Nano Generalized E-Continuous Mappings in Nano Topological Spaces

P. Manivannan, A. Vadivel, M. Seenivasan, V. Chandrasekar

***Abstract:** In this paper, we discuss ng e-continuous and irresoluteness via ng$e$OS in nts's. Also some characterizations are discussed with necessary examples.*

***Keywords and phrases:** nano e-Cts, nano generalized e-Cts, nano generalized e-Irr. AMS (2000) subject classification: 54B05.*

I INTRODUCTION AND PRELIMINARIES

Nano topology (briefly, $\mathfrak{N}\mathfrak{T}$) was introduced by Lellis Thivagar [7] in the year 2013. Also he introduced nano closed ($\mathfrak{N}c$) sets , nano-interior ($\mathfrak{N}int$) and nano-closure ($\mathfrak{N}cl$) in a nano topological spaces (briefly, $\mathfrak{N}ts$). Various forms of sets were discussed in [1] -[13]. Nano regular open (briefly, $\mathfrak{N}ro$) sets, nano θ-interior (resp. nano θ-closure, nano δ-interior, nano δ-closure) (briefly, $\mathfrak{N}int_\theta(A)$ (resp. $\mathfrak{N}cl_\theta(A)$, $\mathfrak{N}int_\delta(A)$, $\mathfrak{N}cl_\delta(A)$)) and also nano θ-open (resp. nano θ-closed, nano δ-open, nano δ-closed) (briefly, $\mathfrak{N}\theta o$ (resp. $\mathfrak{N}\theta c$, $\mathfrak{N}\delta o$, $\mathfrak{N}\delta c$)) sets were introduced in [4, 7, 11, 12]. Nano δ-pre (resp. nano δ-semi, nano e, nano $\mathcal{M}$, nano θ-pre and nano θ-semi) open set (briefly $\mathfrak{N}\delta\mathcal{P}o$ (resp. $\mathfrak{N}\delta\mathcal{S}o$, $\mathfrak{N}eo$, $\mathfrak{N}\mathcal{M}o$, $\mathfrak{N}\theta\mathcal{P}o$ and $\mathfrak{N}\theta\mathcal{S}o$)), nano δ-pre (resp. δ-semi, e, $\mathcal{M}$ and θ-semi) interior (briefly, $\mathfrak{N}\mathcal{P}int_\delta(K)$ (resp. $\mathfrak{N}\mathcal{S}int_\delta(K)$, $\mathfrak{N}eint(K)$, $\mathfrak{N}\mathcal{M}int(K)$ and $\mathfrak{N}\mathcal{S}int_\theta(K)$)), nano δ-pre (resp. δ-semi, e , $\mathcal{M}$ and θ-semi) closure (briefly, $\mathfrak{N}\mathcal{P}cl_\delta(K)$ (resp. $\mathfrak{N}\mathcal{S}cl_\delta(K)$, $\mathfrak{N}ecl(K)$, $\mathfrak{N}\mathcal{M}cl(K)$ and $\mathfrak{N}\mathcal{S}cl_\theta(K)$)) were introduced in [10, 11, 12]. The collection of all $\mathfrak{N}\delta\mathcal{P}o$ (resp. $\mathfrak{N}\delta\mathcal{S}o$, $\mathfrak{N}eo$, $\mathfrak{N}\mathcal{M}o$ and $\mathfrak{N}\theta\mathcal{S}o$) sets is denoted by $\mathfrak{N}\delta\mathcal{P}O(U,\tau_R(P))$ (resp. $\mathfrak{N}\delta\mathcal{S}O(U,\tau_R(P))$, $\mathfrak{N}eO(U,\tau_R(P))$, $\mathfrak{N}\mathcal{M}O(U,\tau_R(P))$ and $\mathfrak{N}\theta\mathcal{S}O(U,\tau_R(P))$) and the collection of all nano δ-pre (resp. δ-semi, e, nano $\mathcal{M}$ and θ-semi) closed (briefly, $\mathfrak{N}\delta\mathcal{P}c$ (resp. $\mathfrak{N}\delta\mathcal{S}c$, $\mathfrak{N}ec$, $\mathfrak{N}\mathcal{M}c$ and $\mathfrak{N}\theta\mathcal{S}c$)) sets is denoted by $\mathfrak{N}\delta\mathcal{P}C(U,\tau_R(P))$ (resp. $\mathfrak{N}\delta\mathcal{S}C(U,\tau_R(P))$, $\mathfrak{N}eC(U,\tau_R(P))$, $\mathfrak{N}\mathcal{M}C(U,\tau_R(P))$ and $\mathfrak{N}\theta\mathcal{S}C(U,\tau_R(P))$). Nano generalized (resp. θ, θ semi, δ, δ semi, δ pre, e and $\mathcal{M}$) closed (briefly, $\mathfrak{N}gc$, $\mathfrak{N}g\theta c$, $\mathfrak{N}g\theta\mathcal{S}c$, $\mathfrak{N}g\delta c$, $\mathfrak{N}g\delta\mathcal{S}c$, $\mathfrak{N}g\delta\mathcal{P}c$, $\mathfrak{N}gec$ and $\mathfrak{N}g\mathcal{M}c$) were introduced in [1, 2, 4, 5, 9].

