# Robust Optimization for Handling Time Uncertainty in Capacitated Vehicle Routing Problem for Post-Disaster Aid Logistical Distribution

**Audi Luqmanul Hakim Achmad, Diah Chaerani, Eman Lesmana**

***Abstract*****:** ***In post-disaster aid logistical distribution, time is the most important thing to minimize the number of victim. But, time travelling become uncertain as the impact of disaster occurrence. In this paper, Robust Capacitated Vehicle Routing Problem (RCVRP) for post-disaster aid logistical distribution under time uncertainty is discussed to handle the uncertain travelling time. The robust optimal solution derivation is presented using Robust Optimization. The time uncertainty is assumed to be lied in a box and a polyhedral uncertainty set. This assumption yields a Robust Counterpart (RC) of the RCVRP model which are computationally tractable. Case study and simulation presented in this paper and shows a robust optimal solution.***

***Keywords: Box and Polyhedral Uncertainnty, Capacitated Vehicle Routing Problem, Computational Tractability, Robust Optimization.***

## I. INTRODUCTION

This paper discuss an aid logistical distribution model, specifically with optimization modelling. Optimization is a method for determining the best solution of problems, considering all conditions that should be fulfilled. In this case, one of the main parameters is travelling time between each locations for distributing the aid logistics. One of the assumptions used is that there is a sufficient amount of logistics that will be distributed. More parameters and assumptions were discussed later in this paper.

In this paper, the Robust Counterpart (RC) of uncertain Capacitated Vehicle Routing Problem (CVRP) for post-disaster aid logistical distribution is presented. The uncertainty in travelling time is assumed to be lied in a box uncertainty set also in a polyhedral uncertainty set. The aims of this approach is to presented the RC is convex and computationally tractable.

Vehicle Routing Problem (VRP) were first introduced by George Dantzig and John Ramser in 1959. They discussed about serving the demand for oil of a several gas stations from a central hub with a minimum total travelled distance using a fleet of homogenous trucks.

Five years later, Clarke and Wright made a generalize formulation of this problem to linear programming which concerned about common logistic and transport problems, e.g. fulfill the demand of customers from a central hub with a number of heterogenous vehicles. This formulation was later known as Vehicle Routing Problem (VRP) [3]. Research development of VRP was going up to this day with varying types of application and problems. One of the research development of VRP in disaster management was discussed by Hsueh, Chen & Chou [8]. Mguis, Zidi, Ghedira & Borne discussed about Dynamic VRP in disaster management with Time Windows [10]. Qin, Ye, Cheng, Zhao & Ni discussed about VRP in disaster management with demand uncertainty [16]. Gharib, Bozorgi-Amiri, Tavakkoli-Moghaddam & Najafi discussed about Cluster-based VRP in disaster management [5]. In this paper, we discussed Robust Capacitated Vehicle Routing Problem (RCVRP) under travelling time uncertainty.

## II. OBJECTIVES

The purposes of the study is to determine the model for aid logistical distribution under time uncertainty and simulate the model for aid logistical distribution under time uncertainty that computationally tractable.

We modified the model that was proposed by Munari [9] as the nominal model. Then, we built the uncertain problem of the model using Robust Optimization as we discussed later in this paper. The uncertain problem of the model then can be formulated into a single deterministic problem called Robust Counterpart that is computationally tractable using Robust Optimization.

## III. METHODS

Method that is used in this research is Robust Optimization. Robust Optimization aims to determine a model, in this case is Capacitated Vehicle Routing Problem (CVRP), that robusts to the uncertain data that is used on the model. The uncertain data could be a result as measurement error (e.g. temperature measurement, length measurement, etc.). Another uncertain data could be a result of the mistake in estimating a data (e.g. demand estimation, profit estimation, cost estimation, etc.).

**Revised Manuscript Received on January 15, 2020**

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In this case, we assume that there's an uncertain travelling time between each location as a result of natural disasters.

According to Ben-Tal & Nemirovski [1] and discussed by Hertog [7], Robust Optimization is a method for determining the solution of optimization problem that robusts to the uncertain data, where the uncertain data are in a set called uncertainty set.

