Fuzzy Cubic Spline Interpolation with Triangular Fuzzy Numbers

A.Karpagam, V.Vijayalakshmi

Abstract : In applied mathematics, the salient and engrossing aspect is how to best approximate a function in a given space. In this paper a cubic spline polynomial approximation as best approximations of fuzzy function on a discrete set of points. In this work a novel approach is adopted to show this method using Triangular fuzzy numbers.

Keywords: Best Approximation, Cubic Spline ,Fuzzy Numbers, Fuzzy Polynomial, , Fuzzy Interpolation, Triangular Fuzzy Numbers.

I. INTRODUCTION

Cubic splines relates each pair of data points are given to third order curves. In all sub intervals it provides property measured by the number of derivates it has that are continuous. A smooth function is a function that has derivatives of all orders everywhere in its domain. Interpolating polynomials may be applied to huge number of data points, since oscillatory behavior gives more (higher) degree polynomials. Spline mainly emerges from the drafting technique of a thin easily bent strip. The Interpolation Problem of fuzzy data was introduced by Zadeh[8] . H.Behforooz, R.Ezzati and S.Abbasbandy[3] have proposed Interpolation of fuzzy data by using E(3) Cubic Splines. Lown [4] gave a fuzzy Lagrange Interpolation Theorem. Kaleva[6] represented some properties of Lagrange and cubic splines of odd degree are introduced. Natural Splines and Complete Splines of odd degree are introduced by Abbasbandy et.al[1,2]. G.J.Klir, U.S.[5] and B.Yuan explained about Fuzzy Set Theory. H.J. Zimmermann[7] authored. the text book Fuzzy Sets Theory and its Application.] L.A. Zadeh[9] explained about Fuzzy Sets. In this paper, some preliminaries presented in section 2. Section 3, illustrates the new approach with numerical examples. Conclusions are discussed in section 4.

II. PRELIMINARIES

Definition 2.1

The objective in cubic splines is to derive a third-order polynomial for each interval between knots as in $f_v\ (l)$ = $a_v l^3$ + $b_v l^2$ +$c_v l$ +d_v.

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Thus, for N+1 data points (v = 0, 1, 2, …), there are N intervals and consequently, 4N unknown constants to evaluate. Just as for quadratic splines, 4N conditions are required to evaluate the unknowns.

Definition 2.2

Given the N data points $(l_1, m_1)\ldots(l_N, m_N)$ where x_i are distinct and in increasing order. A Cubic Spline S(l) through the data points $(l_1, m_1)\ldots(l_N, m_N)$ is a set of cubic polynomials.

$S_1(l)$: $m_1 + b_1(l-l_1) + c_1(l-l_1)^2 + d_1(l-l)^3$ on $[l_1, l_2]$
$S_2(l)$: $m_2 + b_2(l-l_2) + c_2(l-l_2)^2 + d_2(l-l)^3$ on $[l_2, l_3]$
$S_{N-1}(x)$: $m_{N-1} + b_{N-1}(l-l_{N-1}) + c_{N-1}(l-l_{N-1})^2 + d_{N-1}(l-l)^3$ on $[l_{N-1}, l_N]$

With the following conditions:

a) $S_v(l_v) = m\ and\ S_v(l_{v+1}) = m_{v+1}$ for v = 1, 2, …, N-1.
   This property guarantees that the spline S(l) interpolates the data points.
b) $S_{v-1}{}'(l_v) = S_v{}'(l_v)$ for v = 2… N-1.
   $S'(l)$ is continuous on the interval [l_1, l_N] this property forces the slopes of neighboring parts to agree when they meet.
c) $S_{v-1}{}''(l_i) = S_v{}''(l_i)$ for v = 2… N-1.
   $S''(l)$ is continuous on the interval [l_1, l_N] which also forces the neighboring spline to have the same curvature to guarantee the smoothness.

Let S(l) is a cubic polynomial, $S''(l)$ is linear in each interval. In the interval (l_{v-1}, l_v).