A subset K of a nano generalized (resp. θ, θ semi, δ , δ semi and δ pre) open [1, 2, 4, 5] (briefly, $\mathfrak{N}go$(resp. $\mathfrak{N}g\theta o$, $\mathfrak{N}g\theta\mathcal{S}o$, $\mathfrak{N}g\delta o$, $\mathfrak{N}g\delta\mathcal{S}o$ and $\mathfrak{N}g\delta\mathcal{P}o$)) if its complement K^c is nano generalized (resp. θ, θ semi, δ, δ semi and δ pre) closed (briefly, $\mathfrak{N}gc$(resp. $\mathfrak{N}g\theta c$, $\mathfrak{N}g\theta\mathcal{S}c$, $\mathfrak{N}g\delta c$, $\mathfrak{N}g\delta\mathcal{S}c$ and $\mathfrak{N}g\delta\mathcal{P}c$)). The system of all nano generalized (resp. θ, θ semi, δ , δ semi, δ pre, $\mathfrak{N}ge$ and $\mathfrak{N}g\mathcal{M}$) open sets of $(U,\tau_R(P))$ is denoted by $\mathfrak{N}GO(U,P)$ (resp. $\mathfrak{N}G\theta O(U,P)$, $\mathfrak{N}G\theta\mathcal{S}O(U,P)$, $\mathfrak{N}G\delta O(U,P)$, $\mathfrak{N}G\delta\mathcal{S}O(U,P)$, $\mathfrak{N}G\delta\mathcal{P}O(U,P)$, $\mathfrak{N}GeC\ (U,P)$ and $\mathfrak{N}G\mathcal{M}C\ (U,P)$). Nano continuous and irresolute (briefly, $\mathfrak{N}\ Cts$ and $\mathfrak{N}\ Irr$) were introduced by [3, 8].

II NANO GENERALIZED e CONTINUOUS MAPS

Definition 2.1 *A function $f:(U,\tau_R(P)) \to (V,\sigma_{R'}(Q))$ is called nano generalized e (resp. $\mathcal{M}$) continuous (briefly, $\mathfrak{N}ge$ (resp. $\mathfrak{N}g\mathcal{M}$) Cts), if for each $\mathfrak{N}cs$ K of V, $f^{-1}(K)$ is $\mathfrak{N}gecs$ (resp. $\mathfrak{N}g\mathcal{M}cs$) of U.*

Theorem 2.1 *Let $f:(U,\tau_R(P)) \to (V,\sigma_{R'}(Q))$ be a mapping. Then [(i)]*

