Abstract Filter bank multicarrier (FBMC) has been considered as an effective multicarrier communication technique, mainly for communication under doubly dispersive channels as it fulfills the needs of the next generation standards. The prototype filters used in FBMC systems must satisfy localization criteria and dual orthogonality to battle the unfavorable channel effects like intersymbol interference (ISI) and inter carrier interference (ICI). Even if the doubly localized filters which are used conventionally are best possible in the absence of the channel, they fail to satisfy the orthogonally and localization criteria under doubly dispersive channels. In order to maximize the SNR, the prototype pulse shape must match with the time and frequency domain channel dispersions. Also, multiple antenna techniques combined with FBMC can give better performance. An adaptive system using MIMO filter bank is proposed in which time and frequency domain spreading of the pulse is made to match with that of the channel conditions by adaptively varying the subcarrier bandwidth with respect to the maximum delay spread and Doppler spread of the channel. The signal to noise ratio (SNR) and bit error rate (BER) comparisons illustrate the supremacy in the performance of the adaptive system.

Keywords: Adaptive filter system, Ambiguity function, FBMC, SNR maximization.

I. INTRODUCTION

Orthogonal Frequency Division Multiplexing (OFDM) has been considered as the best favorable technique meant for the present day technologies such as LTE, Wi-Max, video broadcasting, etc. Owing to significant advantages like ease of generation, simpler channel equalization, resistance to multipath channels and increased data rate, OFDM has been the favorite amongst multicarrier communication techniques [1]. But under the influence of time varying and time dispersive channels, OFDM suffers many limitations. Time dispersion and frequency dispersion leads to inter symbol interference (ISI) as well as inter carrier interference (ICI) respectively. To battle with the ISI, OFDM uses cyclic prefix which causes a drop in effective spectral efficiency. Also, the OFDM uses a rectangular window filter as the prototype filter with large side lobes in the frequency domain eventually adding to the spectral leakage. These disadvantages make OFDM a poor candidate for next generation communication schemes. To get over the aforementioned limitations of OFDM, Filter bank Multicarrier (FBMC) systems have been considered as a generalized solution to meet the requirements of next generation communication systems. An array of filters, one at the transmitting side (synthesis filter) and the other at the receiving side (analysis filter) constitute an FBMC system. The serial data which is to be transmitted through the synthesis filters are transformed to parallel data and are modulated at a specific subcarrier frequency. The output data from the filters corresponding to each subcarrier is added up and communicated through the channel. Analysis filters on the receiver side which is modulated at each subcarrier frequency as that of the transmitter side separates out the data. In the FBMC systems, the prototype filter plays a pivotal role in overcoming the challenges faced by OFDM. Here the filter arrays must basically satisfy two conditions; the dual Nyquist criterion and the time and frequency domain localization. Thus a class of filters known as isotropic filters which have the same shape and are localized in both the domains are considered in FBMC systems.

FBMC was introduced before the emergence of OFDM. Chang [1] and Saltzberg [2] published the inventive works on FBMC techniques in the 1960s. In [1] Chang proposed the FBMC approach where the parallel set of PAM sequences were allowed to signal through an array of filters which overlap. The signaling was done at minimum bandwidth, using the concept of vestigial sideband signaling. Saltzberg modified the idea proposed by Chang for signaling the QAM sequences by exploiting the double sideband signaling [2]. It was studied in detail by Hirosaki [3]. Later Bellanger [4] presented the method of implementation of Saltzberg’s technique using polyphase structures. Design approaches aimed at prototype filters that permit ISI/ICI free transmission [5, 6] have been discussed and developed by a few authors. Isotropic orthogonal transform algorithm (IOTA) filter design developed by Alard [6] is the most common method. In IOTA filter, doubly orthogonalized Gaussian pulse is used and also has the minimum time- frequency dispersion product. Haas and Belfiore [7] designed Weighted Hermite pulse Filter (WHP) by linearly combining the subset members of the Hermite which satisfy the isotropic properties. The weights are calculated such that the designed filter satisfies orthogonality conditions at both domains. In Modified Hermite pulse design, an isotropic filter design which is an extended version of Haas and Belfiore design is proposed. In [8], the implementation of IOTA using an expression known as Extended Gaussian function is proposed. A modified weighted Hermite pulse (MWHP) design, which shows robust performance in doubly dispersive channel conditions is detailed in [9,10]. The prototype filters and its performance in doubly dispersive channel conditions are analyzed in [11, 12].
According to the theory in [5, 6, 11, 13], it is possible to obtain a subsequent increase in system performance if the prototype pulse spreading ratio matches that of the channel spread in both time and frequency domains. Such a filter can be designed only when the prototype pulse shape gets adapted according to the channel conditions. This can be done either by changing the system parameters or the filter parameters itself. In the present paper, the system parameters are changed accordingly to match the spreadings of the filter in both the domains to the spreadings of the channel. As time and frequency domains are involved simultaneously, the investigations throughout the paper are done by exploiting a two dimensional function named Ambiguity function. The performance analysis for the proposed adaptive system is done using BER plots and SIR plot under the assumption that both the maximum value of delay spread and Doppler spread of the channel is known. Organization of the paper: The paper is organized as follows. The FBMC system model and dual Nyquist criterion are explained in Sect. 2. The proposed adaptive filter system is discussed in Sect. 3. The simulation results and analysis are elaborated in Sect. 4 and the conclusion is given in Sect. 5.

