^c$ is in $[\tau_{\mathfrak{R}}(Y)]^c$.
$[PNL_{\mathfrak{R}}(Y)]^c \cap [PNU_{\mathfrak{R}}(Y)]^c = [PNL_{\mathfrak{R}}(Y) \cup PNU_{\mathfrak{R}}(Y)]^c = [PNU_{\mathfrak{R}}(Y)]^c \in [\tau_{\mathfrak{R}}(Y)]^c$
and
$[PNL_{\mathfrak{R}}(Y)]^c \cap [PNB_{\mathfrak{R}}(Y)]^c = [PNL_{\mathfrak{R}}(Y) \cup PNB_{\mathfrak{R}}(Y)]^c \in [\tau_{\mathfrak{R}}(Y)]^c$
since $PNL_{\mathfrak{R}}(Y) \cup PNB_{\mathfrak{R}}(Y) \in \tau_{\mathfrak{R}}(Y)$,
$[PNB_{\mathfrak{R}}(Y)]^c \cap [PNU_{\mathfrak{R}}(Y)]^c = [PNB_{\mathfrak{R}}(Y) \cup PNU_{\mathfrak{R}}(Y)]^c = [PNU_{\mathfrak{R}}(Y)]^c \in [\tau_{\mathfrak{R}}(Y)]^c$

i.e., finite intersection of elements of $[\tau_{\mathfrak{R}}(Y)]^c$ is in $[\tau_{\mathfrak{R}}(Y)]^c$.
$\therefore$ **$[\tau_{\mathfrak{R}}(Y)]^c$ is a PNT on $\mathcal{V}$.**

Remark 3.6:
$[\tau_{\mathfrak{R}}(Y)]^c$ is called the dual PNT of $\tau_{\mathfrak{R}}(Y)$. Elements of dual PNT are called as Pythagorean nano closed sets (PNCS). We have a result from the above theorem that "a subset A of $\mathcal{V}$ is PNC in $\tau_{\mathfrak{R}}(Y)$ if and only if $\mathcal{V} - A$ is a PNO in $\tau_{\mathfrak{R}}(Y)$ ".

Definition 3.7:
Let the PNT space be $(\mathcal{V}, \tau_{\mathfrak{R}}(Y))$ *w.r.t* $Y \subseteq \mathcal{V}$, then the coalition of all PNO subsets of $A \subseteq \mathcal{V}$ is the Pythagorean nano interior denoted as $\mathcal{PN}int(A)$. Otherwise defined as the largest PNO subset of A.
The intersection of all PNCs having $A \subseteq \mathcal{V}$ is named as Pythagorean nano closure denoted by $\mathcal{PN}c\ell(A)$. Likewise defined as the smallest PNC set containing A.

Remark 3.8:
Let $(\mathcal{V}, \tau_{\mathfrak{R}}(Y))$ be a PNT space *w.r.t* Y, where Y is non-void subset of $\mathcal{V}$. The PNC sets in $\mathcal{V}$ are
$0_P, \mathcal{V}_P, [PNL_{\mathfrak{R}}(Y)]^c, [PNU_{\mathfrak{R}}(Y)]^c, [PNB_{\mathfrak{R}}(Y)]^c$.
$\because PNL_{\mathfrak{R}}(Y) \subseteq Y \Rightarrow [Y]^c \subseteq [PNL_{\mathfrak{R}}(Y)]^c$ and $[PNL_{\mathfrak{R}}(Y)]^c$ is a PNCs but it does not contain Y.
Since $Y \subseteq PNU_{\mathfrak{R}}(Y) \Rightarrow [PNU_{\mathfrak{R}}(Y)]^c \subseteq [Y]^c$ and $[PNU_{\mathfrak{R}}(Y)]^c$ does not contain Y, $[PNU_{\mathfrak{R}}(Y)]^c$ is a PNCs.
Also, $[PNB_{\mathfrak{R}}(Y)]^c$ is a PNCs, but does not contain Y.
Thus, the PNC sets $[PNL_{\mathfrak{R}}(Y)]^c, [PNU_{\mathfrak{R}}(Y)]^c, [PNB_{\mathfrak{R}}(Y)]^c$ in $(\mathcal{V}, \tau_{\mathfrak{R}}(Y))$ but none contains Y.
$\therefore \mathcal{V}$ is the only PNC set that contains Y. i.e., $\mathcal{PN}c\ell(Y) = \mathcal{V}$ in $(\mathcal{V}, \tau_{\mathfrak{R}}(Y))$.

