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Flexural Interpretation of Simply Supported Laminated Composite Beam

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Abstract*: In this Paper, the analysis of simply supported laminated composite beam having uniformly distributed load is performed. The solutions obtained in the form of the displacements and stresses for different layered cross ply laminated composite simply supported beams subjected uniformly distributed to load. Different aspect ratio consider for different results in terms of displacement, bending stress and shear stresses. The shear stresses are calculated with the help of equilibrium equation and constitutive relationship. Using displacement field including trigonometric function of laminated composite beams are derived from virtual displacement principle. There are axial displacement, transverse displacement, bending stress and shear stresses. In addition, Euler-Bernoulli (ETB), First order shear deformation beam theory (FSDT), Higher order shear deformation beam theory (HSDT) and Hyperbolic shear deformation beam theory (HYSDT) solution have been made for comparison and better accuracy of solutions and results of static analyses of laminated composite beams for simply supported laminated composite beam.

Keywords: Laminated composite beam, Principle of virtual work, Trigonometric shear deformation theory, Uniformly distributed load

I. INTRODUCTION

The laminated beam is made of two or more different materials. Laminated beam having support of simply support with different later of angle fly. Laminated beam having more load resisting strength as compare to normal beam.

Timoshenko [1] define the downsides of the essential beam hypothesis by building up a beam hypothesis to incorporate the impact of the transverse shear misshapening. This hypothesis expect a steady shear strain over the thickness of the beam and requires an issue subordinate shear adjustment factor. Nguye *et al*. [2] exhibited twisting practices of overlaid composite beams. Lagrange's conditions are utilized to frame administering conditions. Ritz strategy is applied to proposed another half and half shape capacities. Karamanli Armagan [3] displayed the bowing conduct of overlaid composite bar with various limit condition, for discovering the arrangement of issues use shape capacities for hub, transverse diversions. Ghugal and Shikhare [4] explored the comparable single layer trigonometric shear distortion hypothesis for sandwich beam. In this paper rule of virtual work is utilized to inferred administering condition.

Sayyad *et al.* [5] explored the Stress investigation of covered composite and delicate center sandwich beams utilizing a straightforward higher request shear disfigurement hypothesis. Pawar *et al.* [6] examined novel normal and shear deformation theory (NSDT) for examination of covered composite and sandwich beams in which shear and ordinary disfigurement is considered. Pageno [7] explored by looking at arrangements of a few explicit limit esteem issues in this hypothesis to comparing hypothesis of flexibility arrangements. Si and Basa [8] introduced firmness to weight proportion which builds solidness properties additionally foresee the vibration qualities of these structures appropriately. Chandra and Chopra [9] introduced a relative investigation of trial and hypothetical information to comprehend the static reaction on composite I-types of beams. A Vlasov-type direct hypothesis was produced for open segment bars which incorporated the transverse shear misshapening. They investigated the basic reaction by estimating bowing incline and turn for I-bar under tip stacking and torsional load. Hasim [10] examined the iso geometric static investigation of the overlaid composite plane bars by utilizing refined crisscross hypothesis fit as a fiddle capacities, an isogeometric refined crisscross finite component (IGRZF) has been created. Jun and Hongxing [11] built up the precise powerful firmness network of uniform overlaid composite beams dependent on trigonometric shear misshapening hypothesis. Fereidooni *et al.* [12] examined composite beams exposed to differing time loads utilizing vital type of condition which created limited component model. Reddy [13] had grouped the bar into three classifications, which are Euler-Bernoulli pillar hypothesis, first request shear disfigurement hypothesis and higher request shear distortion speculations. Ferreira *et al.* [14] contemplated outspread premise capacities and higher request shear disfigurement speculations in the investigation of covered composite beams and plates. Bannerjee and Williams [15], exhibited the shear remedy factor is hard to precisely for overlaid composite beams, as it dependants on layer direction, geometric parameters and limit conditions. Ghugal and Sharma [16], built up a variationally steady refined hyperbolic shear disfigurement hypothesis for flexure and free vibration of thick isotropic shaft. This hypothesis considers transverse shear disfigurements impacts.

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II. METHODOLOGY

Fig.1. shows different number of layers in three directions.

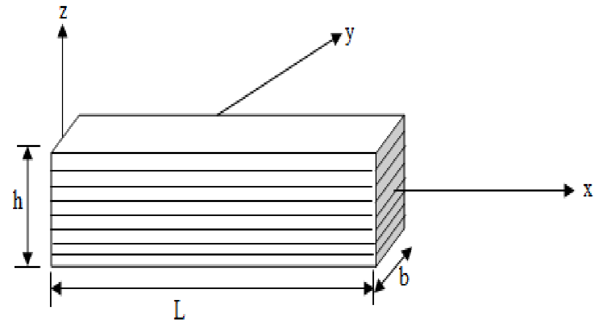

Fig. 1. Beam with number of layers

The displacement field can be given as follows

$$u^{(1)}(x,z) = u(x) - z\frac{dw}{dx} + \left[\frac{h}{\pi} + \sin\left(\frac{\pi z}{h}\right)\right]\phi(x)$$

$$u^{(2)}(x,z) = 0$$

$$u^{(3)}(x,z) = w(x) \tag{1}$$

Where $u^{(1)}$ is the displacement along x directions. $u^{(2)}$ is the displacement along y directions, $u^{(3)}$ is the displacement along z directions of a point in the beam. u is the displacement in the x direction and w is transverse displacement in the y direction of a point on the beam in mid plane. The strain-displacement relations between strain-displacement corresponding to the displacement field are given by

