

# Finite Element Method for Radiation Profile of Heat Transfer Flow in a Vertical Channel Mixed Convective Porous Medium.



P. V. Janardhana Reddy, G. Srinivas Reddy, R Siva Prasad

**Abstract:** A mathematical attempt with FEM technique simulated in the paper is to demonstrate the profile of the free convection flow of an incompressible viscous fluid through a vertical channel of a porous matrix which is bounded by parallel walls of impermeable nature with various physical factors. The flow is considered to take place along the direction in the axis of the channel. The surface of the walls is maintained at uniform temperature. Brinkman model is implemented in framing of conservation equations of momentum for flow model in porous medium. The modifications of equations have been made with the considerations of dissipations of Darcy and viscous parameters to describe heat flow. With the simulations of FEM techniques, local thermal equilibrium conditions are at porous matrix and porous medium and flow occurs in the directions of buoyancy. The analysis and simulations of non-linear equations that governs mass transfer and heat flow.

The evaluations at second order of the Sherwood number, concentration velocity, temperature and Nusselt number and their behavior is exhibited with variations of different parameters modeled in the proposed attempt.

**Keywords :** viscous fluid, Nusselt number, Sherwood Number, Galerkin finite element method.

## I. INTRODUCTION

A wide range of scientific applications involve mixed convection cooling such as electrical heat engines and other electronic appliances. The heat transfer that is from surfaces of heat generating devices is in relation to cooling system has been now most popular area of interest in the domain IC technology in electronics for various applications.

As microminiaturization of design of circuits in electronic equipment there is an enhancement of the densification of circuit fabrications that contains maximum number of electronic components in digital systems. The research in cooling mechanism of electronic equipment has

been an emerging field of study in fluid problems and the situations of heat transfer. Because of miniaturization, density of fabricated components increases causing the effects of heat which is going to be analysed per unit volume of electronic equipment. As air is convenient to handle and comfort in the process of cooling systems. A techniques of air cooling by forced convection methods is implemented at the condition of heat flux which exceeds 1000 W/m<sup>2</sup>. The impact of buoyancy cannot be neglected even with forced convective flow in the situations where heat flux generated by electronic chip containing more density of components. Then in most of such situations flow is come across in convection regime. A precise prediction and analysis of a problem with mixed convection fluid flow, conjugateness the radiation coming from the surfaces must seriously require a the model of mathematical framing with consideration of fluid characteristics of heat flow in cooling systems. The results in the earlier studies with air cooling systems containing sources of three protruding and involve forced convection have been clearly revealed [1]. The substrates of the adiabatic walls are investigated about the impact of conduction. The analysis of the parameters like heat transfer as well as Reynolds number has been made and corresponding results are also shown with respect to Prandtl number. With consideration of uniform inlet profile, a numerical study on conjugate of forced convection of heat transfer which is from protruding heat sources is attempted [2]. Iterations in a large number with the suitable surface boundary conditions outer substrate are periodically performed for each components of circuit boards in transverse arrangements. The earlier studies are also with Protruding heat sources that are mounted on the bottom wall for characteristics of fluid flow and heat transfer [4,5]. The simulations of radiations of surface which is in conjugate mixed convection with two heat sources that are mounted in line with the vertical plate [6,7]. A detailed simulation study on conjugate of natural convection has been executed with certain factors of a surface radiation emanating from a slot in which both the vertical walls are assumed to be in conducting nature and the bottom wall to be isothermal [8]. Different parameters of aspect ratios, Rayleigh numbers, emissivity and other parameters of thermal conductivities', have been taken into account [9]. A correlation has been assigned to the average of Nusselt with the consideration of the pertinent parameters.

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An investigation of numerical simulation has been made on the combined impact of conduction as well as natural convection associated with surface radiation of spherical cavity of discrete uniform flux source which is mounted on the left side wall [10]. Flow visualization patterns have been also made. A numerical study was carried out for a group of geometric parameters considered in the simulation.

**II. FORMULATION OF PROBLEM OF THE POROUS FLUID**

The simulation study of viscous fluid that has been considered under the flow of free convection in the channel of porous medium which is bounded between walls of impermeable nature.

With conditions of uniform temperatures of walls of surface, the flow has been modeled in the problem to be on the axis. The driving equations based on Brinkman model have been framed for the flow in which some modifications in equations governing heat flow are made with considerations of viscous and Darcy dissipation of fluid at local thermal equilibrium

A frame of reference 0 (x, y, z) indicating the x-axis which is in the direction of upward and vertical directions that are in the opposite directions of buoyancy and the vertical walls are given by the planes  $y = \pm b$  that taken in parallel to (y, z). And (u, 0, 0) is assumed to be the velocity field of flow which is unidirectional nature. The equations that govern and heat transfer as well as the flow are as follows

$$\frac{\partial \mathbf{u}}{\partial \mathbf{x}} = \mathbf{0} \tag{2.1}$$

$$\mu \left( \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} - \frac{u}{k} \right) + g\beta(T - T_0) + g\beta^*(C - C_0) = -\nu_0 \frac{\partial u}{\partial y}$$

$$\frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} + \frac{\rho_0 \nu}{k_1} \left[ \left( \frac{\partial u}{\partial y} \right)^2 + \left( \frac{\partial u}{\partial z} \right)^2 \right] + \frac{\rho_0 \nu}{kk_1} u^2 - \frac{\partial}{\partial y} (r q_{cr}) = -\rho_0 c_p \frac{\nu_0}{k_1} \frac{\partial T}{\partial y}$$

