

Estimating the Flexibility Index of the Heat Exchanger Network by using the Confidence Level

M. Kaoud, M. Gouda, S. M. Aly, M. E. Awad

Abstract: *This paper presents a new technique to estimate the heat exchanger network flexibility index by using a statistical method. This method identifies the true value under uncertainty. The flexibility of the heat exchanger network (HEN) is one of factors which affect the operability of the system. This method was applied by the following a sequential procedure. First, the disturbance propagation of HEN was studied by generating the linear model which presents the relationship between the output disturbance of the target temperature and the disturbance in the operation condition in input such as the supply temperature and the flow rate of the heat capacity. The matlab application was used to generate this equation. Second, the stream with low sensitivity to this disturbance was selected. Third, the confidence level for the lower disturbance this stream was calculated. This method was applied on three published cases and their results were compared with the results of the new method. The new method provides a simple way to calculate the flexibility index for large heat exchanger networks..*

Keywords: *heat exchanger network.confidence level.*

I. INTRODUCTION

One of the chemical process operation and design problems is working under the uncertainty condition and how to overcome this problem. There are two types of uncertainty condition; the first one is external such as fluctuation in feedstock, temperature, and pressure. The second type is internal such as fouling and corrosion. Still our question is whether our plant is flexible to stand or handle this change in the operation condition to achieve the outlet target or not, and how we can evaluate the flexibility of the plant. In 1985, Grossmann and Swaney [1] presented the first expression of flexibility index: "it is the evaluation of the maximum fluctuation of uncertainty condition which can be handled by process without effect on the target." Swany and Grossman [2] pointed to the fact that the critical value must be located in uncertainty parameters space according to the assumption of convexity to achieve process flexibility. In 1987, Floudas and Grossman [3] developed the MINLP model to obtain flexibility index by the active constraint which is responsible for determining the flexibility limitation in the heat exchanger network. Studying the disturbance propagation in

heat exchanger network is important to evaluate the cause and effect of disturbance in the inlet condition and its effect on the stream target. Sensitivity tables were presented by Linnhoff and Kotjabaskis [4], but this method was not sufficient in the complex HEN. LI et al. [5] presented the disturbance propagation in the linear equation model, but this model ignores the cross impact of the temperature disturbance and the flow rate of heat capacity. HEN disturbance propagation model was introduced by Yang et al. [6]. This model described the relation between the streams input variable and the output target, and it succeeded to achieve good results within the narrow range of values of disturbance, but in the high mass flow rate this model showed errors. By using the effectiveness NTU method, Hegg and Vizcaíno [7] managed to develop the model of DP for HEN and obtained good results.

In this paper, the statistics tool used to evaluate flexibility index under uncertainty condition is the confidence level. The confidence interval (CI) is a numerical interval that is expected to contain the real value of a statistical parameter to be known to a statistical community. The concept of the confidence.

Domain is associated with another concept, the level of confidence [8, 9, 10], which can be explained as follows: Let's say that required to estimate the value of a parameter for a statistical community. The ratio of the number of trust fields that contain the true value of a parameter is called the confidence level. When we say that we are 99% confident (i.e., the confidence level is 99%), this means that the value of the desired parameter (for example, arithmetic mean) falls within the range of confidence we calculated from a random sample. This is equivalent to saying that 99% of all the areas of confidence can be calculated from samples. The randomness of the studied statistical community will contain the true value of the parameter. So, it is wrong to say that a 99% confidence level means that there is a 99% probability that the real value of the parameter will fall within the calculated confidence interval [11]. What is correct is that the true value of the parameter either falls within or does not fall within the trust field. The word trust here has nothing to do with probability, but rather with the frequency with which the confidence fields from many samples contain the true value of the parameter. The level of confidence is determined by the researcher himself, and therefore it is not a figure derived from the sample data.

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In most of the research, it is customary to use 95% confidence levels, but this can also be calculated with other confidence levels such as 99% and 90%. It should be noted that the size of the field of confidence is affected by several factors such as the size of the sample studied and the dispersion of the statistical community.