Theorem 3.9:

Let $(\mathcal{V}, \tau_{\mathfrak{R}}(Y))$ be a PNT space *w.r.t* Y where $Y \subseteq \mathcal{V}$. Let $A, B \subseteq \mathcal{V}$. Then

a. $A \subseteq \mathcal{PN}c\ell(A)$
b. A *is PNC* $\Leftrightarrow \mathcal{PN}c\ell(A) = A$
c. $\mathcal{PN}c\ell(\emptyset_P) = \emptyset_P$ *and* $\mathcal{PN}c\ell(\mathcal{V}_P) = \mathcal{V}_P$
d. $A \subseteq B \Rightarrow \mathcal{PN}c\ell(A) \subseteq \mathcal{PN}c\ell(B)$
e. $\mathcal{PN}c\ell(A \cup B) = \mathcal{PN}c\ell(A) \cup \mathcal{PN}c\ell(B)$
f. $\mathcal{PN}c\ell(A \cap B) \subseteq \mathcal{PN}c\ell(A) \cap \mathcal{PN}c\ell(B)$
g. $\mathcal{PN}c\ell(\mathcal{PN}c\ell(A)) = \mathcal{PN}c\ell(A)$

Proof:

a. By definition of Pythagorean nano closure (i.e., closure of a PNO set is the minutest PNC set that contains A).
$\Rightarrow \boldsymbol{A \subseteq \mathcal{PN}c\ell(A)}$

b. Consider A, the PNC set, then it is the closest PNC that contains A.
$\therefore$ $\boldsymbol{\mathcal{PN}c\ell(A) = A}$.
Conversely, let $\mathcal{PN}c\ell(A) = A$. By definition $\mathcal{PN}c\ell(A)$ is the tiniest closed set that contains A. This implies **A is PNC set**.

c. It is a known fact that $\emptyset_P$, $\mathcal{V}_P$ are PNC sets then by (b)
$\boldsymbol{\mathcal{PN}c\ell(\emptyset_P) = \emptyset_P}$ and $\boldsymbol{\mathcal{PN}c\ell(\mathcal{V}_P) = \mathcal{V}_P}$.

d. Let $A \subseteq B$, definition of Pythagorean nano closure implies that $B \subseteq \mathcal{PN}c\ell(B)$.
$A \subseteq B$ and $B \subseteq \mathcal{PN}c\ell(B) \Rightarrow A \subseteq \mathcal{PN}c\ell(B)$. Since $\mathcal{PN}c\ell(A)$ is the closest closed set containing A , $A \subseteq \mathcal{PN}c\ell(A)$.
$\therefore$ $\boldsymbol{\mathcal{PN}c\ell(A) \subseteq \mathcal{PN}c\ell(B)}$.

e. $\because$ $A \subseteq A \cup B, B \subseteq A \cup B$, by (d) we get $\mathcal{PN}c\ell(A) \subseteq \mathcal{PN}c\ell(A \cup B)$ and $\mathcal{PN}c\ell(B) \subseteq \mathcal{PN}c\ell(A \cup B)$
$\Rightarrow \mathcal{PN}c\ell(A) \cup \mathcal{PN}c\ell(B) \subseteq \mathcal{PN}c\ell(A \cup B)$
Since $A \cup B \subseteq \mathcal{PN}c\ell(A \cup B)$ and $A \cup B \subseteq \mathcal{PN}c\ell(A) \cup \mathcal{PN}c\ell(B)$
$\Rightarrow \mathcal{PN}c\ell(A \cup B) \subseteq \mathcal{PN}c\ell(A) \cup \mathcal{PN}c\ell(B)$
Thus $\boldsymbol{\mathcal{PN}c\ell(A \cup B) = \mathcal{PN}c\ell(A) \cup \mathcal{PN}c\ell(B)}$