Assume $S''(l) = \frac{1}{h}[(l_v - l)S''(l_{v-1}) + (l - l)S''(l_v)]$

This equation is valid. If $l = l_{v-1}$ and $l = l_v$
Integrating twice,

$$S(l) = \frac{1}{h}\left[\frac{(l_v-l)^3}{3!}S''(l_{v-1}) + \frac{(l-l_{v-1})^3}{3!}S''(l_v)\right] + a_v(l_v - l) + b_v\,(l - l_{v-1}) \quad \ldots\ldots\ldots\ldots\ (2)$$

Where a_v , b_v are constants to be found out by using the conditions

$S(l_v) = m_v, v = 0,1,2, \ldots, N$

Put $l = l_{v-1}$ in (2), we get

$$m_{v-1} = \frac{1}{h}\left[\frac{h^3}{3!}S''(l_{v-1})\right] + h\,a_v$$

$a_v = \frac{1}{h}[m_{v-1} - \frac{h^2}{3!} S''(l_{v-1})]$

Put $l = l_i$ in (2), we get

$b_v = \frac{1}{h}[m_v - \frac{h^2}{3!} S''(l_v)]$

Hence the equation (2) reduces to

$$S(l) = \frac{1}{h}[\frac{(l_v - l)^3}{3!} S''(l_{v-1}) + \frac{(l - l_{v-1})^3}{3!} S''(l_v)] + \frac{1}{h}(l_v - l)[m_{v-1} - \frac{h^2}{3!} S''(l_{v-1})] + \frac{1}{h}(l - l_{v-1})[m_v - \frac{h^2}{3!} S''(l_v)]$$

Put $S''(l) = P_v$, the above equation becomes,

$$S(l) = \frac{1}{6h}[(l_v - l)^3 P_{v-1} + (l - l_{v-1})^3 P_v] + \frac{1}{h}(l_v - l)[m_{v-1} - \frac{h^2}{6} P_{v-1}] + \frac{1}{h}(l - l_{v-1})[m_v - \frac{h^2}{6} P_v] \ldots(3)$$

The quantities P_v are the spline second derivatives which are not yet known. Now we will impose the continuity of $S'(l)$ From (3)

$$S'(l) = \frac{1}{6h}[3(l_v - l)^2(-P_{v-1}) + 3(l - l_{v-1})^2 P_v] + \frac{1}{h}[-m_{v-1} + \frac{h^2}{6} P_{v-1}] + \frac{1}{h}[m_v - \frac{h^2}{6} P_v]$$

$S'(l_v -) = \frac{h}{3} P_v + \frac{h}{6} P_{v-1} + \frac{1}{h}(m_v - m_{v-1})$ ………… (4)

Similarly, $S'(l_v +) = \frac{h}{3} P_v - \frac{h}{6} P_{v-1} + \frac{1}{h}(m_{v+1} - m_v)$ ….. (5)

Equating (4) and (5), we get

$$P_{v-1} + 4P_v + P_{v+1} = \frac{6}{h^2}[m_{v-1} - 2m_v + m_{v+1}]$$

For v = 1, 2, 3… (N-1)

Further, from the first conditions, that

S(l) is linear for $l < l_0$ and $l > l_n$, we have $S''(l_v) = 0$ at $l = l_0$ and $l = l_n$

Hence $P_0 = 0$, $P_N = 0$

Give (N+1) equations in (N+1) unknowns $P_0, P_1, P_2, \ldots, P_N$. Hence we can solve for $P_0, P_1, P_2 \ldots P_N$. Substituting (3), we get the cubic spline in each interval.

III. NUMERICAL EXAMPLES

3.1 Obtain the fuzzy cubic spline approximation for the function m = s(l) from the following data

L:	(4,5,6)	(5,6,7)	(67,8)	(7,8,9)
M:	0	1	1	0

Given that $m_0'' = m_3'' = 0$.

Solution:

Converting the above triangular fuzzy number to crisp number by ranking function.

L:	5	6	7	8
M:	0	1	1	0

h= length of the interval

N = number of intervals

Since the values of l are equally spaced with h = 1 & N = 3.

We have $P_{v-1} + 4P_v + P_{v+1} = 6[m_{v-1} - 2m_v + m_{v+1}]$ (1)

v = 1, 2… (N-1) i.e., v = 1, 2

$P_0 = m_0'' = 0$; $P_3 = m_3'' = 0$.