The principle behind trust periods was formulated to respond to the question raised in the statistical reasoning about how to deal with the inherent uncertainty of the results from data, which is in itself the total of its randomly selected sections of the population. There are other answers, particularly those provided by parallel reasoning in the form of reliable periods. The confidence intervals correspond to the rule chosen to determine the limits of trust, where this rule is determined primarily before obtaining any data, or before conducting the experiment. The rule is defined as all the possible data sets that can be obtained; there is a high probability that the interval set by the rule will include the real value of the quantity under consideration. A parallel approach seems to offer periods, which, subject to the acceptance of the interpretation of "probability" as a parallel probability, can be interpreted as meaning that the specified interval is calculated from a total. A disjunction approach to trust does not allow this, since in this formulation and at this same stage both the time-lapse limits and the true values are fixed values, and there is no random interest. On the other hand, a parallel approach applies only to the previous probability used in the calculation, while its period of confidence does not depend on assumptions about the previous probability.

II. METHODOLOGY

a- DISTURBANCE PROPAGATION MODELING

A disturbance propagation model was adopted from Yang et al. modeling approach [6] and the model is explained below.

$$\delta T^t = D_t \delta T^s + D_m \delta M C p \quad (1)$$

where:

$$\begin{pmatrix} \delta T_h^t \\ \delta T_c^t \end{pmatrix} = \begin{pmatrix} 1 - \alpha & \alpha \\ \alpha_h \beta & 1 - \beta \end{pmatrix} \begin{pmatrix} \delta T_h^s \\ \delta T_c^s \end{pmatrix} + \begin{pmatrix} \alpha_h (2 - \alpha) & -\alpha \alpha_c \\ \alpha_h \beta & -\alpha_c (2 - \beta) \end{pmatrix} \begin{pmatrix} \delta M C p_h \\ \delta M C p_c \end{pmatrix} \quad (2)$$

$$\alpha = \frac{T_h^s - T_h^t}{T_h^s - T_c^s} \quad (3)$$

$$\beta = \frac{T_c^t - T_c^s}{T_h^s - T_c^s} \quad (4)$$

$$\alpha_h = \frac{\Delta T_h}{2M C p_h} \quad (5)$$

$$\alpha_c = \frac{\Delta T_c}{2M C p_c} \quad (6)$$

Where T and δT are the stream temperature and temperature fluctuation respectively. Mcp and $\delta M C p$ are the heat capacity flow rate and heat capacity flow rate fluctuation respectively. Superscripts s and t refer to source and target respectively subscripts h and c refers to hot and cold respectively. If a HEN contains N_e heat exchangers, a system model can be obtained directly by lumping all unit based models in the sequence of exchanger numbers that gives:-

$$\delta T^{*out} = D_{t_{E_i}} \delta T_{E_i}^{in} + D_{m_{E_i}} \delta M C p^* \quad (7)$$

$$\delta T^{*out} = D t_E^* \delta T^{*in} + D m_E^* \delta M C p_E^* \quad (8)$$

Where

$$\delta T^{*in} = [(\delta T_{E_1}^{in})^T (\delta T_{E_2}^{in})^T \dots (\delta T_{E_{N_e}}^{in})^T]^T \quad (9)$$

$$\delta T^{*out} = [(\delta T_{E_1}^{out})^T (\delta T_{E_2}^{out})^T \dots (\delta T_{E_{N_e}}^{out})^T]^T \quad (10)$$

$$\delta M C p_E^* = [(\delta M C p_{E_1})^T (\delta M C p_{E_2})^T \dots (\delta M C p_{E_{N_e}})^T]^T \quad (11)$$

$$D_{t_E}^* = \text{diag} \{D_{t_{E_1}}, D_{t_{E_2}} \dots D_{t_{E_{N_e}}}\} \quad (12)$$

$$D_{m_E}^* = \text{diag} \{D_{m_{E_1}}, D_{m_{E_2}} \dots D_{m_{E_{N_e}}}\} \quad (13)$$

The following equation represents the system disturbance propagation model:

$$\delta T^t = D_t * \delta T^s + D_m * \delta M C P \quad (14)$$

Where

$$D_t = D_{t11} + D_{t12} * (I - D_{t22})^{-1} * D_{t21} \quad (15)$$

$$D_m = D_{m1} + D_{t12} * (I - D_{t22})^{-1} * D_{m2} \quad (16)$$

Where

$$D_t^* = \begin{pmatrix} D_{t11} & D_{t12} \\ D_{t21} & D_{t22} \end{pmatrix} = V1 * D_{t_E}^* * V2 \quad (17)$$

$$D_m^* = \begin{pmatrix} D_{m1} \\ D_{m2} \end{pmatrix} = V1 * D_{m_E}^* * V3 \quad (18)$$

Matrices $V1$, $V2$ and $V3$ are system construction matrices.