II. FBMC SYSTEM MODEL

Fig:1 FBMC System model

The dual Nyquist criterion should be satisfied by the filters in both time and frequency domains in order to perfectly reconstruct the symbol at the receiver. The filters which are modulated with the subcarrier index k at the transmitter and l at the receiver are obtained as,

\[ h_k(t) = h(t)e^{j2\pi ft} \]  
\[ h_l(t) = h(t)e^{j2\pi lt} \] 

If the sampling time indices are m and n at the transmitter and receiver respectively, the dual Nyquist criteria in both domains may be summarized as

\[ \langle h_k(t - mT), h_l(t - nT) \rangle = \delta_{mn}\delta_{kl} \] 

Wh \( \langle h_k(t - mT), h_l(t - nT) \rangle = \int_{-\infty}^{\infty} h_k(t - mT)\cdot h_l^*(t - nT)\,dt \) 

and 

\[ \delta_{mn}\delta_{kl} = \begin{cases} 1, & \text{if } m = n \text{ and } k = l \\ 0, & \text{otherwise} \end{cases} \]

where \( \delta_{kl} \) is the Kronecker delta function.

II.1 Ambiguity function:

The prototype filter must satisfy Nyquist criterion and it must have good localization in both frequency and time domains. Henceforth the designing and analysis of the prototype filter is benefitted by using a two-dimensional function of frequency and time. Here as both the domains are involved simultaneously, ambiguity function-based analysis is used. The continuous ambiguity function is given as

\[ A_{b}(\tau,\nu) = \int_{-\infty}^{\infty} h(t + \tau/2)\cdot h^*(t - \tau/2)\cdot e^{-j2\pi nt}\,dt \] 

where \( \tau \) denotes the time delay and \( \nu \) the frequency shift. The discrete ambiguity function is obtained by discretizing (4) given by

\[ A_{b}(aT'_{r}, bF_{r}) = \sum_{k=-\infty}^{\infty} h(k + \alpha T_{r})h^*(k + \beta F_{r})e^{-j2\pi k\alpha F_{r}} \]

The dual Nyquist criterion for perfect symbol recovery be able to mapped in terms of the Ambiguity function, which is given as follows,

\[ A_{b}(pT', qF) = \begin{cases} 1 , & \text{if } p = q = 0 \\ 0, & \text{otherwise} \end{cases} \]

III. PROPOSED ADAPTIVE MIMO-FBMC SYSTEM

A MIMO-FBMC system with a novel adaptive filter has been proposed. The basic demand for prototype filter that must be used in FBMC systems is that it should follow the Nyquist criteria and also should have optimal localization in both time and frequency. The isotropic filters give improved performance than OFDM systems but are not optimal under the influence of channel.

III.1. Methodology

The criteria for separability and flawless reconstruction of transmitted symbols given by (4) and (8) are true under ideal channel conditions. Actual FBMC communication experiences spreading in time and frequency because of multipath transmission and time variant channel respectively. As a result, under non ideal transmission over a doubly dispersive channel, the transmitted signal encounters inter symbol interference (ISI) as well as inter-carrier interference (ICI). Then a filter satisfying the ambiguity function given by (9) can reconstruct the transmitted signal only if there is no channel.