(2.2)

$$D_1 \left( \frac{\partial^2 C}{\partial y^2} + \frac{\partial^2 C}{\partial z^2} \right) + K_{11} \left( \frac{\partial^2 \theta}{\partial y^2} + \frac{\partial^2 \theta}{\partial z^2} \right) = 0$$

(2.3)

$$\rho = \rho_0 (1 - \beta(T - T_0) - \beta^*(C - C_0))$$

(2.4)

where  $\rho_0$  indicates the density at the  $T_0$  is ambient temperature and concentration  $C_0$  and  $\beta$ ,  $k$ ,  $\nu$  are taken to be the coefficients of viscosity, thermal conductivity and fluid thermal expansion respectively,  $\beta^*$  considered to be with mass fraction concentration over the volumetric coefficient of expansion,  $k$  is the parameter of permeability considered for the porous medium and  $C_p$  at a condition of constant pressure, will be the specific heat, the parameter of molecular diffusivity and  $k_{11}$  is assumed in the model to be the cross diffusivity

in continuation of medium and let  $u = u(y, z)$

The boundary conditions have been taken as

$$u = 0 \text{ on } z = \pm b$$

$$T = \pm T_b, \quad C = \pm C_1$$

(2.5)

$$\frac{\partial u}{\partial z} = 0, \quad \frac{\partial T}{\partial z} = 0 \quad \text{and} \quad \frac{\partial C}{\partial z} = 0 \quad \text{on } Z = 0$$

in view of the symmetry.

non-dimensional variables that are as follows have been chosen

$$z^* = \frac{z}{b}; \quad y^* = \frac{y}{b}; \quad \theta^* = \frac{T - T_0}{T_1 - T_0},$$

$$C^* = \frac{C - C_0}{C_1 - C_0},$$

$$u^* = \frac{\nu u}{\beta g b^2 (T_1 - T_0)}$$

With the process of Boussinesq approximations, certain parameters in the equations considered in dimensionless form (the asterisk which are on dropped) have been as follows

$$\frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} - D^{-1} u + (\theta + NC) = -S \frac{\partial u}{\partial y} \tag{2.7}$$

$$\frac{\partial^2 \theta}{\partial y^2} + N_2 \frac{\partial^2 \theta}{\partial z^2} + Gr_1 Ec \left[ \left( \frac{\partial u}{\partial y} \right)^2 + \left( \frac{\partial u}{\partial z} \right)^2 \right] + D^{-1} u^2 = -Pr S \frac{\partial \theta}{\partial y} \tag{2.8}$$

$$\left( \frac{\partial^2 C}{\partial y^2} + \frac{\partial^2 C}{\partial z^2} \right) + \frac{S_0 Sc}{N} \left( \frac{\partial^2 \theta}{\partial y^2} + \frac{\partial^2 \theta}{\partial z^2} \right) = 0 \tag{2.9}$$

Where

$$D = \frac{k}{b^2} \quad \text{is the Darcy parameter}$$

$$P = \frac{\mu C_p}{k_1} \quad \text{is the Prandtl number}$$

$$Ec = \frac{\beta g b}{C_p} \quad \text{is the Eckert number}$$

$$G = \frac{\beta g b^3 (T_1 - T_0)}{\nu^2} \quad \text{is the Grashof number}$$

$$S = \frac{\nu_0 b}{\nu} \quad \text{is the suction Reynolds number}$$

$$N = \frac{\beta^* (C_1 - C_0)}{\beta (T_1 - T_0)} \quad \text{is the buoyancy ratio}$$

$$S_c = \frac{\nu}{D_1} \quad \text{is the Schmidt number}$$

$$S_o = \frac{\beta^* K_{11}}{\beta \nu} \quad \text{is the Soret number}$$

$$N_1 = \frac{\infty \beta_R K_1}{4 \sigma T_e^3} \quad \text{is the}$$

radiation parameter



$$N_2 = \frac{3N_1}{3N_1 + 4} \quad P_1 = PN_2 \quad \alpha_1 = \alpha N_2$$

The corresponding boundary conditions in the non-dimensional form are

$$u = 0, \theta = 1, C = 1 \text{ on } z = \pm 1$$

$$\frac{\partial u}{\partial z} = 0, \quad \frac{\partial \theta}{\partial z} = 0 \quad \text{and} \quad \frac{\partial C}{\partial z} = 0 \quad \text{On } Z = 0 \quad (2.10)$$

In a symmetric view of profile of the flow with respect to the mid plane of the channel, an investigation is made on the flow in one half of the domain bounded by the impermeable wall to the right and the mid plane. The execution of finite element analysis with consideration of quadratic approximation functions has been carried out in the normal cross sectional plane (y - z) bounded by planes z = 0 and 1 with the technique of eight noded rectangular serendipity element.