f. $A \cap B \subseteq A$ *and* $A \cap B \subseteq B$ that implies $\mathcal{PN}c\ell(A \cap B) \subseteq \mathcal{PN}c\ell(A)$ and $\mathcal{PN}c\ell(A \cap B) \subseteq \mathcal{PN}c\ell(B)$
$\therefore$ $\boldsymbol{\mathcal{PN}c\ell(A \cap B) \subseteq \mathcal{PN}c\ell(A) \cap \mathcal{PN}c\ell(B)}$

g. Consider $\mathcal{PN}c\ell(A)$ is PNC then by (a),
$\boldsymbol{\mathcal{PN}c\ell(\mathcal{PN}c\ell(A)) = \mathcal{PN}c\ell(A)}$.

Theorem 3.10: (Kuratowski closure axiom)

The Pythagorean nano closure in a PNT space $(\mathcal{V}, \tau_{\mathfrak{R}}(Y))$ is the Kuratowski closure operator.
Proof:
The Kuratowski's closure axioms are
$\mathcal{PN}c\ell(\emptyset_P) = \emptyset_P$, $A \subseteq$

$\mathcal{PN}c\ell(A), \mathcal{PN}c\ell(A \cup B) = \mathcal{PN}c\ell(A) \cup \mathcal{PN}c\ell(B)$ and $\mathcal{PN}c\ell(\mathcal{PN}c\ell(A)) = \mathcal{PN}c\ell(A)$, where $A, B \subseteq \mathcal{V}$

By previous theorem, Pythagorean nano closure in PNT space satisfies the above axiom. Thus the Pythagorean nano closure is a Kuratowski closure operator.

Theorem 3.11:

In a PNT space $(\mathcal{V}, \tau_{\mathfrak{R}}(Y))$, $x \in \mathcal{PN}c\ell(A) \Leftrightarrow G \cap A \neq \phi$ for every PNO set G containing x, where $A \subseteq \mathcal{V}$.

Proof:

First consider $x \in \mathcal{PN}c\ell(A)$. Let G be a PNO set containing x. Since G is PNO, $\mathcal{V} - G$ is a PNC set. If $A \cap G = \phi$, then $A \subseteq \mathcal{V} - G$, $\mathcal{V} - G$ is a PNC set holding A. Thus $\mathcal{PN}c\ell(A) \subseteq \mathcal{V} - G$, which is contradiction to the fact that $x \in \mathcal{PN}c\ell(A)$ but $x \notin \mathcal{V} - G$.

Therefore $A \cap G \neq \phi$ for every PNO set G having x.

Conversely, let $A \cap G \neq \phi$ for every PNO set G having x. If $x \notin \mathcal{PN}c\ell(A)$ then $x \in \mathcal{V} - \mathcal{PN}c\ell(A)$, $\mathcal{V} - \mathcal{PN}c\ell(A)$ is PNO set and thus $\mathcal{V} - \mathcal{PN}c\ell(A) \cap A \neq \phi$ (by assumption).

$A \subseteq \mathcal{PN}c\ell(A) \Rightarrow \mathcal{PN}c\ell(A)^c \subseteq A^c \Rightarrow \mathcal{V} - \mathcal{PN}c\ell(A) \subseteq \mathcal{V} - A$

$\Rightarrow (\mathcal{V} - \mathcal{PN}c\ell(A)) \cap A \subseteq (\mathcal{V} - A) \cap A$,

which is contradiction.

$\therefore x \in \mathcal{PN}c\ell(A)$.