Let \( s(t) \) be the transmitted signal and \( c(t, t_{\text{max}}) \) be the impulse response of the doubly dispersive channel with channel delay spread \( t_{\text{max}} \). Considering the channel effect, the received signal is obtained as [14],

\[ r(t) = \int c(t, t_{\text{max}})s(t - t')\,dt' \]
The effect of the channel on the signal transmitted is reflected in the ambiguity function of the filter which is presented in [14-15]. The channel affected ambiguity function is given as [17].

\[ \Delta_k^n(t, f) = \int C(\sigma, \vartheta) A_h(\sigma + t, \vartheta + f) \, d\sigma d\vartheta \]  

(11)

where \( C(t, f) \) is the channel scatter function related with \( c(t, \tau_{\text{max}}) \) which is a two-dimensional function relating delay spread and Doppler spread [20]. Due to the effect of the channel, ambiguity function spreads over the time-frequency lattice and the condition ensuring ISI and ICI free transmission given by (9) is not satisfied. In fast fading multipath channels, the reason for the delocalized ambiguity function in time and frequency domain is owing to the delay spread and Doppler spread respectively. The doubly dispersive channel effect on the transmitted and received symbols for QAM transmission are related to the disturbed ambiguity function [19,20]. The relation of transmitted and received symbols are given by [19]

\[ s_k(n) = s_k(n) A_k^n(0,0) + \sum_p \sum_q s_{k-p}(n - p) A_k^n(p, qT, qF) \]  

(12)

and \( |p| + |q| \neq 0 \).

The data symbol that is transmitted at the nth time instant and for the kth subcarrier is \( s_k(n) \) and \( s_k(n) \) is the corresponding received signal. From (12) it can be inferred that the received symbols along the time-frequency grid can be acquired by carrying out a two dimensional convolution of the transmitted symbols, organized in time-frequency lattice, using a mask having values of the ambiguity function at corresponding normalized symbol-sub-carrier indices [12]. The signal to interference ratio for FBMC transmission under a wide sense stationary uncorrelated scattering (WSSUS) doubly dispersive channels are given as, [14,17,18].

\[ \text{SIR} = \frac{\sigma_t^2}{\sigma_f^2} \]  

(13)

where

\[ \sigma_t^2 = \int_p \int_0 \int_0 C(p, \vartheta) |A_h(\sigma + p, \vartheta)|^2 \, d\sigma d\vartheta \]  

(14)

\[ \sigma_f^2 = \Sigma_{p, q} A_k^n(0,0) \int_p \int_0 C(p, \vartheta) |A_h(\sigma + p, \vartheta + qF)|^2 \, d\sigma d\vartheta \]  

(15)

The multicarrier transmission under doubly dispersive channels are greatly affected by the time-frequency localization of the prototype filter. Heisenberg–Gabor uncertainty principle elucidates the reason for time-limited and band-limited filters being localized either in one of the domains. The localization criteria in both the domains are quantified by the Heisenberg principle in [5] as

\[ H = \frac{1}{4\pi\sigma_t\sigma_f} \leq 1 \]  

(16)

where \( H \) is the Heisenberg parameter and \( \sigma_t, \sigma_f \) are time and frequency dispersions of the prototype filters respectively, which are defined as,

\[ \sigma_t = \sqrt{\int_{-\infty}^{\infty} t^2 |h(t)|^2 \, dt} \]  

(17)

The reason for the delocalized ambiguity function is given by the uncertainty principle in [5], which is quantified as

\[ \sigma_f = \sqrt{\int_{-\infty}^{\infty} f^2 |H(f)|^2 \, df} \]  

(18)

The product of \( H \) and time-frequency spread of the filter are inversely related. The time and frequency dispersion for isotropic filters are nearly equal and the energy of the ambiguity function will be more focused in its main lobe. Immunity of the FBMC system against doubly dispersive channels becomes better when the delay-Doppler product becomes less and isotropicity of the filter becomes more. Gaussian pulses can achieve optimum value for (16) but orthogonality conditions cannot be satisfied. Various filters like IOTA filter [5,6], WHP filter [7], that are derived from the Gaussian filter are perfectly isotropic and are preferred as a compromised choice as prototype filter [16]. For improving SIR performance and for ISI and ICI reduced transmission the shape of the ambiguity function should be harmonized with that of channel scatter function. SIR can be maximized if this ratio is matched with that of the ratio of maximum delay spread and maximum Doppler spread of a channel. The mathematical expression relating the two constraints are given by,

\[ \frac{\sigma_t}{\sigma_f} \approx \frac{T_{\text{max}}}{F_{\text{max}}} \]  

(19)

where \( T_{\text{max}} \) is the ratio of time-frequency spread of the channel and \( F_{\text{max}} \) is the maximum Doppler spread of the channel.