a- Basic concepts in uncertainty and probability

- Measurable features of nature are nearly always random variables – observations of the variable at any place, time and scale will be random numbers drawn from a certain distribution of more or less probable values
- The probability density function (pdf) describes the relative probability (fig.1) that observations of a random variable will fall within a certain range:

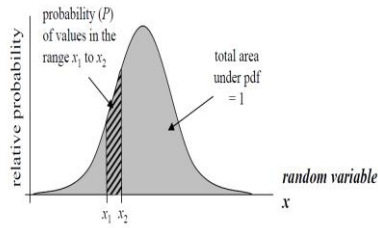


Figure 1 The probability density function (pdf)

- Many variables follow a distribution similar to the Gaussian or normal distribution. For such variables:
- Values above or below the mean are equally likely (i.e. the mean is equal to the median).
- 70% of observations will fall within one standard deviation (σ) of the mean (μ). 95% will fall within two standard deviations (fig. 2).

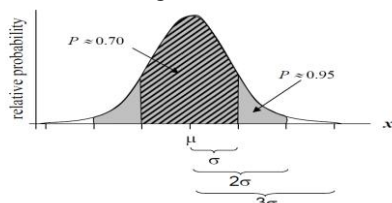


Figure 2 relation between relative probability and standard deviation

b- Sensitivity versus uncertainty

- An output variable from a model is **sensitive** to an input variable (or parameter, or process representation) if variability in the input variable leads to a relatively large amount of variability in the output variable (fig.3).

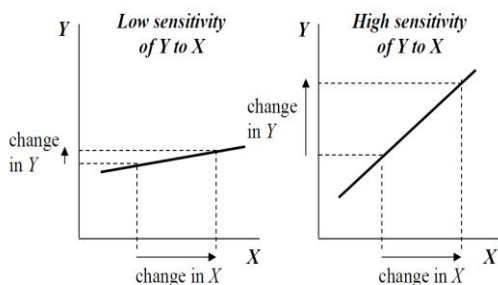


Figure 3 effect of change in input & output variable in degree of sensitivity

The greater the sensitivity of an output variable to an input variable (or parameter), the more uncertainty in the input value propagates to the output variable (fig.4)

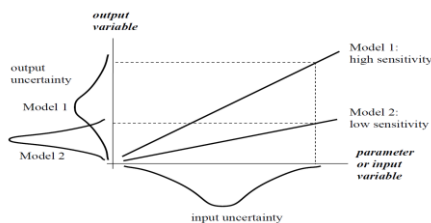


Figure 4 PDF change with degree of sensitivity

III. PROBLEM DESCRIPTION

The system consists of the numbers of hot streams (n_h) which have a certain inlet temperature and target temperature (hot streams will be cooled down), and the

numbers of cold streams (n_c) which have a certain inlet temperature and a target temperature (cold streams will be heated). The two types of streams (hot & cold) are known as the heat capacity flow rate. Streams of this system are exposed to external disturbance in inlet temperature or inlet of heat flow rate or in both. What is required is to evaluate this system and see if this disturbance will be accepted or not and what the HEN flexibility will be.

1- Solution procedure

3.1 Generate the linear equation for the given HEN by using the matlab program [12] to be as in the following equation (eq.1)

$$\delta T^t = Dt\delta T^s + Dm\delta M Cp \quad (1)$$

3.2 -

- a- If the disturbance in the inlet temperature is only the term of $B\delta M c p_{inlet}$, it will be equal to zero and the equation will be $\delta T_{out} = A\delta T_{inlet}$.
- b- If the disturbance in the heat capacity flow rate is only the term of $A\delta T_{inlet}$, it will be equal to zero and the equation will be $\delta T_{out} = B\delta M c p_{inlet}$.
- c- If there is disturbance in the inlet temperature and heat capacity flow rate, the equation will be as follows:
 $\delta T_{out} = A\delta T_{inlet} + B\delta M c p_{inlet}$.

3.3- Compensate for the value of each disturbance in the equation according to step two.

3.4- Select the stream that has the lower value of outlet temperature disturbance and neglect the streams with outlet zero temperature disturbance.

3.5- Assume the confidence interval is [zero, x], where x is the lower value of outlet temperature disturbance.

3.6- Calculate the confidence level at 95% for the confidence interval. (We can use the excel program data analysis – description statistics to find the confidence level at 95%).