1. Every $\mathfrak{N}eCts$ mapping is $\mathfrak{N}geCts$.
2. Every $\mathfrak{N}\mathcal{M}Cts$ mapping is $\mathfrak{N}g\mathcal{M}Cts$.
3. Every $\mathfrak{N}g\theta Cts$ mapping is $\mathfrak{N}gCts$.
4. Every $\mathfrak{N}gCts$ mapping is $\mathfrak{N}g\theta\mathcal{S}Cts$.
5. Every $\mathfrak{N}g\delta Cts$ mapping is $\mathfrak{N}g\delta\mathcal{S}Cts$.
6. Every $\mathfrak{N}g\delta Cts$ mapping is $\mathfrak{N}g\delta\mathcal{P}Cts$.
7. Every $\mathfrak{N}g\delta\mathcal{S}Cts$ mapping is $\mathfrak{N}geCts$.
8. Every $\mathfrak{N}g\delta\mathcal{P}Cts$ mapping is $\mathfrak{N}geCts$.
9. Every $\mathfrak{N}g\theta\mathcal{S}Cts$ mapping is $\mathfrak{N}g\mathcal{M}Cts$.
10. Every $\mathfrak{N}g\delta\mathcal{P}Cts$ mapping is $\mathfrak{N}g\mathcal{M}Cts$.

But not conversely.

Proof. (i) Let $f:(U,\tau_R(P)) \to (V,\sigma_{R'}(Q))$ be $\mathfrak{N}eCts$ and L is a $\mathfrak{N}cs$ in V. Then $f^{-1}(L)$ is $\mathfrak{N}ec$ in U. But every $\mathfrak{N}ecs$ is $\mathfrak{N}gec$, $f^{-1}(L)$ is $\mathfrak{N}gecs$ in U.
The others are proved in a similar way.

Example 2.1 *Let $U = \{o,n,m,l\}$ with $U/R = \{\{l\},\{o\},\{n,m\}\}$, $P = \{o,m\}$. Then, the $\mathfrak{N}t$ is defined as $\tau_R(P) = \{U,\phi,\{o\},\{n,m\},\{n,o,m\}\}$. Then the mapping $f:(U,\tau_R(P)) \to (U,\tau_R(P))$ defined by [(i)]*

1. $f(l) = n$, $f(m) = o$, $f(n) = l$ and $f(o) = m$ is $\mathfrak{N}g\theta\mathcal{S}Cts$ but not $\mathfrak{N}gCts$. The set $\{l\}$ is $\mathfrak{N}c$ in V but $f^{-1}(\{l\}) = \{n\}$ is not $\mathfrak{N}gc$ in U.
2. $f(l) = n$, $f(m) = o$, $f(n) = l$ and $f(o) = m$ is $\mathfrak{N}g\delta\mathcal{S}Cts$ but not $\mathfrak{N}g\delta Cts$. The set $\{l\}$ is $\mathfrak{N}c$ in V but $f^{-1}(\{l\}) = \{n\}$ is not $\mathfrak{N}g\delta c$ in U.
3. $f(l) = n$, $f(m) = m$, $f(n) = l$ and $f(o) = o$ is $\mathfrak{N}geCts$ but not $\mathfrak{N}g\delta\mathcal{S}Cts$. The set $\{l,o\}$ is $\mathfrak{N}c$ in V but $f^{-1}(\{l,o\}) = \{n,o\}$ is not $\mathfrak{N}g\delta\mathcal{S}c$ in U.

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4. $f(l)=n$, $f(m)=o$, $f(n)=l$ and $f(o)=m$ is $\mathfrak{N}geCts$ but not $\mathfrak{N}g\delta\mathcal{P}Cts$. The set $\{l,o\}$ is $\mathfrak{N}c$ in V but $f^{-1}(\{l,o\})=\{n,m\}$ is not $\mathfrak{N}g\delta\mathcal{P}c$ in U.
5. $f(l)=n$, $f(m)=m$, $f(n)=l$ and $f(o)=o$ is $\mathfrak{N}g\mathcal{M}Cts$ but not $\mathfrak{N}g\theta\mathcal{S}Cts$. The set $\{l,o\}$ is $\mathfrak{N}c$ in V but $f^{-1}(\{l,o\})=\{n,o\}$ is not $\mathfrak{N}g\theta\mathcal{S}c$ in U.
6. $f(l)=n$, $f(m)=o$, $f(n)=l$ and $f(o)=m$ is $\mathfrak{N}g\mathcal{M}Cts$ but not $\mathfrak{N}g\delta\mathcal{P}Cts$. The set $\{l,o\}$ is $\mathfrak{N}c$ in V but $f^{-1}(\{l,o\})=\{n,m\}$ is not $\mathfrak{N}g\delta\mathcal{P}c$ in U.