$$\varepsilon_x^0 = \frac{\partial u}{\partial x},\ \mathrm{k}_x^0 = -\frac{\partial^2 w}{\partial x^2},\ \mathrm{k}_x^2 = \frac{\partial \phi}{\partial x},\ \mathrm{k}_{xz}^2 = \frac{h}{\pi}\phi \tag{2}$$

$$\begin{Bmatrix} N_x \\ N_y \\ N_{xy} \\ M_x \\ M_y \\ M_{xy} \\ P_x \\ P_y \\ P_{xy} \end{Bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{16} & B_{11} & B_{12} & B_{16} & E_{11} & E_{12} & E_{16} \\ A_{12} & A_{22} & A_{26} & B_{12} & B_{22} & B_{26} & E_{12} & E_{22} & E_{26} \\ A_{16} & A_{26} & A_{66} & B_{16} & B_{26} & B_{66} & E_{16} & E_{26} & E_{66} \\ B_{11} & B_{12} & B_{16} & D_{11} & D_{12} & D_{16} & F_{11} & F_{12} & F_{16} \\ B_{12} & B_{22} & B_{22} & D_{12} & D_{22} & D_{26} & F_{12} & F_{22} & F_{26} \\ B_{16} & B_{26} & B_{26} & D_{16} & D_{26} & D_{66} & F_{16} & F_{26} & F_{16} \\ E_{11} & E_{12} & E_{16} & F_{11} & F_{12} & F_{16} & H_{11} & H_{12} & H_{16} \\ E_{12} & E_{22} & E_{26} & F_{12} & F_{22} & F_{26} & H_{12} & H_{22} & H_{26} \\ E_{16} & E_{26} & E_{66} & F_{16} & F_{26} & F_{66} & H_{16} & H_{26} & H_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \varepsilon_{xy}^0 \\ k_x^0 \\ k_y^0 \\ k_{xy}^0 \\ k_x^2 \\ k_y^2 \\ k_{xy}^2 \end{Bmatrix} \tag{3}$$

Where N_x, N_y and N_{xy} are the in plane forces, M_x, M_y and M_{xy} the bending and twisting moments, P_x, P_y and P_{xy} the refine bending and twisting moments, ε_x^0, ε_y^0 and ε_{xy}^0 the mid-plane strains, k_x^0, k_y^0 and k_{xy}^0 the bending and twisting curvatures k_x^2, k_y^2 and k_{xy}^2 the refines bending and twisting curvatures, $A_{ij}, B_{ij}, D_{ij}, E_{ij}, F_{ij}, H_{ij}$ (i,j=1,2,6) are the stiffness coefficient. In the above theory, the constitutive equations of laminated composite beam which accounts for the Poisson effect are considered as follows.

Assume N_y, N_{xy}, M_y, M_{xy}, P_y and P_{xy} equal to zero while $\varepsilon_y^0, \varepsilon_{xy}^0, k_y^0, k_{xy}^0,\ k_y^2\ \ k_{xy}^2$ are assume to be non zero.

$$\begin{Bmatrix} N_x \\ M_x \\ P_x \end{Bmatrix} = \begin{bmatrix} \bar{A}_{11} & \bar{B}_{11} & \bar{E}_{11} \\ \bar{B}_{11} & \bar{D}_{11} & \bar{F}_{11} \\ \bar{E}_{11} & \bar{F}_{11} & \bar{H}_{11} \end{bmatrix} \begin{Bmatrix} \varepsilon_x^0 \\ k_x^0 \\ k_x^2 \end{Bmatrix}$$

$$\begin{Bmatrix} N_x \\ M_x \\ P_x \end{Bmatrix} = \begin{bmatrix} \bar{A}_{11} & \bar{B}_{11} & \bar{E}_{11} \\ \bar{B}_{11} & \bar{D}_{11} & \bar{F}_{11} \\ \bar{E}_{11} & \bar{F}_{11} & \bar{H}_{11} \end{bmatrix} \begin{Bmatrix} \dfrac{\partial u}{\partial x} \\ -\dfrac{\partial^2 w}{\partial x^2} \\ \dfrac{\partial \phi}{\partial x} \end{Bmatrix} \tag{4}$$

$$\begin{bmatrix} \bar{A}_{11} & \bar{B}_{11} & \bar{E}_{11} \\ \bar{B}_{11} & \bar{D}_{11} & \bar{F}_{11} \\ \bar{E}_{11} & \bar{F}_{11} & \bar{H}_{11} \end{bmatrix} = \begin{bmatrix} A_{11} & B_{11} & E_{11} \\ B_{11} & D_{11} & F_{11} \\ E_{11} & F_{11} & H_{11} \end{bmatrix} - \begin{bmatrix} A_{12} & A_{16} & B_{12} & B_{16} & E_{12} & E_{16} \\ B_{12} & B_{16} & D_{12} & D_{16} & F_{12} & F_{16} \\ E_{12} & E_{16} & F_{12} & F_{16} & H_{12} & H_{16} \end{bmatrix}$$

$$\begin{bmatrix} A_{22} & A_{26} & B_{22} & B_{26} & E_{22} & E_{26} \\ A_{26} & A_{66} & B_{26} & B_{66} & E_{26} & E_{66} \\ B_{22} & B_{26} & D_{22} & D_{26} & F_{22} & F_{26} \\ B_{26} & B_{66} & D_{26} & D_{66} & F_{26} & F_{66} \\ E_{22} & E_{26} & F_{22} & F_{26} & H_{22} & H_{26} \\ E_{26} & E_{66} & F_{26} & F_{66} & H_{26} & H_{66} \end{bmatrix}^{-1} \begin{bmatrix} A_{12} & B_{12} & E_{12} \\ A_{16} & B_{16} & E_{16} \\ B_{12} & D_{12} & F_{12} \\ B_{16} & D_{16} & F_{16} \\ E_{12} & F_{12} & H_{12} \\ E_{16} & F_{16} & H_{16} \end{bmatrix}$$