2- Case studies

4.1- First case study

The first case study was presented by Marcelo Escobar and Jorge O. Trierweiler [13]. HEN consists of two hot streams and two cold streams as shown in Table 1. The expected variation in inlet temperature is 10 k. HEN structure for nominal parameters was shown in Fig. 6.

Table 1: Streams data

stream	T_{in} (k)	T_{out} (k)	F (KW.K ⁻¹)	h (KW.m ² K ⁻¹)
H1	583±10	323	1.4	0.16
H2	723±10	553	2	0.16
C1	313±10	393	3	0.16
C2	388±10	553	2	0.16
CU	303	323		0.16
HU	573	573		0.16

Cost of heat exchanger ($\$y^{-1}$) = $5500+4333[\text{area}(\text{m}^2)]^{0.6}$

Cost of heating utility = $60.576(\$KW^{-1}.y^{-1})$

Cost of cooling utility = $171.428 (\$KW^{-1}.y^{-1})$



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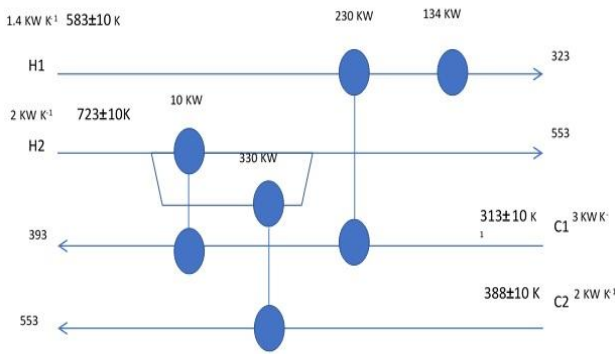


Figure 5: Case 1 HEN configuration

Solution for obtaining the flexibility index by the confidence level:

- Generate the state linear model by using the matlab program.

$$\begin{bmatrix} \delta T_{H1}^{OUT} \\ \delta T_{H2}^{OUT} \\ \delta T_{C1}^{OUT} \\ \delta T_{C2}^{OUT} \end{bmatrix} = \begin{bmatrix} 0.391 & 0 & 0.608 & 0 \\ 0.004 & 0.49 & .011 & .492 \\ 0.281 & 0.01 & 0.709 & 0 \\ 0 & 0.49 & 0 & 0.507 \end{bmatrix} \begin{bmatrix} \delta T_{H1}^{IN} \\ \delta T_{H2}^{IN} \\ \delta T_{C1}^{IN} \\ \delta T_{C2}^{IN} \end{bmatrix} + \begin{bmatrix} 81.61 & 0 & -7.775 & 0 \\ 0.249 & 63.42 & -0.337 & -20.31 \\ 16.493 & 0.425 & -22.813 & 0 \\ 0 & 20.932 & 0 & -62.18 \end{bmatrix} \begin{bmatrix} \delta F_{H1}^{IN} \\ \delta F_{H2}^{IN} \\ \delta F_{C1}^{IN} \\ \delta F_{C2}^{IN} \end{bmatrix}$$

- For no disturbance in the heat capacity flow rate, we can assume the equation as follows:

$$\begin{bmatrix} \delta T_{H1}^{OUT} \\ \delta T_{H2}^{OUT} \\ \delta T_{C1}^{OUT} \\ \delta T_{C2}^{OUT} \end{bmatrix} = \begin{bmatrix} 0.391 & 0 & 0.608 & 0 \\ 0.004 & 0.49 & .011 & .492 \\ 0.281 & 0.01 & 0.709 & 0 \\ 0 & 0.49 & 0 & 0.507 \end{bmatrix} \begin{bmatrix} \delta T_{H1}^{IN} \\ \delta T_{H2}^{IN} \\ \delta T_{C1}^{IN} \\ \delta T_{C2}^{IN} \end{bmatrix}$$

- Select the lower sensitivity output stream disturbance when affected by disturbance 10 k in the inlet temperature of streams.