### III. PROPOSED METHODOLOGY OF FINITE ELEMENT

Assuming  $u^i$  and  $\theta^i$  be the approximations of u and  $\theta$ , the errors (residual)  $E_1^i$  and  $E_2^i$  defined as follows

$$E_1^i = \frac{\partial^2 u^i}{\partial y^2} + \frac{\partial^2 u^i}{\partial z^2} - D^{-1}u^i + (\theta^i + NC^i) + S \frac{\partial u^i}{\partial y} \quad (3.1)$$

$$E_2^i = \frac{\partial^2 \theta^i}{\partial y^2} + N_2 \frac{\partial^2 \theta^i}{\partial z^2} + GP_1 Ec [(\frac{\partial u^i}{\partial y})^2 + (\frac{\partial u^i}{\partial z})^2] + D^{-1}(u^i)^2 + P_1 S \frac{\partial \theta^i}{\partial y} \quad (3.2)$$

$$E_3^i = (\frac{\partial^2 C^i}{\partial y^2} + \frac{\partial^2 C^i}{\partial z^2}) + \frac{S_o S_c}{N} (\frac{\partial^2 \theta^i}{\partial y^2} + \frac{\partial^2 \theta^i}{\partial z^2}) \quad (3.3)$$

where

$$u^i = \sum_{k=1}^8 u_k^i N_k^i \quad (3.4)$$

$$\theta^i = \sum_{k=1}^8 \theta_k^i N_k^i \quad (3.5)$$

$$C^i = \sum_{k=1}^8 C_k^i N_k^i$$

These errors are of orthogonal nature to the weights of  $e^i$ . under Galerkin, the approximation functions are chosen as the weight functions of approximation. Both sides of the equations (3.1) to (3.3) have been multiplied by the weight

function of  $N_j^i$  and integration is made over the surface  $\Omega_i$ , then the following is obtained

$$(3.6)$$

$$\int_{\Omega_i} E_1^i N_j^i d\Omega_i = 0 \quad (j = 1, 2, \dots, 8) \quad (3.7)$$

$$\int_{\Omega_i} E_2^i N_j^i d\Omega_i = 0 \quad (j = 1, 2, \dots, 8) \quad (3.8)$$

$$\int_{\Omega_i} E_3^i N_j^i d\Omega_i = 0 \quad (j = 1, 2, \dots, 8) \quad (3.9)$$

$$\int_{\Omega_i} [\frac{\partial^2 u^i}{\partial y^2} + \frac{\partial^2 u^i}{\partial z^2} - D^{-1}u^i + (\theta^i + NC^i) + S \frac{\partial u^i}{\partial y}] N_j^i d\Omega_i = 0 \quad (3.10)$$

$$\int_{\Omega_i} [\frac{\partial^2 \theta^i}{\partial y^2} + N_2 \frac{\partial^2 \theta^i}{\partial z^2} + GP_1 Ec [(\frac{\partial u^i}{\partial y})^2 + (\frac{\partial u^i}{\partial z})^2] + D^{-1}(u^i)^2] + P_1 S \frac{\partial \theta^i}{\partial y} N_j^i d\Omega_i = 0 \quad (3.11)$$

$$\int_{\Omega_i} [(\frac{\partial^2 C^i}{\partial y^2} + \frac{\partial^2 C^i}{\partial z^2}) + \frac{S_o S_c}{N} (\frac{\partial^2 \theta^i}{\partial y^2} + \frac{\partial^2 \theta^i}{\partial z^2})] N_j^i d\Omega_i = 0 \quad (3.12)$$

$$\int_{\Omega_i} [\frac{\partial N_j^i}{\partial y} \frac{\partial u^i}{\partial y} + \frac{\partial N_j^i}{\partial z} \frac{\partial u^i}{\partial z} - D^{-1}u^i N_j^i + (\theta^i + NC^i) N_j^i + SN_j^i \frac{\partial u^i}{\partial y}] d\Omega_i$$

$$= \int_{\Gamma} [N_j^i \frac{\partial u^i}{\partial y} n_y + N_j^i \frac{\partial u^i}{\partial z} n_z] d\Gamma_i \quad (3.13)$$

$$\int_{\Omega_i} [\frac{\partial N_j^i}{\partial y} \frac{\partial \theta^i}{\partial y} + N_2 \frac{\partial N_j^i}{\partial z} \frac{\partial \theta^i}{\partial z} + GP_1 Ec N_j^i [(\frac{\partial u^i}{\partial y})^2 + (\frac{\partial u^i}{\partial z})^2 + D^{-1}(u^i)^2] + P_1 SN_j^i \frac{\partial \theta^i}{\partial y}] d\Omega_i$$

$$= \int_{\Gamma} [N_j^i \frac{\partial \theta^i}{\partial y} n_y + N_j^i \frac{\partial \theta^i}{\partial z} n_z] d\Gamma_i \quad (3.14)$$

$$\int_{\Omega_i} [N_j^i \frac{\partial N_j^i}{\partial y} (N \frac{\partial C^i}{\partial y} + S_c S_o \frac{\partial \theta^i}{\partial y}) + \frac{\partial N_j^i}{\partial z} (N \frac{\partial C^i}{\partial z})] d\Omega_i$$

$$= \int_{\Gamma} [N_j^i (N \frac{\partial C^i}{\partial y} + S_c S_o \frac{\partial \theta^i}{\partial y}) n_y + N_j^i (N \frac{\partial C^i}{\partial z} + S_c S_o \frac{\partial \theta^i}{\partial z}) n_z] d\Gamma_i \quad (3.15)$$

where  $\Omega_i$  is the serendipity element bounded by  $\Gamma_i$ ,  $n_y$ ,  $n_z$  are the direction cosines normal to  $\Gamma_i$ . Substituting (3.4), (3.5) & (3.6) in L.H.S of (3.13), (3.14) & (3.15) we get