According to Ben-Tal & Nemirovski [2] and discussed by Gorissen, Yanıkoğlu, & Hertog [6], the general formulation for uncertain linear programming was given by (1).

$$\min_{x}\{c^T x : Ax \le b \mid (c, A, b) \in \mathcal{U}\}. \tag{1}$$

Using asumptions that were discussed by Ben-Tal & Nemirovski [2] and discussed by by Gorissen, Yanıkoğlu, & Hertog [7], all uncertain linear programming could be reformulated as (2).

$$\min_{x}\{c^T x : Ax \le b \mid A \in \mathcal{U}\}, \tag{2}$$

where $x \in \mathrm{R}^n$, $b \in \mathbb{R}^m$, $\mathcal{U}$ is primitive uncertainty set dan $A$ is a $(m \times n)$ matrix, $A \in \mathcal{U}$.

The challenge in Robust Optimization is to remove all the uncertain parameters in uncertain problem (2) so that we obtained a single deterministic problem called Robust Counterpart (RC).

Robust Counterpart (RC) for uncertain problem (2) where the uncertain parameters are in Box Uncertainty Set was given by (3).

$$\min_{x}\{c^T x : a_i^T x + \|P^T x\|_1 \le b_i, \forall i = 1,2,3,\ldots,m\}. \tag{3}$$

Robust Counterpart (RC) for uncertain problem (2) where the uncertain parameters are in Polyhedral Uncertainty Set was given by (4).

$$\min_{x}\{c^T x : a_i^T x + a^T y \le b_i, D^T y = P^T x, y \ge 0, \forall i = 1,\ldots,m\}. \tag{4}$$

Robust Counterpart (RC) (3) and (4) are Linear Programming.

**Theorem 1. (Ben-Tal & Nemirovski [1]) and (Chaerani & Roos [4])**

Assume that the uncertainty set $\mathcal{U}$ is given as the affine image of a bounded set $Z = \{\zeta\} \subset \mathbb{R}^N$, and $Z$ is given either:

(1) by a system of linear inequality constraints

$$Q\zeta \le q,$$

or

(2) by a system pf Conic Quadratic constraints

$$\|Q_i\zeta - q_i\|_2 \le q_i^T\zeta - r_i, i = 1,\ldots,M,$$

or

(3) by a system of Linear Matrix Inequalities

$$Q_0 + \sum_{i=1}^{\dim \zeta} \zeta_i Q_i \ge 0.$$

In the cases (2) and (3) assume also that the system of constraints defining $\mathcal{U}$ is strictly feasible. Then the Robust Counterpart (3) ad (4) of the uncertain Linear Programming (2) is equivalent to:

1. a Linear Programming problem in case (1),
2. a Conic Quadratic problem in case (2),
3. a Semidefinite program in case (3).

In all cases, the data of the resulting robust counterpart problem are readily given by $m$, $n$ and the data specifying the uncertainty set. Moreover, the sizes of the resulting problem are polynomial in the size of the data specifying the uncertainty set.

From Theorem 1, linear programming model could be solved computationally within polynomial time. Robust feasible of Robust Counterpart (3) and (4) are the solution that fulfill all the constraints in each problem. Robust optimal solution of Robust Counterpart (3) and (4) are the optimal solution for each problems.

## IV. RESULTS AND DISCUSSION

In this section, mathematical formulation for Capacitated Vehicle Routing Problem (CVRP) in post-disaster aid logistical distribution was discussed. Also, mathematical formulation for Robust Counterpart (RC) of Robust Capacitated Vehicle Routing Problem (RCVRP) in post-disaster aid logistical distribution with time uncertainty in Box Uncertainty Set and Polyhedral Uncertainty Set was discussed. A simulation for all of those 3 models were discussed.

### A. Mathematical Formulation for CVRP in post-disaster aid logistical distribution

Sets that were used in this model are:

$V$ : { $i$ | Locations }
$K$ : { $k$ | Vehicle }

where the distribution center is notated by 1.

Parameters that were used in this model are:

- $t_{ijk}$ : Travelling time from location $i$ to location $j$ with vehicle $k$
- $c_{ijk}$ : Travelling cost from location $i$ to location $j$ with vehicle $k$
- $d_i$ : Amount of supply to location $i$
- $a_k$ : Capacity of vehicle $k$
- $\gamma_{ik}$ : Service time at location $i$ with vehicle $k$
- $T$ : Time allocation to complete the distribution for each vehicle
- $B$ : Budget allocation to complete the distribution
- $n$ : Number of vehicle
- $m$ : Number of locations (with distribution center)

Decision variables that were used in this model are:

- $x_{ijk}$ : Route of vehicle $k$ from location $i$ to location $j$
- $u_{ik}$ : Ratio of supply to location $i$ that fulfilled by vehicle $k$

where $x_{ijk} \in \{0,1\}$ and $u_{ik} \in [0,1]$. The value of $x_{ijk}$ is 1 if vehicle $k$ departs from location $i$ to location $j$ and 0 for else.