$$\begin{bmatrix} \delta T_{H1}^{IN} \\ \delta T_{H2}^{IN} \\ \delta T_{C1}^{IN} \\ \delta T_{C2}^{IN} \end{bmatrix} = \begin{bmatrix} \pm 10 \\ 0 \\ 0 \\ 0 \end{bmatrix} \rightarrow \begin{bmatrix} \delta T_{H1}^{OUT} \\ \delta T_{H2}^{OUT} \\ \delta T_{C1}^{OUT} \\ \delta T_{C2}^{OUT} \end{bmatrix} = \begin{bmatrix} \pm 3.91 \\ \pm 0.04 \\ \pm 2.81 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} \delta T_{H1}^{IN} \\ \delta T_{H2}^{IN} \\ \delta T_{C1}^{IN} \\ \delta T_{C2}^{IN} \end{bmatrix} = \begin{bmatrix} 0 \\ \pm 10 \\ 0 \\ 0 \end{bmatrix} \rightarrow \begin{bmatrix} \delta T_{H1}^{OUT} \\ \delta T_{H2}^{OUT} \\ \delta T_{C1}^{OUT} \\ \delta T_{C2}^{OUT} \end{bmatrix} = \begin{bmatrix} 0 \\ \pm 4.9 \\ \pm 1 \\ \pm 4.9 \end{bmatrix}$$

$$\begin{bmatrix} \delta T_{H1}^{IN} \\ \delta T_{H2}^{IN} \\ \delta T_{C1}^{IN} \\ \delta T_{C2}^{IN} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \pm 10 \\ 0 \end{bmatrix} \rightarrow \begin{bmatrix} \delta T_{H1}^{OUT} \\ \delta T_{H2}^{OUT} \\ \delta T_{C1}^{OUT} \\ \delta T_{C2}^{OUT} \end{bmatrix} = \begin{bmatrix} \pm 6.08 \\ \pm 1.1 \\ \pm 7.09 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} \delta T_{H1}^{IN} \\ \delta T_{H2}^{IN} \\ \delta T_{C1}^{IN} \\ \delta T_{C2}^{IN} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \pm 10 \end{bmatrix} \rightarrow \begin{bmatrix} \delta T_{H1}^{OUT} \\ \delta T_{H2}^{OUT} \\ \delta T_{C1}^{OUT} \\ \delta T_{C2}^{OUT} \end{bmatrix} = \begin{bmatrix} 0 \\ \pm 4.92 \\ 0 \\ \pm 5.07 \end{bmatrix}$$

The lower stream sensitivity to the disturbance is $\delta T_{H2}^{OUT} = \pm 0.04$.

- Apply the calculation of the confidence level 95% on the confidence interval from 0 to .04 by using data analysis in excel (Table 2).

Table 2: Excel data analysis

Mean	0.02
Standard Error	0.02
Median	0.02
Mode	#N/A
Standard Deviation	0.028284
Sample Variance	0.0008
Kurtosis	#DIV/0!
Skewness	#DIV/0!
Range	0.04
Minimum	0
Maximum	0.04
Sum	0.04
Count	2
Confidence Level (95.0%)	0.254124

4.2- Second case study

This is the same case study No. 1, but there is a change in HEN configuration [13] to increase HEN flexibility as shown in Fig. 7.

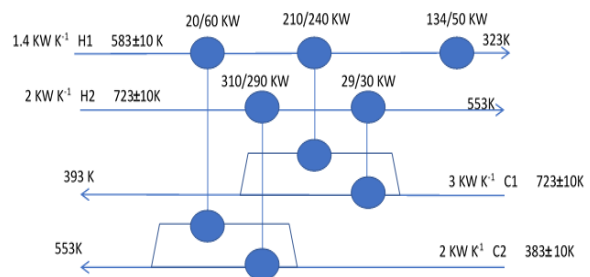


Figure 6: Case 2 HEN configuration

Solution for obtaining the flexibility index by the confidence level:

Generate the state linear model by using the matlab program.

$$\begin{bmatrix} \delta T_{H1}^{OUT} \\ \delta T_{H2}^{OUT} \\ \delta T_{C1}^{OUT} \\ \delta T_{C2}^{OUT} \end{bmatrix} = \begin{bmatrix} 0.383 & 0 & 0.586 & 0.030 \\ 0 & 0.506 & 0.058 & 0.435 \\ 0.253 & 0.021 & 0.687 & 0.038 \\ 0.051 & 0.463 & 0 & 0.486 \end{bmatrix} \begin{bmatrix} \delta T_{H1}^{IN} \\ \delta T_{H2}^{IN} \\ \delta T_{C1}^{IN} \\ \delta T_{C2}^{IN} \end{bmatrix} +$$

$$\begin{bmatrix} 79.782 & 0 & -7.821 & -1.249 \\ 0 & 63.346 & -0.784 & -17.963 \\ 17.356 & 2.483 & -22.493 & -1.576 \\ 0.262 & 17.929 & 0 & -61.299 \end{bmatrix} \begin{bmatrix} \delta F_{H1}^{IN} \\ \delta F_{H2}^{IN} \\ \delta F_{C1}^{IN} \\ \delta F_{C2}^{IN} \end{bmatrix}$$