$$\int_{\Omega_i} \sum_{k=1}^8 u_k^i \left[ \frac{\partial N_j^i}{\partial y} \frac{\partial N_k^i}{\partial y} + \frac{\partial N_j^i}{\partial z} \frac{\partial N_k^i}{\partial z} - D^{-1} N_k^i N_j^i + S N_j^i \frac{\partial N_k^i}{\partial y} \right] d\Omega_i + \int_{\Omega_i} \sum_{k=1}^8 (\theta_k^i + N C_k^i) N_j^i N_k^i d\Omega_i = Q_j^i \quad (3.16)$$

$$\int_{\Omega_i} \sum_{k=1}^8 \theta_k^i \left[ \frac{\partial N_j^i}{\partial y} \frac{\partial N_k^i}{\partial y} + N_2 \frac{\partial N_j^i}{\partial z} \frac{\partial N_k^i}{\partial z} + P_1 S N_j^i \frac{\partial N_k^i}{\partial y} \right] d\Omega_i + \int_{\Omega_i} \sum_{k=1}^8 u_k^i G P_1 E c \left[ \left( \frac{\partial N_k^i}{\partial y} \right)^2 + \left( \frac{\partial N_k^i}{\partial z} \right)^2 + D^{-1} (N_k^i)^2 \right] d\Omega_i = (Q^T)_j^i \quad (3.17)$$

$$\int_{\Omega_i} \sum_{k=1}^8 C_k^i \left\{ \frac{\partial N_k^i}{\partial y} \frac{\partial N_j^i}{\partial y} + \frac{\partial N_k^i}{\partial z} \frac{\partial N_j^i}{\partial z} \right\} d\Omega_i - \int_{\Omega_i} N_2 S c \sum_{k=1}^8 u_k^i (N_j^i N_k^i) d\Omega_i - S c S_o \int_{\Omega_i} \sum_{k=1}^8 \theta_k^i \left\{ \frac{\partial N_k^i}{\partial y} \frac{\partial N_j^i}{\partial y} + \frac{\partial N_k^i}{\partial z} \frac{\partial N_j^i}{\partial z} \right\} d\Omega_i = (Q^C)_j^i \quad (3.18) \text{ where}$$

$$Q_j^i = \oint_{\Gamma_i} (N_j^i) \left( \frac{\partial u^i}{\partial y} n_y + N_j^i \frac{\partial u^i}{\partial z} n_z \right) d\Gamma_i$$

$$(Q^T)_j^i = \oint_{\Gamma_i} (N_j^i) \left( \frac{\partial T^i}{\partial y} n_y + N_j^i \frac{\partial T^i}{\partial z} n_z \right) d\Gamma_i$$

$(j = 1, 2, \dots, 8)$

$$(Q^C)_j^i = \oint_{\Gamma_i} \left[ N_j^i \left\{ N \frac{\partial C^i}{\partial y} + S_o S c \frac{\partial \theta^i}{\partial y} \right\} n_y + N_j^i \left\{ N \frac{\partial C^i}{\partial z} + S_o S c \frac{\partial \theta^i}{\partial z} \right\} n_z \right] d\Gamma_i \quad j=1,2,\dots,8.$$

Choosing different  $N_k^i$ 's corresponding to each element  $e_i$  (3.11) leads to 16 equations in 2 sets having

unknown  $(u_k^i)$  and  $(\theta_k^i)$  viz

$$(a_{kj}^i)(u_k^i) = Q_j^i \quad (3.19)$$

$$(b_{kj}^i)(\theta_k^i) + (c_{kj}^i)u_k^i = (Q^T)_j^i \quad (j = 1, 2, \dots, 8) \quad (3.20)$$

$$(m_{kj}^i)(C_k^i) + (l_{kj}^i)(u_k^i) = (n_{kj}^i)(\theta_k^i) + (Q^C)_j^i \quad (j, k=1,2,\dots,8) \quad (3.21)$$

where

$$(a_{kj}^i), (b_{kj}^i), (c_{kj}^i), (m_{kj}^i), (n_{kj}^i) \text{ and } (l_{kj}^i)$$

are  $8 \times 8$  stiffness matrices and  $Q_j^i, (Q^T)_j^i$  and  $(Q^C)_j^i$  are  $8 \times 1$  column matrices. At each iteration, each element in the simulation  $mn$  with boundary conditions will assemble for global matrices with the unknowns  $u, \theta$  and  $C$  at respective nodes causing the determination through matrix equations.

In order to simulate, a serendipity element at points or nodes (0,0), (0,1) (1,0) and (1,1) at vertices. The eight nodes in the formulation are clearly given in Fig.(a) and the corresponding functions of quadratic interpolation at these nodes are as follows

$$N_1 = -2(y-1)(z-1)(z+y-\frac{1}{2}) \quad ; \quad N_2 = -4z(z-1)(y-1)$$

$$N_3 = -2z(y-1)(z-y+\frac{1}{2}) \quad ; \quad N_4 = -4yz(y-1)$$

$$N_5 = 2yz(z+y-\frac{3}{2}) \quad ; \quad N_6 = -4yz(z-1)$$

$$N_7 = 2y(z-1)(z-y+\frac{1}{2}) \quad ; \quad N_8 = -4y(z-1)(y-1)$$

shape functions need to be placed in the equations of (3.19) and the process of integration is performed over the matrix for the global nodes  $u_i$  ( $i=1,2,\dots$ ) which reduce to the order of a  $8 \times 8$  matrix equations.