The laminated stiffness coefficients $A_{ij}, B_{ij}, D_{ij}, E_{ij}, F_{ij}, H_{ij}$ (i, j=1, 2, 6) and the transverse shear stiffness F_{55}, which are function of laminate ply orientation, material property and stack sequence , are given by

$$(A_{ij}, B_{ij}, D_{ij}) = \int_{-h/2}^{h/2} \bar{Q}_{ij}\ (1,\ z,\ z^2)\ dz,$$

$$(E_{ij}, F_{ij}, H_{ij}) = \int_{-h/2}^{h/2} \bar{Q}_{ij}\ f(z)\ (1,\ z,\ f(z))\ dz,$$

$$G_{55} = \int_{-h/2}^{h/2} \bar{Q}_{55}\left[g\ (z)\right]^2\ dz, \qquad g\ (z)\ = 1 - f\ '(z) \tag{5}$$

The transformed reduced stiffness constants $\bar{Q}_{ij}$ (i,j=1,2,6) and $\bar{Q}_{55}$ are given by

$$\bar{Q}_{11} = Q_{11}\cos^4\theta + 2(Q_{12} + 2Q_{66})\sin^2\theta\cos^2\theta + Q_{22}\sin^4\theta \tag{6a}$$

$$\bar{Q}_{12} = (Q_{11} + Q_{22} - 4Q_{66})\sin^2\theta\cos^2\theta + Q_{12}(\sin^4\theta + \cos^4\theta) \tag{6b}$$

$$\bar{Q}_{22} = Q_{11}\sin^4\theta + 2(Q_{12} + 2Q_{66})\sin^2\theta\cos^2\theta + Q_{22}\cos^4\theta \tag{6c}$$

$$\bar{Q}_{16} = (Q_{11} - Q_{12} - 2Q_{66})\sin\theta\cos^3\theta + (Q_{12} - Q_{22} + 2Q_{66})\sin^3\theta\cos\theta \tag{6d}$$

$$\bar{Q}_{26} = (Q_{11} - Q_{12} - 2Q_{66})\sin^3\theta\cos\theta + (Q_{12} - Q_{22} + 2Q_{66})\sin\theta\cos^3\theta \tag{6e}$$

$$\bar{Q}_{66} = (Q_{11} + Q_{22} - 2Q_{12} - 2Q_{66})\sin^2\theta\cos^2\theta + Q_{66}(\sin^4\theta\cos^4\theta) \tag{6f}$$

$$\bar{Q}_{55} = G_{13}\cos^2\theta + G_{23}\sin^2\theta \tag{6g}$$

Where θ is the angle between the fiber direction and longitudinal axis of the beam and reduced stiffness constants Q_{11}, Q_{22}, Q_{12} and Q_{66} can be obtained in terms of constant

$$Q_{11} = \frac{E_1}{1-\upsilon_{12}\upsilon_{21}},\ Q_{12} = \frac{\upsilon_{12}E_2}{1-\upsilon_{12}\upsilon_{21}} = \frac{\upsilon_{21}E_1}{1-\upsilon_{12}\upsilon_{21}},$$

$$Q_{22} = \frac{E2}{1-\upsilon_{12}\upsilon_{21}},\ Q_{66} = G_{12}$$

All laminates made same orthotropic material, which properties are assumed

$$\frac{E_1}{E_2}=25,\ E_2=1,\ \upsilon_{12}=0.25,\ G_{12}=G_{13}=0.5E_2, G_{23}=0.2E_2$$

$$N=\int_{-h/2}^{h/2}\sigma_x^k dz,\ M=\int_{-h/2}^{h/2}\sigma_x^k z\, dz, P=\int_{-h/2}^{h/2}\sigma_x^k f(z)\, dz,$$

$$Q=\int_{-h/2}^{h/2}\tau_{zx}^k\, g(z)\, dz \qquad (7)$$

The virtual work principle is given as

$$b\int_0^L\int_{-h/2}^{h/2}\left(\sigma_x^k\delta\varepsilon_x+\tau_{zx}^k\delta\gamma_{zx}\right)dz\, dx-\int_0^L q\,(\delta w)\, dx=0 \qquad (8)$$

The force and moment resultants are as follows

$$\frac{dN}{dx}=0\ :\ \delta u,\quad \frac{d^2M}{dx^2}+q=0\ :\ \delta w,\quad \frac{dP}{dx}=0\ :\ \delta\phi$$

$$\frac{dN}{dx}=0\ : For\ N_x=\delta u,$$

$$-\bar{A}_{11}\frac{d^2u}{dx^2}+\bar{B}_{11}\frac{d^3w}{dx^3}+\bar{E}_{11}\frac{d^2\phi}{dx^2}=0 \qquad (9a)$$

$$\frac{d^2M}{dx^2}+q=0\ For\ M_x=\delta w,$$

$$-\bar{B}_{11}\frac{d^3u}{dx^3}+\bar{D}_{11}\frac{d^4w}{dx^4}+\bar{F}_{11}\frac{d^3\phi}{dx^3}=q \qquad (9b)$$

$$\frac{dP}{dx}=0\ : For\ P_x=\delta\phi,$$

$$-\bar{E}_{11}\frac{d^2u}{dx^2}+\bar{F}_{11}\frac{d^3w}{dx^3}-\bar{H}_{11}\frac{d^2\phi}{dx^2}+G=0 \qquad (9c)$$

for symmetrical angle ply $\bar{E}_{11}$ and $\bar{B}_{11}$ is zero.