The direction parameter \( \eta = \frac{\sigma_t}{\sigma_f} \) is the ratio between the dispersion of pulse in time-frequency domain. Using this equality condition, an adaptive filter can be formulated by adopting either pulse shape adaptation or time frequency grid adaptation. Rectangular and hexagonal lattices are the two popular grid adaptation strategies which have been studied in [13,17,24]. For a fixed lattice geometry, time pulse adaptation is performed if the pulse in either domain is adaptively dilated in accordance with the spreading of channel. The channel scatter function for a WSSUS doubly dispersive channel hiving exponential power delay profile and U shaped Doppler profile, is given by,

\[ C(t, f) = \frac{e^{- \frac{t^2}{2\tau_{\text{max}}^2}} \frac{1}{\tau_{\text{max}} \sqrt{\pi}}} {\tau_{\text{max}} \sqrt{\pi}} \]  

(20)

and the optimum pulse adaptation conditions are given by [11]

\[ \frac{\sigma_t}{\sigma_f} = \frac{T_{\text{max}}}{F_{\text{max}}} \]  

(21)

The parameter \( \eta \) determines the spread of the pulse in both the domains. Actual ratio of the maximum delay spread and Doppler spread is the ratio of their normalised values.

\[ T_{\text{max}} = \frac{T_{\text{max}}}{T} \quad \text{and} \quad F_{\text{max}} = \frac{F_{\text{max}}}{F} \]  

(22)
where \( T \) and \( F \) are symbol time instant and subcarrier spacing respectively. Also \( T = 1/F \). Thus \( \eta \) becomes,

\[
\eta = \frac{F^2 \times \tau_{\text{max}}}{\nu_{\text{max}}} = F^2 \times \eta' \tag{23}
\]

Thus even though the actual ratio, \( \eta' \), is not matching with the ratio of the time spread and frequency spread of the pulse, the effective ratio \( \eta \) can be matched to that of the pulse dispersions in both the domains by varying the subcarrier bandwidth. In this work a MWHP is designed as in [9-10]. For conventional MWHP which gives optimum performance under doubly dispersive channel among the fixed conventional designs, the ratio \( \sigma_T \) is 1. Therefore, for SIR maximization

\[
F^2 \times \frac{\tau_{\text{max}}}{\nu_{\text{max}}} = 1 \tag{24}
\]

ie,

\[
F = \sqrt{\frac{\nu_{\text{max}}}{\tau_{\text{max}}} \tag{25}
\]

Thus for improved performance, the pulse shape is adaptively varied by changing the system parameter, the subcarrier bandwidth, according to varying channel information.

In this work, a 2x2 MIMO FBMC system is considered. FBMC systems, when combined with multiple antenna configurations can achieve better performance. Simulation results show that MIMO FBMC system using the proposed adaptive filter perform better than any other filters in terms of BER.

### IV. RESULTS AND DISCUSSIONS

So as to prove the superiority of the adaptive system over other existing filter designs various aspects have been demonstrated using numerical simulations plots. The orthogonalization parameter for all the filter designs is chosen as \( \sqrt{2} \). For MWHP and WHP designs length is chosen as 3. Fig. 2 shows the ambiguity function plot of IOTA filter. The magnitudes of the ambiguity function are given in the colour bar adjacent to it in the figure. It is seen from the figure that the shape of the ambiguity function is isotropic in both time and frequency dimensions. The ambiguity function has a peak at the center and it has a low magnitude at all other points. The ambiguity function plot of WHP is given in Fig. 3. Apart from IOTA, the unwanted side lobes are decreased in the WHP pulse design. In Fig. 4, the ambiguity function plot of MWHP design is plotted. It can be inferred from the figure that MWHP design has the most localized design than that of the conventional filters designed so far with fewer side lobes. The adaptive filter system is designed by modifying the MWHP design by allocating different subcarrier bandwidths.