Objective function that gives minimum total time needed to complete all the distribution was given by (5).

$$\min\left(\sum_{i\in V}\sum_{j\in V}\sum_{k\in K} t_{ijk}x_{ijk} + \sum_{i\in V}\sum_{k\in K} u_{ik}\gamma_{ik}\right) \tag{5}$$

Constraint that guarantee all the vehicles depart from the distribution center to a particular location was given by (6).

$$\sum_{j\in V} x_{1jk} \geq 1, (\forall k \in K) \quad (6)$$

Constraint that controls the flow of each vehicles was given by (7).

$$\sum_{p\in V} x_{ipk} - \sum_{j\in V} x_{pjk} = 0, (\forall p \in V, k \in K) \quad (7)$$

Constraint that guarantee the whole supply for each location (except the distribution center) will be fulfilled by the available vehicles was given by (8).

$$\sum_{k\in K} u_{ik} = 1, (\forall i \in V - \{1\}) \quad (8)$$

Constraint that guarantee the distribution will not exceed the capacity of each vehicles was given by (9).

$$\sum_{i\in V-\{1\}} d_i u_{ik} \leq a_k, (\forall k \in K) \quad (9)$$

Constraint that guarantee the supply for each location would be fulfilled only if there is a vehicle(s) that arrive to that location was given by (10).

$$u_{ik} \leq \sum_{j\in V} x_{jik}, (\forall i \in V - \{1\}, k \in K) \quad (10)$$

Constraint that guarantee the total time needed by each vehicle to complete the distribution will not exceed the given time allocation was given by (11).

$$\sum_{i\in V}\sum_{j\in V} t_{ijk}x_{ijk} + \sum_{i\in V} u_{ik}\gamma_{ik} \leq T, (\forall k \in K) \quad (11)$$

Constraint that guarantee the total cost needed by all of the vehicles will not exceed the given budget allocation was given by (12).

$$\sum_{i\in V}\sum_{j\in V}\sum_{k\in K} c_{ijk}x_{ijk} \leq B \quad (12)$$

Constraint that guarantee there will be no vehicle that departs from and arrive to the same location was given by (13).

$$x_{iik} = 0, (\forall i \in V, k \in K) \quad (13)$$

Constraint that guarantee there will be no subtour for each vehicle was given by (14).

$$\sum_{j\in S}\sum_{i\in S} x_{ijk} \leq |S| - 1, (S \subset V - \{1\}, 2 \leq |S| \leq n-2, k \in K) \quad (14)$$

Bound that guarantee the ratio of supply to each location by all vehicles are nonnegative was given by (15).

$$u_{ik} \geq 0, (\forall i \in V - \{1\}, k \in K) \quad (15)$$

Bound that guarantee the variable $x_{ijk}$ are binary was given by (16).

$$x_{ijk} \in \{0,1\}, (\forall i, j \in V, k \in K) \quad (16)$$

The overall CVRP model for post-disaster aid logistical distribution was given by $(P1)$

$$\begin{aligned}
&\min\left(\sum_{i\in V}\sum_{j\in V}\sum_{k\in K} t_{ijk}x_{ijk} + \sum_{i\in V}\sum_{k\in K} u_{ik}\gamma_{ik}\right), \\
&s.t. \\
&\sum_{j\in V} x_{1jk} \geq 1, (\forall k \in K), \\
&\sum_{p\in V} x_{ipk} - \sum_{j\in V} x_{pjk} = 0, (\forall p \in V, k \in K), \\
&\sum_{k\in K} u_{ik} = 1, (\forall i \in V - \{1\}), \\
&\sum_{i\in V-\{1\}} d_i u_{ik} \leq a_k, (\forall k \in K), \\
&u_{ik} \leq \sum_{j\in V} x_{jik}, (\forall i \in V - \{1\}, k \in K), \\
&\sum_{i\in V}\sum_{j\in V} t_{ijk}x_{ijk} + \sum_{i\in V} u_{ik}\gamma_{ik} \leq T, (\forall k \in K), \\
&\sum_{i\in V}\sum_{j\in V}\sum_{k\in K} c_{ijk}x_{ijk} \leq B, \\
&x_{kii} = 0, (\forall i \in V, k \in K), \\
&\sum_{j\in S}\sum_{i\in S} x_{ijk} \leq |S| - 1, (S \subset V - \{1\}, 2 \leq |S| \leq n-2, k \in K), \\
&u_{ik} \geq 0, (\forall i \in V - \{1\}, k \in K), \\
&x_{ijk} \in \{0,1\}, (\forall i, j \in V, k \in K).
\end{aligned} \quad (P1)$$