$$\bar{D}_{11}\frac{d^4w}{dx^4}+\bar{F}_{11}\frac{d^3\phi}{dx^3}=q,$$

$$\bar{F}_{11}\frac{d^3w}{dx^3}-\bar{H}_{11}\frac{d^2\phi}{dx^2}+G=0 \qquad (9d)$$

Associated boundary condition are as follows

Along edges x= 0 and x=L

$$\bar{B}_{11}\frac{d^2u}{dx^2}-\bar{D}_{11}\frac{d^3w}{dx^3}+\bar{F}_{11}\frac{d^2\phi}{dx^2}=0\ or\ w\ is\ prescribed,$$

$$\bar{B}_{11}\frac{du}{dx}-\bar{D}_{11}\frac{d^2w}{dx^2}+\bar{F}_{11}\frac{d\phi}{dx}=0\ or\ \frac{dw}{dx}\ is\ prescribed$$

$$\bar{E}_{11}\frac{du}{dx}-\bar{F}_{11}\frac{d^2w}{dx^2}+\bar{H}_{11}\frac{d\phi}{dx}=0\ or\ \phi\ is\ prescribed,$$

$$\bar{A}_{11}\frac{du}{dx}-\bar{B}_{11}\frac{d^2w}{dx^2}+\bar{E}_{11}\frac{d\phi}{dx}=0\ or\ u\ is\ prescribed$$

Example: Simply supported beam with uniformly distributed load $q=q_0x$

Non dimensional transverse displacement $\bar{w}$

$$\bar{w}=\left[100\ \frac{E_2}{\bar{D}_{11}}\ L\left\{\frac{1}{120}\frac{x^5}{L^5}-\frac{1}{36}\frac{x^3}{L^3}+\frac{7}{360}\frac{x}{L}\right\}\right]+$$
$$\left\{\begin{matrix}\left[100\ \frac{E_2}{\bar{D}_{11}}\ L\ \frac{h^2}{L^2}\frac{1}{G_{55}}\ \frac{1}{\lambda^2}\left(\frac{\bar{H}_{11}}{\Omega}\lambda^2-G_{55}\right)\right]\\ \left[\frac{1}{\lambda^2}\frac{h^2}{L^2}\frac{\text{Sinh}\,\lambda x}{L}-\frac{1}{\lambda^2}\frac{h^2}{L^2}\frac{x}{L}-\frac{1}{6}\frac{x^3}{L^3}+\frac{1}{6}\frac{x}{L}\right]\end{matrix}\right\}$$

$$where\ \ \Omega=\left[1+\left(\frac{\pi}{\lambda L}\right)^2\right]$$

Non dimensional axial displacement $\bar{u}$

$$\bar{u}=\left\{\begin{matrix}\left[-E_2\frac{z}{h}\ \frac{L^3}{h^3}\frac{1}{\bar{D}_{11}}L\left\{\frac{1}{24}\frac{x^4}{L^4}-\frac{1}{12}\frac{x^2}{L^2}+\frac{7}{360}\right\}\right]\\ +\left[\begin{matrix}-E_2\frac{z}{h}\frac{1}{G_{55}}\frac{1}{\bar{D}_{11}}\frac{1}{\lambda^2}\frac{L}{h}h\left(\frac{\bar{H}_{11}}{\Omega}\lambda^2-G_{55}\right)\\ \left(\left(\left[\frac{1}{\lambda}\frac{h}{L}\frac{\text{Cosh}\,\lambda x}{\text{Sinh}\,\lambda L}-\frac{1}{\lambda^2}\frac{h^2}{L^2}-\frac{1}{2}\frac{x^2}{L^2}+\frac{1}{6}\right]\right)\right)\end{matrix}\right]\end{matrix}\right\}$$
$$+\left[\begin{matrix}E_2\ \frac{L}{\pi}\frac{\bar{F}_{11}}{G_{55}}\frac{1}{\bar{D}_{11}}\frac{L}{h}\ \frac{1}{\Omega}\ \text{Sin}\left(\frac{\pi z}{h}\right)\\ \left(\frac{1}{\lambda}\frac{h}{L}\frac{\text{Cosh}\,\lambda x}{\text{Sinh}\,\lambda L}-\frac{1}{\lambda^2}\frac{h^2}{L^2}-\frac{1}{2}\frac{x^2}{L^2}+\frac{1}{6}\right)\end{matrix}\right]$$

Non dimensional axial stresses $\bar{\sigma}_x$

$$\bar{\sigma}_x=\left\{\begin{matrix}\left[\frac{z}{h}\ \frac{L^2}{h^2}\frac{1}{\bar{D}_{11}}L\left\{\frac{1}{6}\frac{x^3}{L^3}-\frac{1}{6}\frac{x}{L}\right\}\right]\\ -\left[\begin{matrix}\frac{z}{h}\frac{1}{G_{55}}\frac{1}{\bar{D}_{11}}\frac{1}{\lambda^2}\frac{z}{h}L\left(\frac{\bar{H}_{11}}{\Omega}\lambda^2-G_{55}\right)\\ \left(\left(\left[\frac{\text{Sinh}\,\lambda x}{\text{Sinh}\,\lambda L}-\frac{x}{L}\right]\right)\right)\end{matrix}\right]\end{matrix}\right\}$$
$$+\left[\begin{matrix}\frac{h}{\pi}\frac{\bar{F}_{11}}{G_{55}}\frac{1}{\bar{D}_{11}}\frac{L}{h}\ \frac{1}{\Omega}\ L\,\text{Sin}\left(\frac{\pi z}{h}\right)\\ \left(\left[\frac{\text{Sinh}\,\lambda x}{\text{Sinh}\,\lambda L}-\frac{x}{L}\right]\right)\end{matrix}\right]$$