$8 \times 8$  matrix equations will be divided (partitioned) in the following form

$$\begin{bmatrix} A^{11} & A^{12} \\ A^{21} & A^{22} \end{bmatrix} \begin{bmatrix} \Delta_U^1 \\ \Delta_U^2 \end{bmatrix} = \begin{bmatrix} F_U^1 \\ F_U^2 \end{bmatrix} \quad (3.22)$$

Where  $\Delta_U^1, \Delta_U^2, F_U^1, F_U^2$  are column matrices given by

$$\Delta_U^1 = \begin{bmatrix} U_1 \\ U_2 \\ U_3 \\ U_4 \end{bmatrix}, \quad \Delta_U^2 = \begin{bmatrix} U_5 \\ U_6 \\ U_7 \\ U_8 \end{bmatrix}$$

Equation (3.20) results in the following two sets of equations in the form of matrices in partitioned form

$$[S^{11}] [\Delta_U^1] + [S^{12}] [\Delta_U^2] = [F_U^1]$$

$$[S^{21}] [\Delta_U^1] + [S^{22}] [\Delta_U^2] = [F_U^2] \quad (3.23)$$

Similarly the  $8 \times 8$  matrix equations for  $\theta_j, C_j$  ( $j=1,2,\dots,8$ ) in the partitioned form are

$$\begin{bmatrix} B^{11} & B^{12} \\ B^{21} & B^{22} \end{bmatrix} \begin{bmatrix} \Delta_\theta^1 \\ \Delta_\theta^2 \end{bmatrix} = \begin{bmatrix} F_\theta^1 \\ F_\theta^2 \end{bmatrix} \quad (3.24)$$

$$\begin{bmatrix} L^{11} & L^{12} \\ L^{21} & L^{22} \end{bmatrix} \begin{bmatrix} \Delta_C^1 \\ \Delta_C^2 \end{bmatrix} = \begin{bmatrix} F_C^1 \\ F_C^2 \end{bmatrix} \quad (3.25)$$

Where  $\Delta_\theta^1, \Delta_\theta^2, F_\theta^1, F_\theta^2, \Delta_C^1, \Delta_C^2, F_C^1, F_C^2$  are column matrices which follow as given below

$$\Delta_{\theta}^1 = \begin{bmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \\ \theta_4 \end{bmatrix} ; \quad \Delta_{\theta}^2 = \begin{bmatrix} \theta_5 \\ \theta_6 \\ \theta_7 \\ \theta_8 \end{bmatrix} ;$$

$$\Delta_C^1 = \begin{bmatrix} C_1 \\ C_2 \\ C_3 \\ C_4 \end{bmatrix} ; \quad \Delta_C^2 = \begin{bmatrix} C_5 \\ C_6 \\ C_7 \\ C_8 \end{bmatrix}$$

The boundary conditions that essentially for the primary variables

$$u_3 = u_4 = u_5 = 0;$$

$$\theta_3 = \theta_4 = \theta_5 = 1 \text{ and}$$

$$C_3 = C_4 = C_5 = 1 \text{ on } y = 1$$

(3.26)

the symmetry conditions for convenience, the followings are obtained

$$Q_1 = Q_2 = Q_6 = Q_7 = Q_8 = 0$$

$$Q_1^T = Q_2^T = Q_6^T = Q_7^T = Q_8^T = 0$$

(3.27)

$$Q_1^C = Q_2^C = Q_3^C = Q_4^C = Q_8^C = 0$$

Solving the ultimate  $8 \times 8$  matrices, and the unknown global nodal values of  $u_i, \theta_i$

( $i = 1, 2, \dots, 8$ ), are obtained.

The solution for  $u, \theta$  may now be represented as

$$u = \sum_{k=1}^8 u_k N_k, \quad \theta = \sum_{j=1}^8 \theta_j N_j \text{ and } C = \sum_{i=1}^8 C_i N_i$$

#### IV. RESULTS ANALYSIS OF NUMERICAL SIMULATION

The velocity profiles along the planes  $y=0$  &  $1$  that are normal and planes  $Z=0$  &  $Z=1/2$  that are parallel are respectively demonstrated by Figures 1 & 2 and Figures 3 & 4. It is noticed from the figures that for variations in  $D^{-1}$ , flow takes place in the normal directions of  $y=0$  to  $1$  with movement of flow in upward directions as indicated by velocity variations on  $Z=0$  &  $Z=1/2$ .

It is observed in profile that at  $y=0.4$ , a maximum is attained whereas at  $Z=1/2$  level level is across  $y=1$ . The mathematical observations of flow pattern will be different in patterns along normal and parallel directions of fluid flow in medium. These patterns are clearly investigated with the technique of the finite element method. From mid to boundary, velocity is observed to decrease in its pattern with the enhanced pattern of  $D^{-1}$ . And therefore minimum level or order of permeability of porous fluid medium causes the low velocity phenomenon as indicated clearly with profile  $u$  with  $D^{-1}$  along the directions of  $y=0$  &  $1$  which are normal planes shown in figure 1 and figure 2.

This clearly conveys that the minimum is the permeability of the porous medium the lesser is the amount of velocity. The values of velocity at the point or location  $y=0$  have been found to be higher than those that are at  $y=1$  level. Figures 5, 6, 9 and 10 depict the the profile of non-dimensional temperature variations at the horizontal levels of  $y=0$  &  $1/2$  with considerations of  $\alpha$ , and  $N_1$ .