For no disturbance in the heat capacity flow rate, we can assume the equation as follows:

$$\begin{bmatrix} \delta T_{H1}^{OUT} \\ \delta T_{H2}^{OUT} \\ \delta T_{C1}^{OUT} \\ \delta T_{C2}^{OUT} \end{bmatrix} = \begin{bmatrix} 0.383 & 0 & 0.586 & 0.030 \\ 0 & 0.506 & 0.058 & 0.435 \\ 0.253 & 0.021 & 0.687 & 0.038 \\ 0.051 & 0.463 & 0 & 0.486 \end{bmatrix} \begin{bmatrix} \delta T_{H1}^{IN} \\ \delta T_{H2}^{IN} \\ \delta T_{C1}^{IN} \\ \delta T_{C2}^{IN} \end{bmatrix}$$

Select the lower sensitivity output stream disturbance when affected by disturbance 10 k in the inlet temperature of streams.

$$\begin{bmatrix} \delta T_{H1}^{IN} \\ \delta T_{H2}^{IN} \\ \delta T_{C1}^{IN} \\ \delta T_{C2}^{IN} \end{bmatrix} = \begin{bmatrix} \pm 10 \\ 0 \\ 0 \\ 0 \end{bmatrix} \rightarrow \begin{bmatrix} \delta T_{H1}^{OUT} \\ \delta T_{H2}^{OUT} \\ \delta T_{C1}^{OUT} \\ \delta T_{C2}^{OUT} \end{bmatrix} = \begin{bmatrix} \pm 3.83 \\ 0 \\ \pm 2.53 \\ 0.51 \end{bmatrix}$$

$$\begin{bmatrix} \delta T_{H1}^{IN} \\ \delta T_{H2}^{IN} \\ \delta T_{C1}^{IN} \\ \delta T_{C2}^{IN} \end{bmatrix} = \begin{bmatrix} 0 \\ \pm 10 \\ 0 \\ 0 \end{bmatrix} \rightarrow \begin{bmatrix} \delta T_{H1}^{OUT} \\ \delta T_{H2}^{OUT} \\ \delta T_{C1}^{OUT} \\ \delta T_{C2}^{OUT} \end{bmatrix} = \begin{bmatrix} 0 \\ \pm 5.06 \\ \pm 0.21 \\ \pm 4.63 \end{bmatrix}$$

$$\begin{bmatrix} \delta T_{H1}^{IN} \\ \delta T_{H2}^{IN} \\ \delta T_{C1}^{IN} \\ \delta T_{C2}^{IN} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \pm 10 \\ 0 \end{bmatrix} \rightarrow \begin{bmatrix} \delta T_{H1}^{OUT} \\ \delta T_{H2}^{OUT} \\ \delta T_{C1}^{OUT} \\ \delta T_{C2}^{OUT} \end{bmatrix} = \begin{bmatrix} \pm 5.86 \\ \pm .58 \\ \pm 6.87 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} \delta T_{H1}^{IN} \\ \delta T_{H2}^{IN} \\ \delta T_{C1}^{IN} \\ \delta T_{C2}^{IN} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \pm 10 \end{bmatrix} \rightarrow \begin{bmatrix} \delta T_{H1}^{OUT} \\ \delta T_{H2}^{OUT} \\ \delta T_{C1}^{OUT} \\ \delta T_{C2}^{OUT} \end{bmatrix} = \begin{bmatrix} 0.3 \\ \pm 4.35 \\ 0.38 \\ \pm 4.86 \end{bmatrix}$$

The lower stream sensitivity to the disturbance is $\delta T_{C1}^{OUT} = \pm 0.21$.

Apply the calculation of the confidence level 95% on the confidence interval from 0 to .21 by using data analysis in excel (Table 3).

Table 3: Excel data analysis

0.105	Mean
0.105	Standard Error
0.105	Median
#N/A	Mode
0.148492	Standard Deviation
0.02205	Sample Variance
#DIV/0!	Kurtosis
#DIV/0!	Skewness
0.21	Range
0	Minimum
0.21	Maximum
0.21	Sum
2	Count
1.334151	Confidence Level (95.0%)

4.1- Third case study

This problem was presented by Xiaoling Zhang, Hongchao Yin, and Zhaoyi Huo [14]. The heat exchanger network consists of two hot streams and two cold streams (Table 4) HEN structure for nominal parameters was shown in Fig. 8.