Non dimensional transverse shear stresses $\bar{\tau}_{zx}^{EE}$ using equilibrium equation

$$\bar{\tau}_{zx}^{EE}=\left[\begin{matrix}\frac{h^2}{8}\left[\frac{4z^2}{h^2}-1\right]\frac{1}{\bar{D}_{11}}\\ \left\{\frac{1}{G_{55}}\frac{1}{\lambda L}\left(\frac{\bar{H}_{11}}{\Omega}\lambda^2-G_{55}\right)\left(\frac{\lambda\,\text{Cosh}\,\lambda x}{\text{Sin}\,\lambda L}-1\right)+\left(L^2\left(\frac{1}{2}\frac{x^2}{L^2}-\frac{1}{6}\right)\right)\right\}\end{matrix}\right]$$
$$+\left[\frac{1}{G_{55}}\frac{L}{\bar{D}_{11}}\frac{\bar{F}_{11}}{\Omega}\left(\frac{\lambda\,\text{Cosh}\,\lambda x}{\text{Sin}\,\lambda L}-1\right)\right]$$

Non dimensional Transverse shear stresses $\bar{\tau}_{zx}^{CR}$ using constitutive relationship

$$\bar{\tau}_{zx}^{CR} = \left[-\frac{L}{h}\frac{1}{\Omega}\frac{\bar{F}_{11}}{\bar{D}_{11}} L \left(\left[\frac{1}{\lambda}\frac{h}{L}\frac{\text{Cosh}\,\lambda x}{\text{Sinh}\,\lambda L} - \frac{1}{\lambda^2}\frac{h^2}{L^2} - \frac{1}{2}\frac{x^2}{L^2} + \frac{1}{6} \right] \right) \right]$$

III. NUMERICAL RESULT

$$\bar{u}\left(0, \frac{h}{2}\right) = \frac{buE_2}{q_0 h}, \quad \bar{w}\left(\frac{L}{2}, 0\right) = \frac{100wh^3E_2}{q_0 L^4},$$

$$\bar{\sigma}_x\left(0, -\frac{h}{2}\right) = \frac{b\sigma_x}{q_0}, \quad \bar{\tau}_{zx}(0,0) = \frac{b\tau_{zx}}{q_0}, \quad E_2 = 1$$

TABLE I

Collation of non dimensional axial displacement $(\bar{u})$,transverse displacement $(\bar{w})$, bending stresses $(\bar{\sigma}_x)$, transverse shear stress $(\bar{\tau}_{zx})$ for single layer (0^0) laminated beam subjected uniformly distributed load for aspect ratio (AS) 4 and 10.

AS	Theory	$\bar{u}$	$\bar{w}$	$\bar{\sigma}_x$	$\bar{\tau}_{zx}^{CR}$	$\bar{\tau}_{zx}^{EE}$
4	TSDT [11]	2.700	2.376	4.456	0.507	0.512
	HYSDT[16]	3.602	2.791	5.284	0.523	0.546
	HSDT [15]	4.466	2.801	5.303	0.524	0.549
	FSDT [14]	0.341	0.624	0.375	0.267	0.040
	ETB [13]	0.341	0.312	0.375	-----	0.040

AS	Theory	$\bar{u}$	$\bar{w}$	$\bar{\sigma}_x$	$\bar{\tau}_{zx}^{CR}$	$\bar{\tau}_{zx}^{EE}$
10	TSDT [11]	12.93	0.642	13.64	1.270	9.51
	HYSDT [16]	14.93	0.709	16.07	1.310	9.76
	HSDT [15]	15.83	0.710	16.12	1.312	9.85
	FSDT [14]	5.333	0.361	1.500	0.668	0.10
	ETB [13]	5.333	0.312	1.500	----	0.10

TABLE II

Collation of non dimensional axial displacement $(\bar{u})$,transverse displacement $(\bar{w})$, bending stresses $(\bar{\sigma}_x)$, transverse shear stress $(\bar{\tau}_{zx})$ for single layer (90^0) laminated beam subjected uniformly distributed load for aspect ratio (AS) 4 and 10

AS	Theory	$\bar{u}$	$\bar{w}$	$\bar{\sigma}_x$	$\bar{\tau}_{zx}^{CR}$	$\bar{\tau}_{zx}^{EE}$
4	Present	14.75	13.14	16.88	0.515	15.45
	HYSDT [16]	17.00	14.11	18.80	0.533	15.76
	HSDT [15]	19.22	14.13	18.85	0.533	15.90
	FSDT [14]	8.536	8.578	9.378	0.262	1.000
	ETB [13]	8.536	7.812	9.378	-----	1.000

AS	Theory	$\bar{u}$	$\bar{w}$	$\bar{\sigma}_x$	$\bar{\tau}_{zx}^{CR}$	$\bar{\tau}_{zx}^{EE}$
10	Present	153.1	8.668	68.86	1.288	245.1
	HYSDT [16]	157.9	8.823	74.51	1.332	249.7
	HSDT [15]	160.2	8.826	74.63	1.333	250.2
	FSDT [14]	133.3	7.921	37.51	0.634	2.501
	ETB [13]	133.3	7.812	37.51	-----	2.501