**B.** ***Mathematical Formulation for RC of RCVRP with time uncertainty in Box Uncertainty Set and Polyhedral Uncertainty Set***

Robust Counterpart (RC) of Robust Capacitated Vehicle Routing Problem (RCVRP) with time uncertainty in Box Uncertainty Set was given by $(P2)$.

$$\begin{aligned}
&\min\left(w + \sum_{i\in V}\sum_{k\in K} u_{ik}\gamma_{ik}\right), \\
&s.t. \\
&\sum_{j\in V} x_{1jk} \geq 1, (\forall k \in K), \\
&\sum_{p\in V} x_{ipk} - \sum_{j\in V} x_{pjk} = 0, (\forall p \in V, k \in K), \\
&\sum_{k\in K} u_{ik} = 1, (\forall i \in V - \{1\}), \\
&\sum_{i\in V-\{1\}} d_i u_{ik} \leq a_k, (\forall k \in K), \\
&u_{ik} \leq \sum_{j\in V} x_{jik}, (\forall i \in V - \{1\}, k \in K), \\
&\sum_{i\in V}\sum_{j\in V} \overline{t_{ijk}}x_{ijk} + \sum_{i\in V} u_{ik}\gamma_{ik} + \sum_{p}\sum_{i\in V}\sum_{j\in V}\sum_{k\in K} A_{pijk}x_{ijk} \leq T, (k \in K), \\
&\sum_{i\in V}\sum_{j\in V}\sum_{k\in K} \overline{t_{ijk}}x_{ijk} + \sum_{p}\sum_{i\in V}\sum_{j\in V}\sum_{k\in K} A_{pijk}x_{ijk} \leq w, \\
&\sum_{i\in V}\sum_{j\in V}\sum_{k\in K} c_{ijk}x_{ijk} \leq B, \\
&x_{kii} = 0, (\forall i \in V, k \in K), \\
&\sum_{j\in S}\sum_{i\in S} x_{ijk} \leq |S| - 1, (S \subset V - \{1\}, 2 \leq |S| \leq n-2, k \in K), \\
&u_{ik} \geq 0, (\forall i \in V - \{1\}, k \in K), \\
&x_{ijk} \in \{0,1\}, (\forall i, j \in V, k \in K).
\end{aligned} \quad (P2)$$

Robust Counterpart (RC) of Robust Capacitated Vehicle Routing Problem (RCVRP) with time uncertainty in Polyhedral Uncertainty Set was given by $(P3)$.

$$\begin{aligned}
&\min\left(\sum_{i\in V}\sum_{j\in V}\sum_{k\in K} t_{ijk}x_{ijk} + \sum_{i\in V}\sum_{k\in K} u_{ik}\gamma_{ik}\right), \\
&s.t. \\
&\sum_{j\in V} x_{1jk} \geq 1, (\forall k \in K), \\
&\sum_{p\in V} x_{ipk} - \sum_{j\in V} x_{pjk} = 0, (\forall p \in V, k \in K), \\
&\sum_{k\in K} u_{ik} = 1, (\forall i \in V - \{1\}), \\
&\sum_{i\in V-\{1\}} d_i u_{ik} \leq a_k, (\forall k \in K), \\
&u_{ik} \leq \sum_{j\in V} x_{jik}, (\forall i \in V - \{1\}, k \in K),
\end{aligned} \quad (P3)$$

$$\sum_{i\in V}\sum_{j\in V} t_{ijk}x_{ijk} + \sum_{i\in V} u_{ik}y_{ik} \le T, (\forall k\in K),$$
$$\sum_{i\in V}\sum_{j\in V}\sum_{k\in K} c_{ijk}x_{ijk} \le B,$$
$$x_{kii} = 0, (\forall i\in V, k\in K),$$
$$\sum_{j\in S}\sum_{i\in S} x_{ijk} \le |S| - 1, (S\subset V - \{1\}, 2\le |S| \le n-2, k\in K),$$
$$u_{ik} \ge 0, (\forall i\in V - \{1\}, k\in K),$$
$$x_{ijk} \in \{0,1\}, (\forall i,j\in V, k\in K).$$