Example 2.2 *Let* $U=\{o,n,m,l,p\}$ *with* $U/R=\{\{p\},\{m,l\},\{o,n\}\}$, $P=\{l,o,n\}$ and $V=\{l,n,m\}$ *with* $V/R'=\{\{l\}\{n,m\}\}$, $Q=\{l,n\}$. *Then,* $\tau_R(P)=\{U,\{m,l\},\phi,\{o,n\},\{o,n,m,l\}\}$ *and* $\tau_{R'}(Q)=\{V,\{l\},\phi,\{n,m\}\}$. *Then the mapping* $f:(U,\tau_R(P))\to(V,\tau'_R(Q))$ *defined by* $f(l)=n$, $f(m)=l$, $f(n)=m$, $f(o)=m$ *and* $f(p)=m$ *is* $\mathfrak{N}gCts$ but *not* $\mathfrak{N}g\theta Cts$. *The set* $\{l\}$ *is* $\mathfrak{N}c$ *in* V *but* $f^{-1}(\{l\})=\{m\}$ *is not* $\mathfrak{N}g\theta c$ *in* U.

Example 2.3 *Let* $U=\{o,n,m,l\}$ *with* $U/R=\{\{o\},\{l\},\{n,m\}\}$, $P=\{o,m\}$ *and* $V=\{o,n,m,l\}$ *with* $V/R'=\{\{m,l\}\{o,n\}\}$, $Q=\{l,o,n\}$. *Then,* $\tau_R(P)=\{U,\{o\},\phi,\{n,m\},\{o,n,m\}\}$ *and* $\tau_{R'}(Q)=\{V,\phi,\{m,l\},\{o,n\}\}$. *Then the mapping* $f:(U,\tau_R(P))\to(V,\tau'_R(Q))$ *defined by [(i)]*

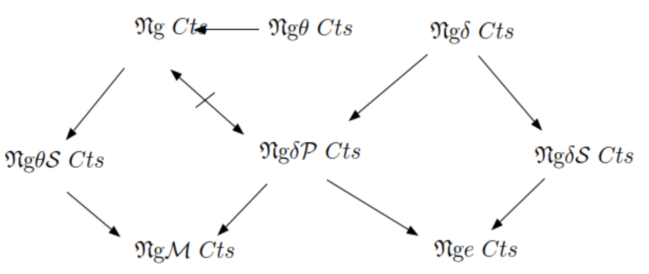
1. $f(l)=l$, $f(m)=m$, $f(n)=n$ and $f(o)=o$ is $\mathfrak{N}geCts$ (resp. $\mathfrak{N}geCts$) but not $\mathfrak{N}eCts$ (resp. $\mathfrak{N}g\mathcal{M}Cts$). The set $\{o,n\}$ is $\mathfrak{N}c$ in V but $f^{-1}(\{o,n\})=\{o,n\}$ is not $\mathfrak{N}ec$ (resp. $\mathfrak{N}\mathcal{M}c$) in U.
2. $f(l)=l$, $f(m)=m$, $f(n)=n$ and $f(o)=o$ is $\mathfrak{N}g\delta\mathcal{P}Cts$ but not $\mathfrak{N}g\delta\mathcal{S}Cts$. The set $\{o,n\}$ is $\mathfrak{N}c$ in V but $f^{-1}(\{o,n\})=\{o,n\}$ is not $\mathfrak{N}g\delta c$ in U.