TABLE III

Collation of non dimensional axial displacement $(\bar{u})$,transverse displacement $(\bar{w})$, bending stresses $(\bar{\sigma}_x)$, transverse shear stress $(\bar{\tau}_{zx})$ for three-layer symmetric ($0^0/90^0/0^0$) laminated beam subjected uniformly distributed load for aspect ratio (AS) 4 and 10

AS	Theory	$\bar{u}$	$\bar{w}$	$\bar{\sigma}_x$	$\bar{\tau}_{zx}^{CR}$	$\bar{\tau}_{zx}^{EE}$
4	Present	2.681	2.362	4.399	0.515	0.539
	HYSDT [16]	3.564	2.769	5.215	0.519	0.544
	HSDT [15]	4.436	2.781	5.235	0.521	0.551
	FSDT [14]	0.353	0.713	0.388	0.256	0.0414
	ETB [13]	0.353	0.323	0.388	-----	0.0414

AS	Theory	$\bar{u}$	$\bar{w}$	$\bar{\sigma}_x$	$\bar{\tau}_{zx}^{CR}$	$\bar{\tau}_{zx}^{EE}$
10	Present	13.036	0.650	13.53	1.264	9.882
	HYSDT [16]	14.987	0.715	15.92	1.299	10.06
	HSDT [15]	15.905	0.717	15.98	1.305	10.12
	FSDT [14]	5.520	0.385	1.552	0.675	0.103
	ETB [13]	5.520	0.323	1.552	----	0.103

TABLE IV

Collation of non dimensional axial displacement $(\bar{u})$, transverse displacement $(\bar{w})$, bending stresses $(\bar{\sigma}_x)$, transverse shear stress $(\bar{\tau}_{zx})$ for three-layer symmetric ($90^0/0^0/90^0$) laminated composite beam subjected uniformly distributed load for aspect ratio (AS) 4 and 10

AS	Theory	$\bar{u}$	$\bar{w}$	$\bar{\sigma}_x$	$\bar{\tau}_{zx}^{CR}$	$\bar{\tau}_{zx}^{EE}$
4	Present	2.681	10.761	18.32	0.576	8.121
	HYSDT [16]	3.564	11.966	20.84	0.581	8.353
	HSDT [15]	4.4362	11.283	19.50	0.582	8.410
	FSDT [14]	4.502	4.6462	4.946	0.234	0.528
	ETB [13]	4.502	4.1222	4.946	------	0.528

TABLE V

Collation of non dimensional axial displacement $(\bar{u})$, transverse displacement $(\bar{w})$, bending stresses $(\bar{\sigma}_x)$, transverse shear stress $(\bar{\tau}_{zx})$ for four-layer symmetric ($0^0/90^0/90^0/0^0$) laminated composite beam subjected uniformly distributed load for aspect ratio (AS) 4 and 10

AS	Theory	$\bar{u}$	$\bar{w}$	$\bar{\sigma}_x$	$\bar{\tau}_{zx}^{CR}$	$\bar{\tau}_{zx}^{EE}$
4	Present	2.664	2.355	4.318	0.499	0.498
	HYSDT [16]	3.561	2.768	5.146	0.514	0.542
	HSDT [15]	4.416	2.777	5.147	0.514	0.548
	FSDT [14]	0.386	0.800	0.424	0.261	0.045
	ETB [13]	0.386	0.353	0.424	----	0.045

AS	Theory	$\bar{u}$	$\bar{w}$	$\bar{\sigma}_x$	$\bar{\tau}_{zx}^{CR}$	$\bar{\tau}_{zx}^{EE}$
10	Present	13.41	0.674	13.43	1.247	10.21
	HYSDT [16]	15.39	0.740	15.85	1.287	10.88
	HSDT [15]	16.27	0.425	15.85	1.286	11.02
	FSDT [14]	6.039	0.404	1.698	0.648	0.113
	ETB [13]	6.039	0.353	1.698	-----	0.113

TABLE VI

Collation of non dimensional axial displacement $(\bar{u})$, transverse displacement $(\bar{w})$, bending stresses $(\bar{\sigma}_x)$, transverse shear stress $(\bar{\tau}_{zx})$ for four-layer symmetric ($90^0/0^0/0^0/90^0$) laminated composite beam subjected uniformly distributed load for aspect ratio (AS) 4 and 10

AS	Theory	$\bar{u}$	$\bar{w}$	$\bar{\sigma}_x$	$\bar{\tau}_{zx}^{CR}$	$\bar{\tau}_{zx}^{EE}$
4	Present	10.25	8.561	15.44	0.618	4.781
	HYSDT [16]	12.82	9.735	17.82	0.614	4.933
	HSDT [15]	15.19	9.735	17.82	0.618	4.988
	FSDT [14]	2.125	2.394	2.335	0.313	0.249
	ETB [13]	2.125	1.946	2.335	----	0.249

AS	Theory	$\bar{u}$	$\bar{w}$	$\bar{\sigma}_x$	$\bar{\tau}_{zx}^{CR}$	$\bar{\tau}_{zx}^{EE}$
10	Present	54.24	3.008	48.70	1.493	63.12
	HYSDT [16]	63.88	3.195	55.59	1.537	64.74
	HSDT [15]	66.28	3.195	55.58	1.546	65.12
	FSDT [14]	33.21	2.019	9.341	0.734	0.623
	ETB [13]	33.21	1.946	9.341	----	0.623