The impact of source heat on the parameter  $\theta$  is depicted as shown in figures 5 and 6. It is found that depreciation in  $\theta$  at a marginal level with the enhancement in the parameter  $\alpha$  of heat source. The profile of parameter  $\theta$  with respect to radiation parameter  $N_1$  reveals that an enhanced  $N_1$  causes an increase in the actual parameter of temperature at planes level of  $y=0$  &  $1$ . The variation profile in the parameter of  $\theta$  with respect to both  $\alpha$  and  $N_1$  at the both the levels  $Z=0$  &  $1/2$  in vertical direction is depicted in figures 7, 8, 11 and 12. It is noticed by both the figures 7 and 8 that depreciation is caused by the actual temperature with the enhanced heat source parameter  $\alpha$ . It reveals that the heat generating source which is present in the fluid region causes the reduction in the actual temperature. The profile of parameter of  $\theta$  with respect to radiation parameter  $N_1$  reveals the enhancement in tendency of  $N$  with radiative heat flux in both the figures 11 and 12. In a common practicality, it is observed that the actual levels of parameter of temperature at the horizontal level is found to be minimum than that at the vertical levels.

#### V. CONCLUSIONS

The simulation which includes modifications of equations have been made with the considerations of dissipations of Darcy and viscous parameters was successfully executed to deal with and describe heat flow. Assumptions that local thermal equilibrium conditions are at porous matrix and porous medium and flow occurs in the directions of buoyancy have been perfectly fitted to proposed FEM technique implemented in the present work. It is interested to note that evaluations at second order of the Sherwood number, concentration velocity, temperature and Nusselt number and their behavior have been demonstrated with variations of different parameters modeled in the proposed attempt.

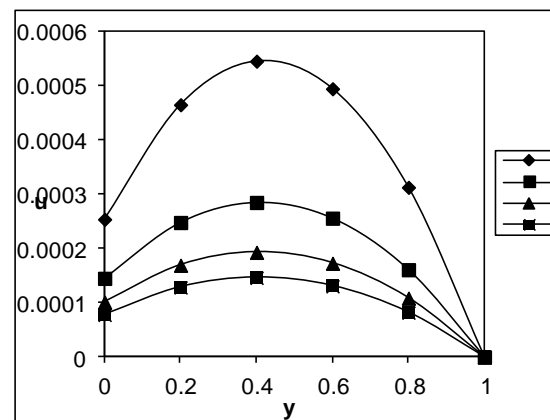


Fig. 1: A change profile of  $u$  with  $D^{-1}$  at the level of the plane

$y=0$   $M=5$ ;  $G=200$ ;  $N_1=0.5$ ;  $S=0.8$ ;  $k=0.5$ ;  $P=0.71$ ;  $\beta=2$ ;  
 $D^{-1}=2000$ ;  $z=0.5$  I II III IV  
 $D^{-1}$   $2 \times 10^3$   $4 \times 10^3$   $6 \times 10^3$   $8 \times 10^3$

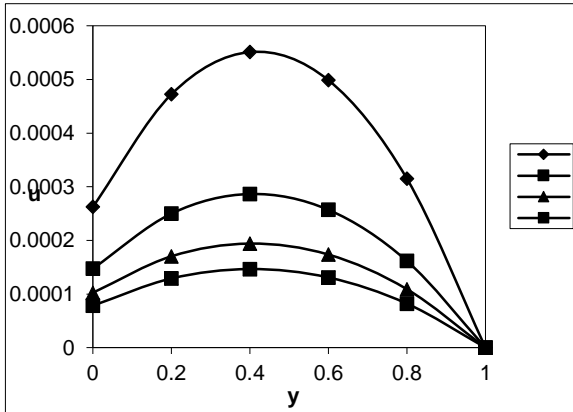


Fig. 2:

A change profile of  $u$  with  $D^{-1}$  at the level of the plane  $y=0.5$   
 $M=5$ ;  $G=200$ ;  $N_1=0.5$ ;  $S=0.8$ ;  $k=0.5$ ;  $P=0.71$ ;  $D^{-1}=2000$ ;  $z=0.5$   
I II III IV  
 $D^{-1}$   $2 \times 10^3$   $4 \times 10^3$   $6 \times 10^3$   $8 \times 10^3$

$M=5$ ;  $G=200$ ;  $cn=0.5$ ;  $S=0.8$ ;  $k=0.5$ ;  $P=0.71$ ;  $D^{-1}=2000$ ;  $z=0.5$   
I II III IV  
 $D^{-1}$   $2 \times 10^3$   $4 \times 10^3$   $6 \times 10^3$   $8 \times 10^3$

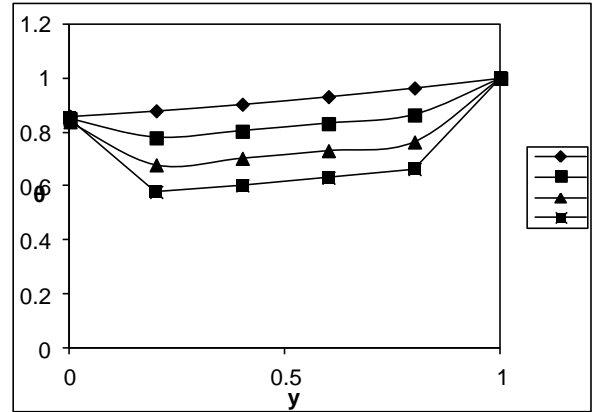


Fig. 5:

Profile or change in  $\theta$  with  $\alpha$  at the plane  $y=0$   
 $M=5$ ;  $G=200$ ;  $N_1=0.5$ ;  $S=0.8$ ;  $k=0.5$ ;  $P=0.71$ ;  $\beta=2$ ;  $D^{-1}=2000$ ;  
 $z=0.5$   
I II III IV

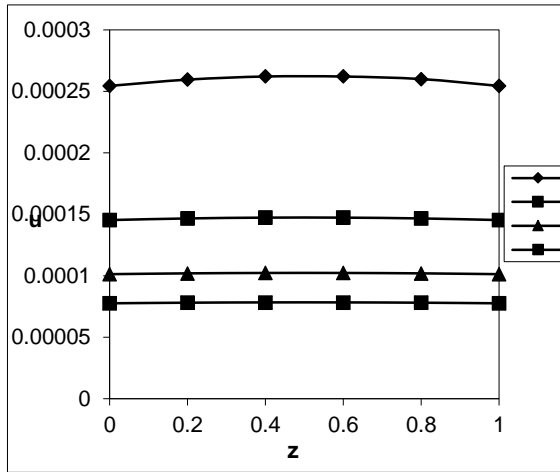


Fig. 3:

A change profile of  $u$  with  $D^{-1}$  at the level of the plane  $Z=0$   
 $M=5$ ;  $G=200$ ;  $N_1=0.5$ ;  $S=0.8$ ;  $k=0.5$ ;  $P=0.71$ ;  $\beta=2$ ;  $D^{-1}=2000$ ;  
 $z=0.5$   
I II III IV  
 $D^{-1}$   $2 \times 10^3$   $4 \times 10^3$   $6 \times 10^3$   $8 \times 10^3$

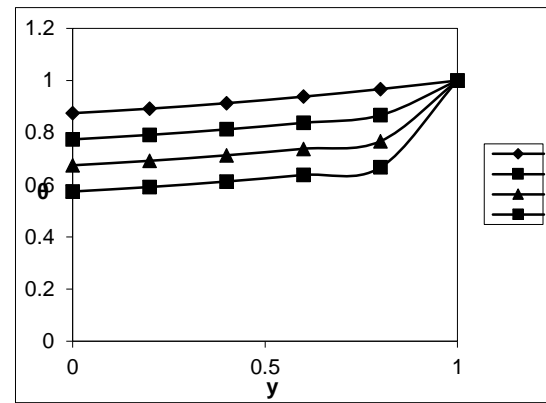


Fig. 6:

A change in (Profile) of  $\theta$  with  $\alpha$  at the plane  $y=0.5$   
 $M=5$ ;  $G=200$ ;  $N_1=0.5$ ;  $S=0.8$ ;  $k=0.5$ ;  $P=0.71$ ;  $D^{-1}=2000$ ;  $z=0.5$   
I II III IV  
 $\alpha$  0 2 4 10

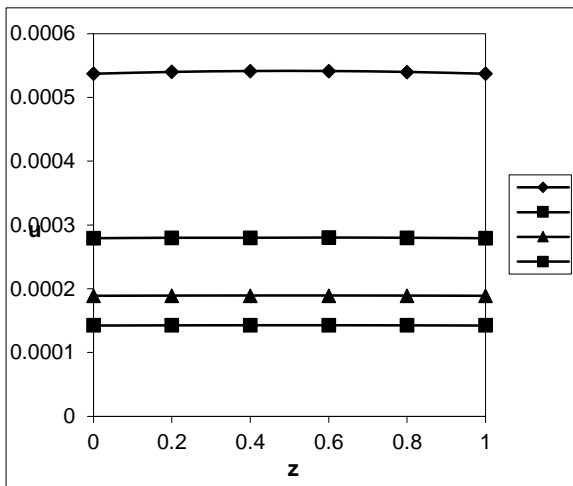


Fig. 4:

A change profile of  $u$  with  $D^{-1}$  at the level of the plane  $z=0.5$

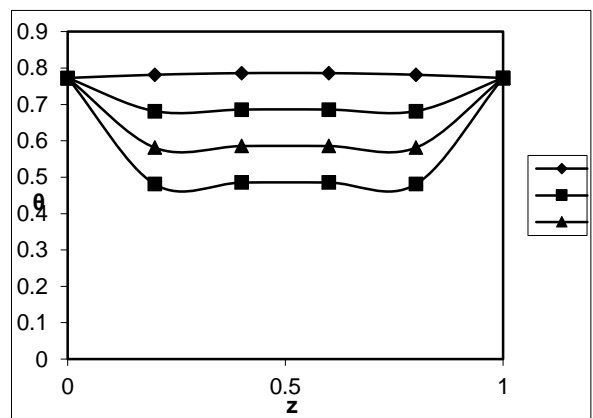
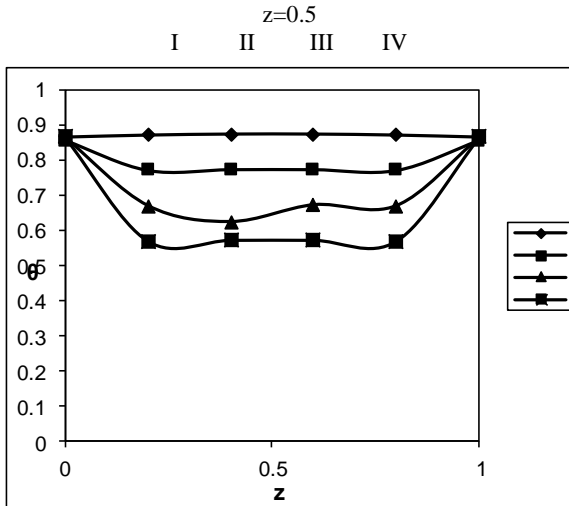