Theorem 3.12:

Let $(\mathcal{V}, \tau_{\mathfrak{R}}(Y))$ be a PNT space *w.r.t* Y where $Y \subseteq \mathcal{V}$. Let $A \subseteq \mathcal{V}$,

1. $\mathcal{V}_P - \mathcal{PN}int(A) = \mathcal{PN}c\ell(\mathcal{V}_P - A)$
2. $\mathcal{V}_P - \mathcal{PN}c\ell(A) = \mathcal{PN}int(\mathcal{V}_P - A)$

Proof:

1. Let $x \in \mathcal{V}_P - \mathcal{PN}int(A) \Rightarrow x \notin \mathcal{PN}int(A)$.

Thus, $G \nsubseteq A$ for every PNO set G containing x.

$\because G \nsubseteq A \Rightarrow G \subseteq \mathcal{V} - A, G \cap (\mathcal{V} - A) \neq \phi$ for each PNO set G having x.

By previous theorem, $G \cap (\mathcal{V} - A) \neq \phi$ for every PNO set G having

$x \Leftrightarrow x \in \mathcal{PN}c\ell(\mathcal{V} - A)$

$x \in \mathcal{V}_P - \mathcal{PN}int(A) \Rightarrow x \in \mathcal{PN}c\ell(\mathcal{V}_P - A)$

$\therefore \mathcal{V}_P - \mathcal{PN}int(A) \subseteq \mathcal{PN}c\ell(\mathcal{V}_P - A)$

Now let $x \in \mathcal{PN}c\ell(\mathcal{V}_P - A)$, then $G \cap (\mathcal{V} - A) \neq \phi$ for each PNO set G containing x . i.e., $G \nsubseteq A$ for every PNO set G containing x. i.e., $x \notin \mathcal{PN}int(A)$.

Therefore,

$x \in \mathcal{V}_P - \mathcal{PN}int(A) \Rightarrow \mathcal{PN}c\ell(\mathcal{V}_P - A) \subseteq \mathcal{V}_P - \mathcal{PN}int(A)$

$\boldsymbol{\mathcal{V}_P - \mathcal{PN}int(A) = \mathcal{PN}c\ell(\mathcal{V}_P - A)}$

2. Let $x \in \mathcal{V}_P - \mathcal{PN}c\ell(A) \Rightarrow x \in \mathcal{V}_P$ and $x \notin \mathcal{PN}c\ell(A)$.

Since $x \notin \mathcal{PN}c\ell(A)$, by previous theorem for every PNO set G containing x , $G \cap A = \phi$, $G \subseteq \mathcal{V}$ & $A \subseteq \mathcal{V}$.

$\Rightarrow x \notin A, x \in G \subseteq \mathcal{V}$

$\Rightarrow x \notin A$ & $x \in \mathcal{V} \Rightarrow x \in \mathcal{V} - A$

$\therefore x \in \mathcal{PN}int(\mathcal{V}_P - A)$

$\Rightarrow \mathcal{V}_P - \mathcal{PN}c\ell(A) \subseteq \mathcal{PN}int(\mathcal{V}_P - A)$.

Conversely, if $x \in \mathcal{PN}int(\mathcal{V}_P - A)$

$\Rightarrow x \in \mathcal{V} - A \Rightarrow x \notin A$ & $x \in \mathcal{V}$, then for each PNO set G containing x , $G \cap A = \phi$ for $G \subseteq \mathcal{V}$ & $A \subseteq \mathcal{V}$.

By theorem $x \notin \mathcal{PN}c\ell(A)$, but $x \in G \subseteq \mathcal{V}$.