AS	Theory	$\bar{u}$	$\bar{w}$	$\bar{\sigma}_x$	$\bar{\tau}_{zx}^{CR}$	$\bar{\tau}_{zx}^{EE}$
10	Present	85.20	4.529	60.29	1.444	108.1
	HYSDT [16]	90.93	4.722	67.46	1.454	110.7
	HSDT [15]	90.78	4.612	63.48	1.984	111.2
	FSDT [14]	70.35	4.209	19.78	0.968	1.321
	ETB [13]	70.35	4.122	19.78	-----	1.321

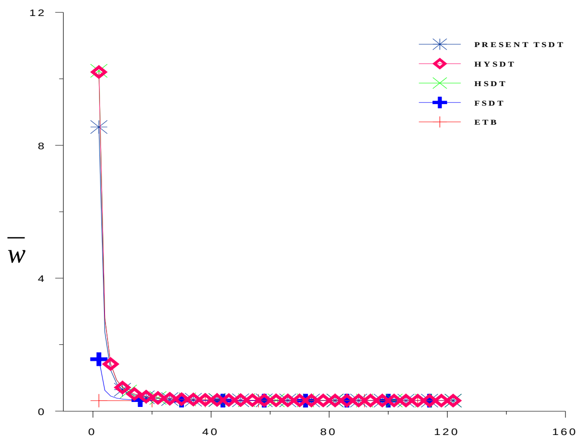

Fig. 2. Dispersal of transverse displacement for single layer (0^0) laminated beam in terms of non dimensional

AS

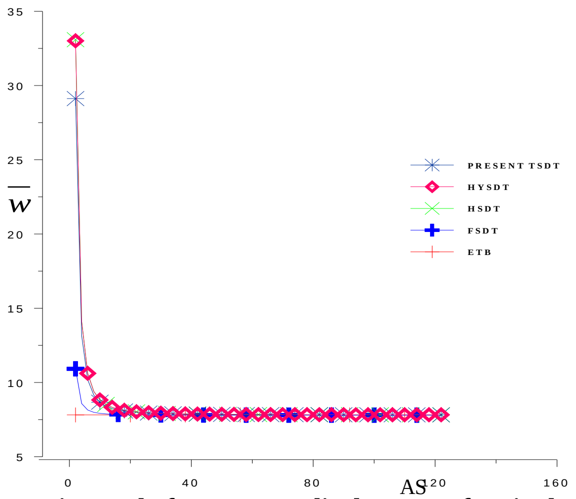

Fig. 3. Dispersal of transverse displacement for single layer (90^0) laminated beam in terms of non dimensional

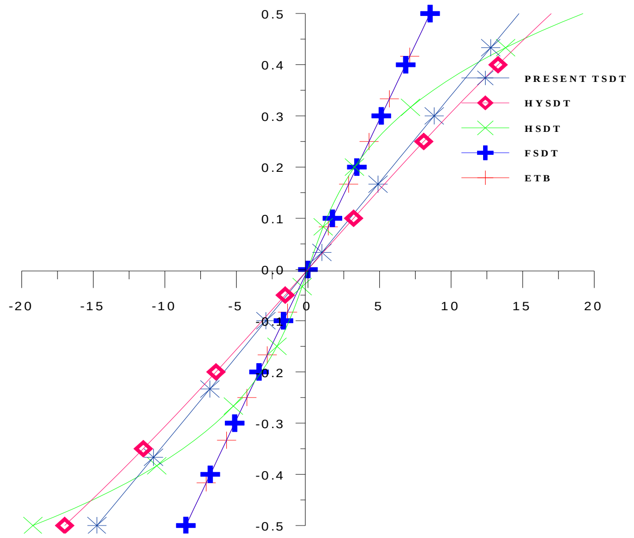

Fig. 6. Dispersal of axial displacement for single layer (90^0) laminated beam in terms of non dimensional

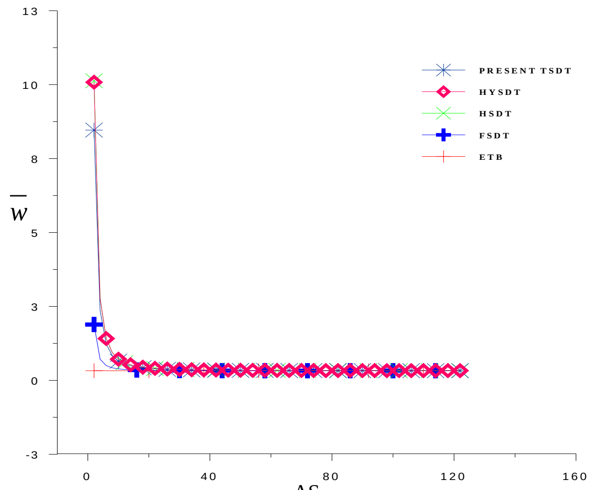

Fig. 4. Dispersal of transverse displacement for three layer (0^0/90^0/0^0) laminated beam in terms of non dimensional