### C. Case Study

An earthquake occurred in City A so that a number of troops were prepared to the disaster-affected location for the evacuation process. The whole troops were dispatched from Command Centre, which notated by Location 1. The troops then were dispatched to 9 disaster-affected location, which notated by Location 2 up to Location 10, with a specific number of troops on each location. The number of troops that will be dispatched to each disaster-affected location was given by Table-I. To sent the troops, 3 vehicles with different type were prepared. Vehicle I has a maximum capacity of 40 people, while Vehicle II and Vehicle III have a maximum capacity of 50 and 45 people. Total 1 million monetary budget were allocated for this troops dispatch. Vehicle I has an average fuel cost of 4000 monetary/km, while Vehicle II and Vehicle III have 4800 monetary/km and 5000 monetary/km. The distance between each location was given by Table-II. Based on average fuel cost of each vehicles and distance between each location, we could calculate the cost that will be needed by each vehicle to travel between locations. Vehicle I can reach an average speed of 40 km/h, while Vehicle II and Vehicle III can reach an average speed of 50 km/h and 60 km/h. Based on average speed of each vehicles and distance between each location, we could calculate the travelling time of each vehicle between each location. Also, service time on each location by each vehicle was given by Table-III. Total time 120 minutes were set as the limit to complete this troops dispatch. Furthermore, the travelling time between locations were uncertain due to the earthquake. The simulation for the certain and uncertain time was given later in this paper. The uncertain travelling time were assumed to be in Box Uncertainty Set and Polyhedral Uncertainty Set.

**Table-I. The number of troops that will be sent to each disaster-affected locations**

| No | Disaster-affected Location | Number of Troops (people) |
|---|---|---|
| 1 | Lokasi 2 | 15 |
| 2 | Lokasi 3 | 12 |
| 3 | Lokasi 4 | 10 |
| 4 | Lokasi 5 | 10 |
| 5 | Lokasi 6 | 10 |
| 6 | Lokasi 7 | 12 |
| 7 | Lokasi 8 | 11 |
| 8 | Lokasi 9 | 11 |
| 9 | Lokasi 10 | 14 |

**Table-II. Distance between each location**

| Loc. | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|
| **1** | 0 | 6.5 | 5.9 | 6.8 | 6.9 | 5.1 | 5.8 | 10.2 | 10.7 | 6.7 |
| **2** | - | 0 | 5 | 11.2 | 14.3 | 7.6 | 4.9 | 12.8 | 13.4 | 12 |
| **3** | - | - | 0 | 6.3 | 9 | 10.9 | 9.1 | 16.2 | 16.1 | 12.6 |
| **4** | - | - | - | 0 | 4.3 | 9 | 14.3 | 13.3 | 13.8 | 10 |
| **5** | - | - | - | - | 0 | 7.9 | 11.7 | 12.6 | 12.7 | 8.6 |
| **6** | - | - | - | - | - | 0 | 4.4 | 6 | 5.8 | 4.4 |
| **7** | - | - | - | - | - | - | 0 | 9.7 | 9.6 | 8.9 |
| **8** | - | - | - | - | - | - | - | 0 | 4.5 | 5 |
| **9** | - | - | - | - | - | - | - | - | 0 | 7.9 |
| **10** | - | - | - | - | - | - | - | - | - | 0 |

**Table-III. Service time on each location by each vehicle (minute)**

| Loc. | Vehicle I | Vehicle II | Vehicle III |
|---|---|---|---|
| 2 | 12 | 10 | 11 |
| 3 | 11 | 14 | 15 |
| 4 | 10 | 15 | 14 |
| 5 | 14 | 13 | 14 |
| 6 | 15 | 13 | 10 |
| 7 | 12 | 13 | 10 |
| 8 | 12 | 15 | 10 |
| 9 | 15 | 12 | 14 |
| 10 | 11 | 12 | 12 |

### D. *Numerical Simulation for Certain Time*

Result shown that an optimal route for the troops dispatch was obtained, with an optimal total time needed to complete the dispatch is 72.1 minutes and an optimal costs is 329,420 monetary. The optimal route was given by Fig.1.