Remark 2.1 *The concept of* $\mathfrak{N}gCts$ *and* $\mathfrak{N}g\delta\mathcal{P}Cts$ *are independent.*

Example 2.4 *In Example 2.3, [(i)]*

1. $f(l)=l$, $f(m)=m$, $f(n)=n$ and $f(o)=o$ is $\mathfrak{N}g\delta\mathcal{P}Cts$ but not $\mathfrak{N}gCts$. The set $\{o,n\}$ is $\mathfrak{N}c$ in V but $f^{-1}(\{o,n\})=\{o,n\}$ is not $\mathfrak{N}gc$ in U.
2. $f(l)=n$, $f(m)=m$, $f(n)=l$ and $f(o)=o$ is $\mathfrak{N}gCts$ but not $\mathfrak{N}g\delta\mathcal{P}Cts$. The set $\{m,l\}$ is $\mathfrak{N}c$ in V but $f^{-1}(\{m,l\})=\{n,m\}$ is not $\mathfrak{N}g\delta\mathcal{P}c$ in U.

From Examples 2.1 to 2.4 & Theorem 2.1, we have



Note: $K\to L$ denotes K implies L, L does not implies K.

Theorem 2.2 *A map* $f:(U,\tau_R(P))\to(V,\sigma_{R'}(Q))$ *is* $\mathfrak{N}ge$ *(resp.* $\mathfrak{N}g\mathcal{M}$*)* Cts *iff the inverse image* of *every* $\mathfrak{N}o$ *set in* V *is* $\mathfrak{N}geo$ *(resp.* $\mathfrak{N}g\mathcal{M}o$*) in* U.

Theorem 2.3 *A map* $f:(U,\tau_R(P))\to(V,\sigma_{R'}(Q))$ *is* $\mathfrak{N}geCts$ *iff* $f(\mathfrak{N}gecl(K))\subseteq\mathfrak{N}cl(f(K))\ \forall$ *subset* K *of* U.

Proof. We prove only the necessity part. Let $K\subseteq U$ & f be $\mathfrak{N}geCts$ implies $f(K)\subseteq V$. Implies $\mathfrak{N}cl(f(K))$ is $\mathfrak{N}c$ in V $\Rightarrow$ $f^{-1}(\mathfrak{N}cl(f(K)))$ is $\mathfrak{N}gec$ in U. Clearly, $f(K)\subseteq\mathfrak{N}cl(f(K))\Rightarrow f^{-1}(f(K))\subseteq f^{-1}(\mathfrak{N}cl(f(K)))\Rightarrow K\subseteq f^{-1}(\mathfrak{N}cl(f(K)))$. $\mathfrak{N}gecl(K)\subseteq\mathfrak{N}gecl[f^{-1}(\mathfrak{N}cl(f(K)))]=f^{-1}(\mathfrak{N}cl(f(K)))$. Hence $\mathfrak{N}gecl(K)\subseteq f^{-1}(\mathfrak{N}cl(f(K)))$. $\Rightarrow f(\mathfrak{N}gecl(K))\subseteq\mathfrak{N}cl(f(K))\ \forall$ subset K of U.

The Theorem 2.3 is also satisfy the map $\mathfrak{N}g\mathcal{M}Cts$ for their respective closure.

Remark 2.2 *A map* $f:(U,\tau_R(P))\to(V,\sigma_{R'}(Q))$ *is* $\mathfrak{N}geCts$ *then* $f(\mathfrak{N}gecl(K))$ *is not necessarily equal to* $\mathfrak{N}cl(f(K))$ *where* $K\subseteq U$.

Example 2.5 *In Example 2.1, Define an identity map, then* f *is* $\mathfrak{N}geCts$. *Let* $K=\{m,n\}\subset U$. *Then* $f(\mathfrak{N}gecl(K))=f(\mathfrak{N}gecl(\{m,n\}))=f(\{m,n\})=\{m,n\}$. *But* $\mathfrak{N}cl(f(K))=\mathfrak{N}cl(\{m,n\})=\{l,m,n\}$. *Thus* $f(\mathfrak{N}gecl(K))\neq\mathfrak{N}cl(f(K))$, *even though* f *is* $\mathfrak{N}ge\ cts$. *That is equality does not hold.*

Theorem 2.4 *A map* $f:(U,\tau_R(P))\to(V,\sigma_{R'}(Q))$ *is* $\mathfrak{N}geCts$ *iff* $\mathfrak{N}gecl(f^{-1}(L))\subseteq f^{-1}(\mathfrak{N}cl(L))\ \forall$ *subset* L *of* V.