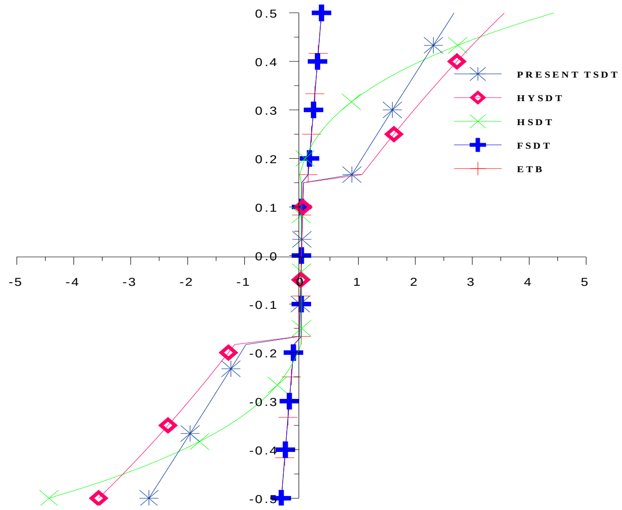

Fig. 7. Dispersal of axial displacement for three layer (0^0/90^0/0^0) laminated beam in terms of non dimensional

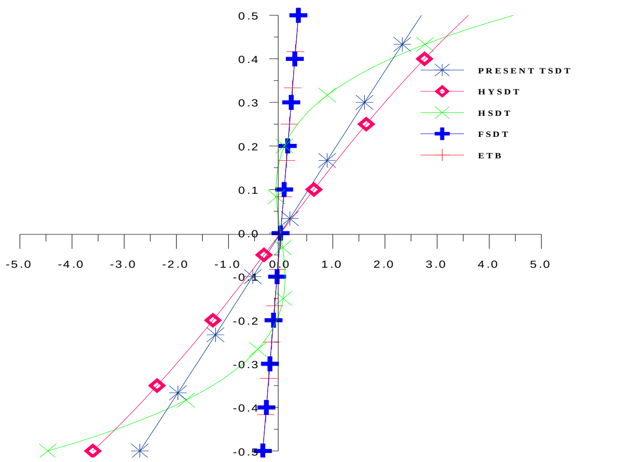

Fig. 5. Dispersal of axial displacement for single layer (0^0) laminated beam in terms of non dimensional

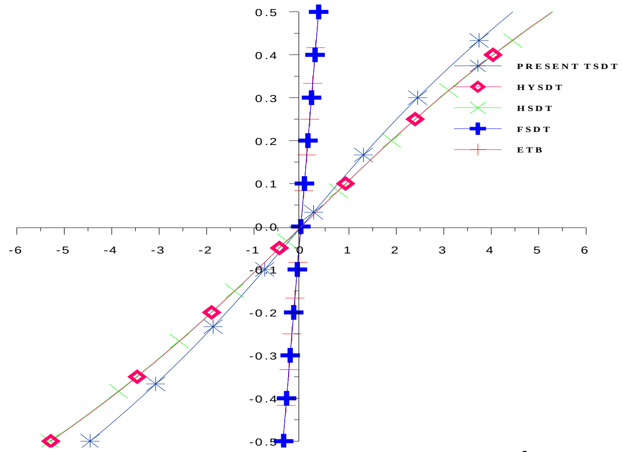

Fig. 8. Dispersal of stress for single layer (0^0) laminated beam in terms of non dimensional

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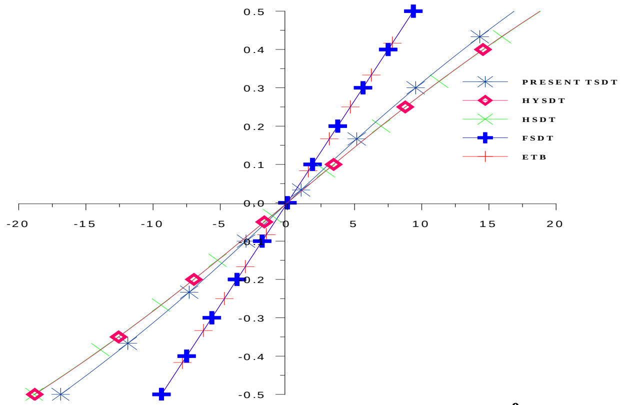

Fig. 9. Dispersal of stress for single layer (90^0) laminated beam in terms of non dimensional

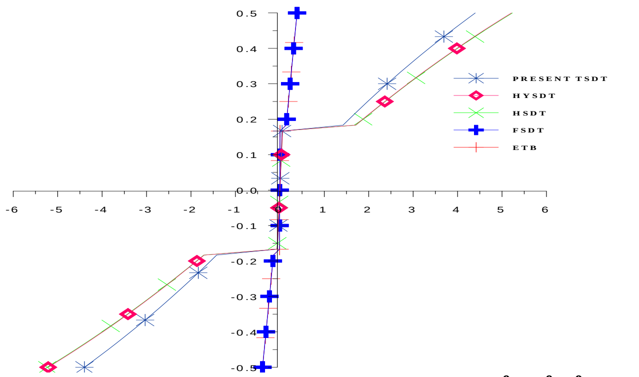

Fig. 10. Dispersal of stress for three layer ($0^0/90^0/0^0$) laminated beam in terms of non dimensional

The non dimensional properties of simply supported laminated beam are shown in Table I to VI. The aspect ratio (AS) are considered as 4 and 10. The properties of simply supported laminated beam are non dimensional axial, transverse displacement , non dimensional bending stresses and non dimensional shear stresses

Fig. 2-10 shows when aspect ratio increases the displacement become constant, the bending stresses are maximum at top and bottom but zero at centre.

IV.CONCLUSIONS

The comparison of various shear deformation theories are considered. The properties of are five different theories are calculated. The theories are Eular- Bernoulli (ETB) theory, First order shear deformation theory (FSDT), Higher order shear deformation theory (HSDT), Trigonometric shear deformation theory (TSDT) and Hyperbolic shear deformation theory (HYSDT) are compared. The HSDT and HYSDT gives similar result and TSDT gives lesser results as compared to above two theories.

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