**Fig.1 Optimal route for troops dispatch in City A under certain travelling time**

The time needed by Vehicle I to complete the dispatch is 70.1 minutes, with the optimal route:

1. Location 1 (Command Centre)
2. Location 5
3. Location 4
4. Location 3
5. Location 1 (Command Centre)

The time needed by Vehicle II to complete the dispatch is 49.38 minutes, with the optimal route:

1. Location 1 (Command Centre)
2. Location 2
3. Location 7
4. Location 6
5. Location 1 (Command

Centre)

The time needed by Vehicle III to complete the dispatch is 72.1 minutes, with the optimal route:

1. Location 1 (Command Centre)
2. Location 10
3. Location 8
4. Location 9
5. Location 6
6. Location 1 (Command Centre)

**Table-IV. The number of troops dispatched by each vehicle on each location**

| Loc. | Vehicle I (people) | Vehicle 2 (people) | Vehicle 3 (people) | Number of Troops Assigned (people) |
|---|---|---|---|---|
| Location 2 | - | 10 | - | 10 |
| Location 3 | 15 | - | - | 15 |
| Location 4 | 12 | - | - | 12 |
| Location 5 | 10 | - | - | 10 |
| Location 6 | - | 1 | 9 | 10 |
| Location 7 | - | 12 | - | 12 |
| Location 8 | - | - | 11 | 11 |
| Location 9 | - | - | 11 | 11 |
| Location 10 | - | - | 14 | 14 |
| Total Troops Sent by Each Vehicle | 37 | 23 | 45 | |
| Max. Capacity | 40 | 50 | 45 | |

**E. Numerical Simulation for Uncertain Time lied in Box Uncertainty Set**

Results shown that a robust optimal route was obtained, with an optimal total time needed to complete the dispatch is 110.65 minutes and an optimal costs is 336,580 monetary. The optimal route was given by Fig.2.

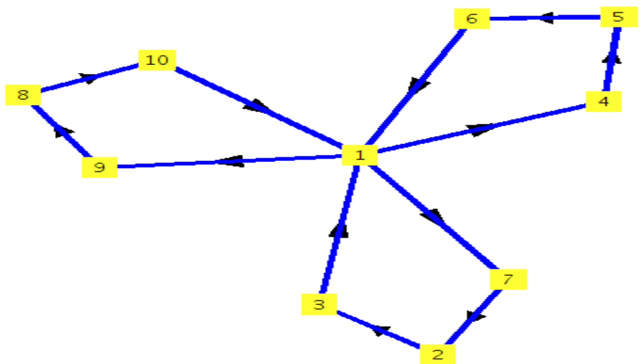


**Fig. 2. Robust optimal route for troops dispatch in City A under travelling time uncertainty lied in Box Uncertainty Set**

The time needed by Vehicle I to complete the dispatch is 108.13 minutes, with the optimal route:

1. Location 1 (Command Centre)
2. Location 7
3. Location 2
4. Location 3
5. Location 1 (Command Centre)

The time needed by Vehicle II to complete the dispatch is 110.65 minutes, with the optimal route:

1. Location 1 (Command Centre)
2. Location 4
3. Location 5
4. Location 6
5. Location 1 (Command Centre)

The time needed by Vehicle III to complete the dispatch is 103.63 minutes, with the optimal route:

1. Location 1 (Command Centre)
2. Location 9
3. Location 8
4. Location 10
5. Location 1 (Command Centre)

**Table-V. The number of troops dispatched by each vehicle on each location**

| Loc. | Vehicle I (people) | Vehicle 2 (people) | Vehicle 3 (people) | Number of Troops Assigned (people) |
|---|---|---|---|---|
| Location 2 | 10 | - | - | 10 |
| Location 3 | 15 | - | - | 15 |
| Location 4 | - | 12 | - | 12 |
| Location 5 | - | 10 | - | 10 |
| Location 6 | - | 10 | - | 10 |
| Location 7 | 12 | - | - | 12 |
| Location 8 | - | - | 11 | 11 |
| Location 9 | - | - | 11 | 11 |
| Location 10 | - | - | 14 | 14 |
| Total Troops Sent by Each Vehicle | 37 | 32 | 36 | |
| Max. Capacity | 40 | 50 | 45 | |

**F. Numerical Simulation for Uncertain Time lied in Polyhedral Uncertainty Set**

Results shown that a robust optimal route was obtained, with an optimal total time needed to complete the dispatch is 110.65 minutes and an optimal costs is 336,580 monetary. The optimal route was given by Fig.3.