For v = 1

$P_0 + 4P_1 + P_2 = 6[m_0 - 2m_1 + m_2]$

$4P_1 + P_2 = 6[0 - 2(1) + 1]$

$4P_1 + P_2 = 6(-1)$

$4P_1 + P_2 = -6$

For v = 2

$P_1 + 4P_2 + P_3 = 6[m_1 - 2m_2 + m_3]$

$P_1 + 4P_2 = 6[1 - 2(1) + 0]$

$P_1 + 4P_2 = 6(-1)$

$P_1 + 4P_2 = -6$

Solving, we get $P_1 = -\frac{6}{5}$, $P_2 = -\frac{6}{5}$

The Cubic Spline in $l_{v-1} \le l \le l_v$, is given by

$M = S(l) = \frac{1}{6}[(l_v - l)^3 P_{v-1} + (l - l_{v-1})^3 P_v] + (l_v - l)[m_{v-1} - \frac{1}{6} P_{v-1}] + (l - l_{v-1})[m_v - \frac{1}{6} P_v]$

Put v =1 in (3), the cubic spline, for $5 \le l \le 6$ is given by

$M = \frac{1}{6}[(6 - l)^3(0) + (l - 5)^3(-\frac{6}{5})] + (6 - l)[0 - \frac{1}{6}(0)] + (l - 5)[1 - \frac{1}{6}(-\frac{6}{5})]$

$= \frac{1}{6}\left(-\frac{6}{5}\right)[x^3 - 125 - 3l^2(5) + 3l(25)] + (l - 5)\left(\frac{6}{5}\right)$

$= -\frac{1}{5}[l^3 - 125 - 15l^2 + 75l - 6l + 30]$

$= -\frac{1}{5}[l^3 - 15l^2 + 69l - 95]$

Put v =2 in (3), the cubic spline, for $6 \le l \le 7$ is given by

$M = \frac{1}{6}[(7 - l)^3(-\frac{6}{5}) + (l - 6)^3(-\frac{6}{5})] + (7 - l)[1 - \frac{1}{6}(-\frac{6}{5})] + (l - 6)[1 - \frac{1}{6}(-\frac{6}{5})]$

$= \frac{1}{6}\left(-\frac{6}{5}\right)[(7^3) - l^3 - 3l(7^2) + 3(7)l^2 + l^3 -$

(63)−3 l26+3 l62+7−l) (65+l−6(65)

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$$= -\frac{1}{5}\,[3\,l^2 - 39l + 127 - 42 + 6l - 6l + 36]$$

$$= -\frac{1}{5}\,[3\,l^2 - 39l + 121]$$

Put v = 3 in (3), the cubic spline, for $7 \le l \le 8$ is given by

$$M = \frac{1}{6}[(8-l)^3(-\frac{6}{5}) + (l-7)^3\,(0)\,] + (8-l)$$

$$[1 - \frac{1}{6}(-\frac{6}{5})] + (l-7)\,[0 - \frac{1}{6}(0)]$$

$$= \frac{1}{6}\left(-\frac{6}{5}\right)[8^3 - l^3 - 3l8^2 - 3\,l^2(8)] + \left(8-l\right)\left(\frac{6}{5}\right)$$

$$= -\frac{1}{5}\,[-l^3 + 24\,l^2 - 192l + 512 - 48 + 6l]$$

$$= -\frac{1}{5}\,[-\,l^3 + 24\,l^2 - 186l + 464]$$

Hence the required cubic spline approximation for the given function is

M =

$$\begin{cases} -\frac{1}{5}[l^3 - 15\,l^2 + 69l - 95] & for\ \ 5 \le l \le 6 \\ -\frac{1}{5}[\,3\,l^2 - 39l + 121 & for\ \ 6 \le l \le 7 \\ -\frac{1}{5}[-\,l^3 + 24\,l^2 - 186l + 464] & for\ \ 7 \le l \le 8 \end{cases}$$

IV. CONCLUSION

In this paper, a new set of cubic spline functions to interpolate given Triangular fuzzy data is defined. We propose a method to find a fuzzy cubic spline polynomial as a best approximation of a fuzzy function on a discrete set of points. Hence this method can be regarded as a best approximation cubic spline polynomial Some numerical examples are presented in a new way.

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