**Fig 2 Ambiguity function plot of IOTA**

**Fig 3 Ambiguity function plot of WHP**

The channel scatter function is assumed to have Jake’s Doppler profile and exponential delay profile. The SIR and BER performances of conventional filters with adaptive system are evaluated under doubly dispersive channels. The BER performance versus SNR, under doubly dispersive channel with \( \mu_{\text{max}} = 700 \text{Hz} \) and maximum delay spread chosen as \( 5 \mu\text{s} \) using 16 PSK and 16 QAM are depicted in Fig. 5 and Fig. 6 respectively.

**Fig 4 Ambiguity function plot of MWHP**

**Fig 5 BER verses SNR for \( \mu_{\text{max}} = 700 \text{Hz} \) and \( \tau_{\text{max}} = 5 \mu\text{s} \) using 16-PSK**

The proposed filter design has been implemented in a 2x2 MIMO FBMC system with LTE specifications.
The BER performance versus SNR is plotted in Fig. 7. It can be concluded from the figure that the MIMO FBMC system having the proposed filter design outperforms the MIMO FBMC system using conventional filter banks.

Table 1. below gives the comparison of BER values obtained for an SNR of 20dB in a 2x2 MIMO-FBMC system using the conventional filters and the proposed one. The value shows that the MIMO FBMC system with the proposed filter outperforms the same system using conventional filters giving a very low BER for a given SNR.

![Fig. 6 BER versus SNR for $\mu_{\text{max}} = 700\text{Hz}$ and $\tau_{\text{max}} = 5\mu$s using 16-QAM](image)

![Fig. 7 BER versus SNR for MIMO-FBMC system using different filters](image)

Table 1: BER values for MIMO FBMC system using different filters for an SNR of 20dB.

<table>
<thead>
<tr>
<th>MIMO-FBMC system with different filters</th>
<th>Bit Error Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>MIMO OFDM</td>
<td>$2 \times 10^{-4}$</td>
</tr>
<tr>
<td>RECANGULAR</td>
<td>$2 \times 10^{-4}$</td>
</tr>
<tr>
<td>WHF</td>
<td>$7 \times 10^{-5}$</td>
</tr>
<tr>
<td>IOTA</td>
<td>$3 \times 10^{-5}$</td>
</tr>
<tr>
<td>MWHF</td>
<td>$2 \times 10^{-5}$</td>
</tr>
<tr>
<td>PROPOSED</td>
<td>$3 \times 10^{-6}$</td>
</tr>
</tbody>
</table>

V. CONCLUSION

In this paper, the performance of a 2x2 MIMO-FBMC system has been compared using conventional filters and a proposed adaptive filter. The simulation results show that the proposed system gives better performance compared to conventional filters. So the Proposed MIMO FBMC system using adaptive filter is ascertained to be a superior candidate for future development in wireless communications.

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AUTHORS PROFILE

Mrs. Luxy Mathews
Luxy Mathews received her M.Tech degree in VLSI and Embedded Systems from ER&DC-IT, Trivandrum, Kerala in the year 2008. She graduated in Electronics and Communication Engineering from College of Engineering, door in the year 2001. She is currently working as an Assistant Professor in the department of Electronics and Communication Engineering at Mar Baselios College of Engineering, Trivandrum, Kerala and pursuing her Ph. D in MIMO-OFDM systems under the guidance of Dr. Sakuntala S. Pillai and Dr.N.Vijayakumar in Kerala University.
Email: luxy.mathews@mbcet.ac.in

Dr. Sakuntala S Pillai
Dr. Sakuntala S. Pillai obtained Ph. D degree from University of Kerala in 1989. She was the Head of the Department of Electronics & Communication, College of Engineering, and Trivandrum from 1996-1998 and later worked as Director, LBS Centre for Science and Technology, Trivandrum. She worked as Head of the Department of Electronics & Communication and as the Dean (R&D) at Mar Baselios College of Engineering & Technology, Trivandrum. Her research interests include OFDM, OFDMA, MIMO wireless systems, Error Correction Coding etc. She is a Senior Member of IEEE, Fellow of IETE and Fellow of Institution of Engineers (India).

Dr.N.Vijayakumar
Dr.N.Vijayakumar obtained Ph.D. degree from Indian Institute of Technology, Kharagpur in 2000.He is currently working as the Principal of Govt.College of Engineering, Kannur. He worked as Professor and Dean of the Department of Electronics & Communication in Govt. Engineering College, Barton Hill and as Professor in College of Engineering, Trivandrum. His research interests include Microwave and optical communication