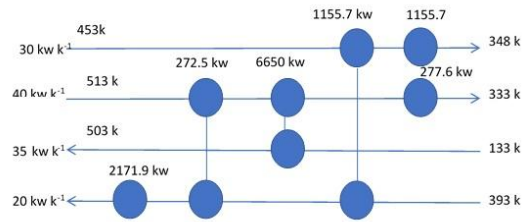


Figure 7 case study no 3

Table 4: Streams data

stream	T _{in} (k)	T _{out} (k)	F (KW.K ⁻¹)	h(KW.m ² K ⁻¹)
H1	453±20	348	30±1	0.15
H2	513	333	40	0.10
C1	313±10	503	35	0.20
C2	393	573	20±2	0.10
CU	598	598		2.0
HU	298	313		0.5

Cost of heat exchanger (\$y⁻¹) = 15000+60[area(m²)]^{0.8}

Cost of heating utility = 110(\$KW⁻¹.y⁻¹)

Cost of cooling utility = 10 (\$KW⁻¹.y⁻¹)

The expected operating disturbances are ±20K for the input temperature of H1; ±1kw/K for the heat capacity flow rate of H1; ±10K for the input temperature of C1; ±2kw/K for the heat capacity flow rate of C2.

Solution for obtaining the flexibility index by the confidence level: Generate the state linear model by using the matlab program.

$$\begin{bmatrix} \delta T_{H1}^{OUT} \\ \delta T_{H2}^{OUT} \\ \delta T_{C1}^{OUT} \\ \delta T_{C2}^{OUT} \end{bmatrix} = \begin{bmatrix} 0.3579 & 0 & 0 & 0.6421 \\ 0.0147 & 0.1242 & 0.8606 & 5.6365e-04 \\ 0.1037 & 0.8758 & 0.0165 & .0040 \\ 0.7522 & 0.2190 & 0 & .0288 \end{bmatrix} \begin{bmatrix} \delta T_{H1}^{IN} \\ \delta T_{H2}^{IN} \\ \delta T_{C1}^{IN} \\ \delta T_{C2}^{IN} \end{bmatrix} + \begin{bmatrix} .8719 & 0 & 0 & -.9275 \\ 0.0094 & 2.3903 & -2.3358 & -0.0281 \\ .0666 & 2.2022 & -2.7591 & -0.1980 \\ 0.4829 & .0186 & 0 & -1.7766 \end{bmatrix} \begin{bmatrix} \delta F_{H1}^{IN} \\ \delta F_{H2}^{IN} \\ \delta F_{C1}^{IN} \\ \delta F_{C2}^{IN} \end{bmatrix}$$

Select the lower sensitivity output stream disturbance when affected by disturbance 10 k in the inlet temperature of streams.

$$\begin{bmatrix} \delta T_{H1}^{IN} \\ \delta T_{H2}^{IN} \\ \delta T_{C1}^{IN} \\ \delta T_{C2}^{IN} \end{bmatrix} = \begin{bmatrix} \pm 20 \\ 0 \\ 0 \\ 0 \end{bmatrix} \rightarrow \begin{bmatrix} \delta T_{H1}^{OUT} \\ \delta T_{H2}^{OUT} \\ \delta T_{C1}^{OUT} \\ \delta T_{C2}^{OUT} \end{bmatrix} = \begin{bmatrix} \pm 7.1567 \\ \pm 0.2940 \\ \pm 2.0740 \\ 15.0424 \end{bmatrix}$$

$$\begin{bmatrix} \delta T_{H1}^{IN} \\ \delta T_{H2}^{IN} \\ \delta T_{C1}^{IN} \\ \delta T_{C2}^{IN} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \pm 10 \\ 0 \end{bmatrix} \rightarrow \begin{bmatrix} \delta T_{H1}^{OUT} \\ \delta T_{H2}^{OUT} \\ \delta T_{C1}^{OUT} \\ \delta T_{C2}^{OUT} \end{bmatrix} = \begin{bmatrix} 0 \\ \pm 8.6057 \\ \pm 0.1650 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} \delta F_{H1}^{IN} \\ \delta F_{H2}^{IN} \\ \delta F_{C1}^{IN} \\ \delta F_{C2}^{IN} \end{bmatrix} = \begin{bmatrix} \pm 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} \rightarrow \begin{bmatrix} \delta T_{H1}^{OUT} \\ \delta T_{H2}^{OUT} \\ \delta T_{C1}^{OUT} \\ \delta T_{C2}^{OUT} \end{bmatrix} = \begin{bmatrix} \pm 0.872 \\ \pm 0.0094 \\ \pm 0.0666 \\ \pm 0.4829 \end{bmatrix}$$