$\Rightarrow x \in \mathcal{V}_P - \mathcal{PN}c\ell(A) \Rightarrow \mathcal{PN}int(\mathcal{V}_P - A) \subseteq \mathcal{V}_P - \mathcal{PN}c\ell(A)$

$\therefore \boldsymbol{\mathcal{V}_P - \mathcal{PN}c\ell(A) = \mathcal{PN}int(\mathcal{V}_P - A)}$

Remark 3.13:

We get $\mathcal{PN}int(A) = \mathcal{V}_P - \mathcal{PN}c\ell(\mathcal{V}_P - A)$ and $\mathcal{PN}c\ell(A) = \mathcal{V}_P - \mathcal{PN}int(\mathcal{V}_P - A)$ by taking complement on above theorem.

Theorem 3.14:

Let $(\mathcal{V}, \tau_{\mathfrak{R}}(Y))$ be a PNT space *w.r.t* Y where $Y \subseteq \mathcal{V}$ and A, B are subsets of $\mathcal{V}$, then

a. A is PNO $\Leftrightarrow \mathcal{PN}int(A) = A$
b. $\mathcal{PN}int(\emptyset_P) = \emptyset_P$ and $\mathcal{PN}int(\mathcal{V}_P) = \mathcal{V}_P$
c. $A \subseteq B \Rightarrow \mathcal{PN}int(A) \subseteq \mathcal{PN}int(B)$
d. $\mathcal{PN}int(A) \cup \mathcal{PN}int(B) \subseteq \mathcal{PN}int(A \cup B)$
e. $\mathcal{PN}int(A \cap B) = \mathcal{PN}int(A) \cap \mathcal{PN}int(B)$
f. $\mathcal{PN}int(\mathcal{PN}int(A)) = \mathcal{PN}int(A)$

Proof:

a. Let A be a PNO set
$\Leftrightarrow \mathcal{V}_P - A$ is a PNC in $\mathcal{V}$
$\Leftrightarrow \mathcal{PN}c\ell(1_P - A) = \mathcal{V}_P - A$ (By result A is PNC $\Leftrightarrow \mathcal{PN}c\ell(A) = A$)
$\Leftrightarrow \mathcal{PN}c\ell(\mathcal{V}_P - A)^c = (\mathcal{V}_P - A)^c$
$\Leftrightarrow \mathcal{V}_P - \mathcal{PN}c\ell(\mathcal{V}_P - A) = A$
By remark, $\mathcal{V}_P - \mathcal{PN}c\ell(\mathcal{V}_P - A) = \mathcal{PN}int(A)$
$\Leftrightarrow \boldsymbol{\mathcal{PN}int(A) = A}$.

b. $\emptyset_P$ & $\mathcal{V}_P$ are PNO sets, thus $\boldsymbol{\mathcal{PN}int(\emptyset_P) = \emptyset_P}$ and $\boldsymbol{\mathcal{PN}int(\mathcal{V}_P) = \mathcal{V}_P}$.

c. $A \subseteq B \Rightarrow \mathcal{V}_P - B \subseteq \mathcal{V}_P - A$
$\Rightarrow \mathcal{PN}c\ell(\mathcal{V}_P - B) \subseteq \mathcal{PN}c\ell(\mathcal{V}_P - A)$ (by $A \subseteq B \Rightarrow \mathcal{PN}c\ell(A) \subseteq \mathcal{PN}c\ell(B)$)
$\Rightarrow \mathcal{PN}c\ell(\mathcal{V}_P - A)^c \subseteq \mathcal{PN}c\ell(\mathcal{V}_P - B)^c$
$\Rightarrow \mathcal{V}_P - \mathcal{PN}c\ell(\mathcal{V}_P - A) \subseteq \mathcal{V}_P - \mathcal{PN}c\ell(1_P - B)$
$\Rightarrow \boldsymbol{\mathcal{PN}int(A) \subseteq \mathcal{PN}int(B)}$

d. $A \subseteq A \cup B, B \subseteq A \cup B \Rightarrow \mathcal{PN}int(A) \subseteq \mathcal{PN}int(A \cup B)$ and $\mathcal{PN}int(B) \subseteq \mathcal{PN}int(A \cup B)$
Thus
$\boldsymbol{\mathcal{PN}int(A) \cup \mathcal{PN}int(B) \subseteq \mathcal{PN}int(A \cup B)}$.