The time needed by Vehicle I to complete the dispatch is 108.13 minutes, with the optimal route:

1. Location 1 (Command Centre)
2. Location 7
3. Location 2
4. Location 3
5. Location 1 (Command Centre)

The time needed by Vehicle II to complete the dispatch is 110.65 minutes, with the optimal route:

1. Location 1 (Command Centre)
2. Location 4
3. Location 5
4. Location 6
5. Location 1 (Command Centre)

The time needed by Vehicle III to complete the dispatch is 103.63 minutes, with the optimal route:

6. Location 1 (Command Centre)
7. Location 9
8. Location 8
9. Location 10
10. Location 1 (Command Centre)


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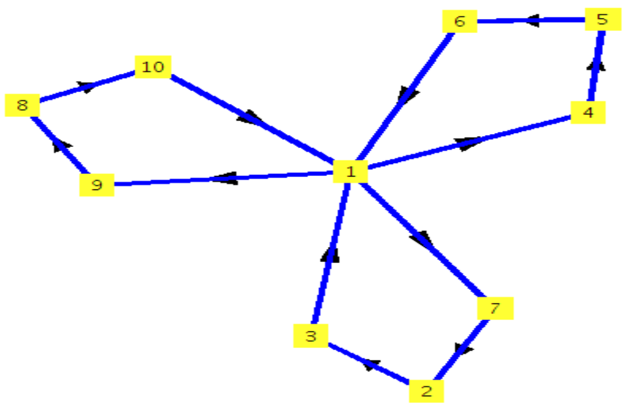


**Fig. 3. Robust optimal route for troops dispatch in City A under travelling time uncertainty lied in Polyhedral Uncertainty Set**

**Table-VI. The number of troops dispatched by each vehicle on each location**

| Loc. | Vehicle I (people) | Vehicle 2 (people) | Vehicle 3 (people) | Number of Troops Assigned (people) |
|---|---|---|---|---|
| Location 2 | 10 | - | - | 10 |
| Location 3 | 15 | - | - | 15 |
| Location 4 | - | 12 | - | 12 |
| Location 5 | - | 10 | - | 10 |
| Location 6 | - | 10 | - | 10 |
| Location 7 | 12 | - | - | 12 |
| Location 8 | - | - | 11 | 11 |
| Location 9 | - | - | 11 | 11 |
| Location 10 | - | - | 14 | 14 |
| Total Troops Sent by Each Vehicle | 37 | 32 | 36 | |
| Max. Capacity | 40 | 50 | 45 | |

The optimal solutions for each case (e.g. certain travelling time, uncertain travelling time lied in Box Uncertainty Set, uncertain travelling time lied in Polyhedral Uncertainty Set) was summarized in Table-VII.

**Table-VII. Summarize of the optimal solution (minutes)**

| | Certain Time | Uncertain Time (Box Uncertainty Set) | Uncertain Time (Polyhedral Uncertainty Set) |
|---|---|---|---|
| Minimum Total Time | 72.1 | 110.65 | 110.65 |

Although the optimal time in case of uncertain time lied in Box and Polyhedral Uncertainty Set was higher than the certain time, but it was robust against the uncertain travelling time caused by the natural disaster occured.

## V. CONCLUSIONS

Based on the discussion, we can conclude that Robust Capacitated Vehicle Routing Problem (RCVRP) for post-disaster aid logistical distribution under uncertain travelling time in both Box Uncertainty Set and Polyhedral Uncertainty Set yields a Robust Counterpart (RC) which robust against the uncertain travelling time. Numerical simulation of Robust Counterpart (RC) of Capacitated Vehicle Routing Problem (RCVRP) for post-disaster aid logistical distribution under uncertain travelling time in both Box Uncertainty Set and Polyhedral Uncertainty Set are computationally tractable. It means that the model could be solved computationally within polynomial time. Numerical simulation also shows that RC of RCVRP for post-disaster aid logistical distribution gives a robust optimal solution of the distribution route with minimum time.

## ACKNOWLEDGMENT

This research is part of the Basic Research Grant to the second author, funded by Indonesian Ministry of Research Technology and Higher Education for the year of 2019, under contract number 2923/UND.6/LT/2019

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