$$\begin{bmatrix} \delta F_{H1}^{IN} \\ \delta F_{H2}^{IN} \\ \delta F_{C1}^{IN} \\ \delta F_{C2}^{IN} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \pm 2 \end{bmatrix} \rightarrow \begin{bmatrix} \delta T_{H1}^{OUT} \\ \delta T_{H2}^{OUT} \\ \delta T_{C1}^{OUT} \\ \delta T_{C2}^{OUT} \end{bmatrix} = \begin{bmatrix} \pm 1.8552 \\ \pm 0.0561 \\ \pm 0.3960 \\ \pm 3.5532 \end{bmatrix}$$

$$\begin{bmatrix} \delta F_{H1}^{IN} \\ \delta F_{H2}^{IN} \\ \delta F_{C1}^{IN} \\ \delta F_{C2}^{IN} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \pm 2 \end{bmatrix} \rightarrow \begin{bmatrix} \delta T_{H1}^{OUT} \\ \delta T_{H2}^{OUT} \\ \delta T_{C1}^{OUT} \\ \delta T_{C2}^{OUT} \end{bmatrix} = \begin{bmatrix} \pm 1.8552 \\ \pm 0.0561 \\ \pm 0.3960 \\ \pm 3.5532 \end{bmatrix}$$

To select the lower sensitivity stream to all disturbances, see Table 5.

Table 5: Sensitivity analysis

	Dist. in outlet stream H1	Dist. in outlet stream H2	Dist. in outlet stream C1	Dist. in outlet stream C2
$\delta T_{H1}^{IN}=20$	7.156	.2940	2.074	15.0424
$\delta T_{C1}^{IN}=10$	0	8.607	.1650	0
$\delta F_{H1}^{IN}=\pm 1$	0.872	.0094	.0666	.4829
$\delta F_{C2}^{IN}=\pm 2$	1.855	.0561	.3960	3.5532
sum	9.883	8.9665	2.7016	19.078

The lower stream sensitivity is C1 .

- Select the lower temperature disturbance for c1 is which .1650.
- Select the lower heat capacity flow rate for c1 which is .0666

• Apply the calculation of the confidence level 95% on the confidence interval from .0666 to .1650 by using data analysis in excel (Table 6).

Table 6: Excel data analysis

Mean	0.1155
Standard Error	0.0495
Median	0.1155
Mode	#N/A
Standard Deviation	0.070004
Sample Variance	0.004901
Kurtosis	#DIV/0!
Skewness	#DIV/0!

Range	0.099
Minimum	0.066
Maximum	0.165
Sum	0.231
Count	2
Confidence Level (95.0%)	0.628957

IV. DISCUSSION OF RESULTS

- For first case, after applying the confidence level to find the flexibility index for HEN, we got .254 .When comparing this result with the same case but with the flexibility index with another method (.25), we found that the difference between the two results is 1.6% (Table 7).
- For the second case, after applying the confidence level to find the flexibility index for HEN, we got 1.33. When comparing this result with the same case but with the flexibility index with another method (1.42), we found that the difference between the two results is 9.5% (Table 8).
- For the third case, after applying the confidence level to find the flexibility index for HEN, we got 0.628. When comparing this result with the same case but with the flexibility index with another method (0.63), we found that the difference between two results is .31 (Table 9).

Table 7: Results for case 1

	Case 1	
	Mercelo Escobar [13]	This work
Flexibility index	0.250	0.254
Error %	1.6	

Table 8: Results for case 2

	Case 2	
	Mercelo Escobar [13]	This work
Flexibility index	1.429	1.334
Error %	9.5	

Table 9: Results for case 3

	Case 3	
	Xiaoling Zhang[14]	This work
Flexibility index	0.63	.6289
Error %	0.001	

V. CONCLUSION

When comparing the results of the three published cases to evaluate the flexibility index of HEN by using the confidence level and the confidence interval with the flexibility index by the equality and inequality equation for HEN (using MILP & MINLP), we found that the results of both methods matched, and found the error due to the equation of the overall disturbance through HEN is linear. In the other method, the nonlinear equation was used and there may be differences in the program solver. The new method provides a simple way to calculate the flexibility index for large heat exchanger networks.

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