e. $A \cap B \subseteq A$ & $A \cap B \subseteq B \Rightarrow \mathcal{PN}int(A \cap B) \subseteq \mathcal{PN}int(A)$ & $\mathcal{PN}int(A \cap B) \subseteq \mathcal{PN}int(B)$
$\Rightarrow \mathcal{PN}int(A \cap B) \subseteq \mathcal{PN}int(A) \cap \mathcal{PN}int(B)$
$\mathcal{PN}int(A \cap B)$ is the biggest PNO set enclosed in $A \cap B$.
$\Rightarrow \mathcal{PN}int(A \cap B) \subseteq A \cap B$
$\Rightarrow \mathcal{PN}int(A) \cap \mathcal{PN}int(B) \subseteq \mathcal{PN}int(A \cap B)$
$\boldsymbol{\mathcal{PN}int(A \cap B) = \mathcal{PN}int(A) \cap \mathcal{PN}int(B)}$

f. $\mathcal{PN}int(A)$ is a PNO set, thus $\boldsymbol{\mathcal{PN}int(\mathcal{PN}int(A)) = \mathcal{PN}int(A)}$ (by (a)).

IV. CONCLUSION

Herein the idea of Pythagorean nano topological space has been established. Some of its properties have been discussed. Further, continuity of Pythagorean nano topological spaces, applications of Pythagorean nano topological space may be studied.

REFERENCES

1. L. A. Zadeh Fuzzy sets, Information and Control 8(1965), 338-353.
2. K. T. Atanassov Intuitionstic fuzzy sets, Fuzzy sets and systems, 20(1), (1986), 87-96.
3. Yager, R. R., & Abbasov, A. M., Pythagorean membership grades, complex numbers, and decision making, International Journal of Intelligent Systems, 28(5), (2013), 436-452.
4. Yager RR, Pythagorean membership grades in multicriteria decision making, IEEE Trans Fuzzy Syst 22(4), (2014), 958–965.
5. Chang CL, Fuzzy topological spaces. J Math Anal Appl 24(1), (1968), 182–190
6. Lowen R, Fuzzy topological spaces and fuzzy compactness, J Math Anal Appl 56(3), (1976), 621–633.
7. Coker D, Haydar Es, A On fuzzy compactness in intuitionistic fuzzy topological spaces, J Fuzzy Math 3, (1995), 899–910.
8. Lellis Thivagar M, Richard C. On Nano Continuity, Mathematical Theory and Modelling, 3(7), 2013, 32–7.
9. Ramachandran, M., & Raj, A. S. A., Intuitionistic Fuzzy Nano Topological Space: Theory and Applications. Sciexplore, International Journal of Research in Science, 4(1), (2017), 1-6.
10. M. Lellis Thivagar, S. Jafari, V. Sutha Devi, V. Antonysamy, A novel approach to nano topology via neutrosophic sets, Neutrosophic Sets and Systems, 20, (2018),86-94.
11. Coker, D., An introduction to intuitionistic topological spaces, Busefal, 81(2000), 51-56.
12. Kim, J. H., P. K. Lim, J. G. Lee, and K. Hur, Intuitionistic topological spaces, Infinite Study, 2018.

AUTHORS PROFILE

Dr. D. Ajay, Assistant professor in Sacred Heart College, Tirupattur Dt. Have published remarkable 21 research articles in reputed journals, with wonderful six years of teaching experience and co-author of two books. Area of interest is fuzzy logic, fuzzy subsets, fuzzy graphs and specialized in fuzzy decision making. e-mail: dajaypravin@gmail.com

J. Joseline Charisma, pursuing Ph. D in Sacred Heart College, Tirupattur Dt. An active researcher with interest in fuzzy subsets, fuzzy graphs, fuzzy topology and Pythagorean topology. e-mail: joecharish@gmail.com