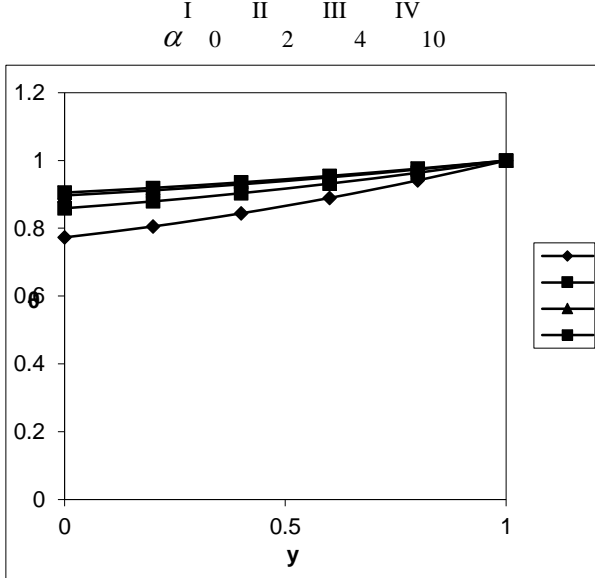
Fig. 7:

**A profile of change in  $\theta$  with  $\alpha$  at the plane  $z=0$**   
 $M=5; G=200; N_1=0.5; S=0.8; k=0.5; P=0.71; \beta=2; D^{-1}=2000;$



**Fig. 8:**

**A profile of change in  $\theta$  with  $\alpha$  at the plane  $z=0.5$**   
 $M=5; G=200; N_1=0.5; S=0.8; k=0.5; P=0.71; D^{-1}=2000; z=0.5$

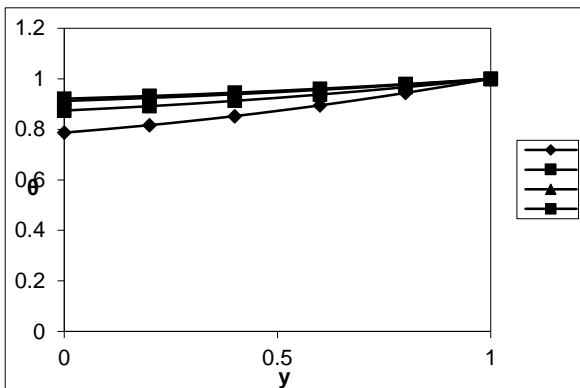


**Fig. 9:**

**Profile of changes in  $\theta$  with  $N_1$  at the plane**

$M=5; G=200; S=0.8; k=0.5; P=0.71; \beta=2; D^{-1}=2000; z=0.5$

I II III IV  
 $N_1$  0.5 1.5 5

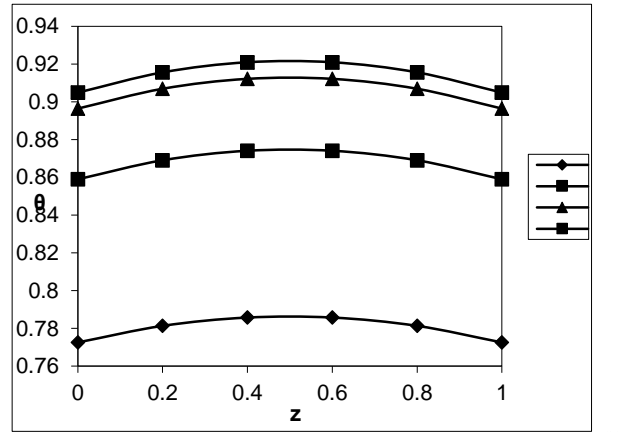


**Fig. 10:**

**Profile of changes in  $\theta$  with  $N_1$  at the plane  $Z=0.5$**

$M=5; G=200; S=0.8; k=0.5; P=0.71; D^{-1}=2000; z=0.5$

I II III IV  
 $N_1$  0.5 1.5 5 10

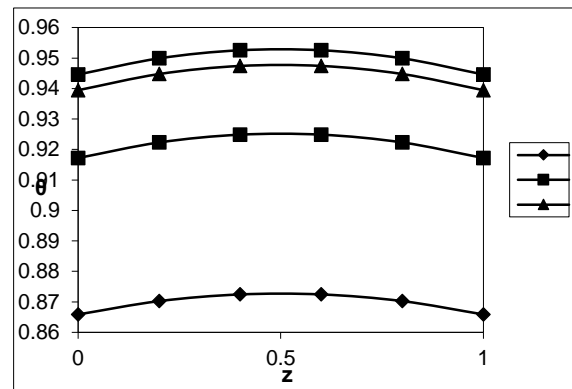


**Fig. 11:**

**Profile of changes in  $\theta$  with  $N_1$  at the plane  $Z=0$**

$M=5; G=200; S=0.8; k=0.5; P=0.71; \beta=2; D^{-1}=2000; z=0.5$

I II III IV  
 $N_1$  0.5 1.5 5 10



**Fig. 12:**

**Profile of changes in  $\theta$  with  $N_1$  at the plane  $Z=0.5$**

$M=5; G=200; S=0.8; k=0.5; P=0.71; D^{-1}=2000; z=0.5$

I II III IV  
 $N_1$  0.5 1.5 5 10

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