Remark 2.3 *A map* $f:(U,\tau_R(P))\to(V,\sigma_{R'}(Q))$ *is* $\mathfrak{N}geCts$ *then* $\mathfrak{N}gecl(f^{-1}(L))$ *is not necessarily* equal *to* $f^{-1}(\mathfrak{N}cl(L))$ *where* $L\subseteq V$.

Example 2.6 *In Example 2.1, Define an identity map, then* f *is* $\mathfrak{N}geCts$. *Let* $L=\{m\}\subset L$. *Then* $\mathfrak{N}gecl(f^{-1}(L))=\mathfrak{N}gecl(f^{-1}(\{m\}))=\mathfrak{N}gecl(\{m\})=\{m\}$. *But* $f^{-1}(\mathfrak{N}cl(f(L)))=f^{-1}(\mathfrak{N}cl(\{m\}))=f^{-1}(\{l,m,n\})=\{l,m,n\}$. *Thus* $\mathfrak{N}gecl(f^{-1}(L))\neq f^{-1}(\mathfrak{N}cl(f(L)))$, *even though* f *is* $\mathfrak{N}ge\ cts$. *That is equality does not hold.*

Theorem 2.5 *A map* $f:(U,\tau_R(P))\to(V,\sigma_{R'}(Q))$ *is* $\mathfrak{N}geCts$ *iff* $f^{-1}(\mathfrak{N}int(L))\subseteq\mathfrak{N}geint(f^{-1}(L))\ \forall$ subset L *of* V.

The map $\mathfrak{N}g\mathcal{M}Cts$ satisfy the Theorems 2.4 & 2.5 for their respective interior.

Remark 2.4 *A map* $f:(U,\tau_R(P))\to(V,\sigma_{R'}(Q))$ *is* $\mathfrak{N}geCts$ *then* $f^{-1}(\mathfrak{N}int(L))$ *is not* necessarily *equal to* $\mathfrak{N}geint(f^{-1}(L))$ *where* $L\subseteq V$.

Example 2.7 In Example *2.1, Define an identity map, then* f *is* $\mathfrak{N}geCts$. *Let* $L=\{m\}\subset V$.
Then

$\mathfrak{N}geint(f^{-1}(L)) = \mathfrak{N}geint f^{-1}(\{m\}) = \mathfrak{N}geint(\{m\}) = \{m\}$.

But $f^{-1}(\mathfrak{N}int(L)) = f^{-1}(\mathfrak{N}int(\{m\})) = f^{-1}(\{\phi\}) = \phi$. *Thus* $\mathfrak{N}geint\ (f^{-1}(L)) \neq f^{-1}(\mathfrak{N}int(L))$, *even though* f *is* $\mathfrak{N}ge\ cts$. *That is equality does not hold.*

Theorem 2.6 *In a* $\mathfrak{N}ts$ $(U, \tau_R(P))$, *if the collection of* $\mathfrak{N}GeO(U, P)$ *is closed under arbitrary union & let* $f:(U, \tau_R(P)) \to (V, \sigma_{R'}(Q))$ *be a function. Then [(i)]*

1. f is $\mathfrak{N}geCts$
2. $\forall\ l \in U$ and each $\mathfrak{N}os$ H in V with $f(l) \in H$ $\exists$ a $\mathfrak{N}geos$ G in $U \ni l \in G$ & $f(G) \subset H$.
3. $\forall\ l \in U$, the inverse of every $\mathfrak{N}Nbd$ of $f(l)$ is $\mathfrak{N}geNbdS(l)$

are equivalent.

The map $\mathfrak{N}gMCts$ satisfy the Theorem 2.6 for their respective family of open sets.

Remark 2.5 *The composition of two* $\mathfrak{N}geCts$ *functions need not be* $\mathfrak{N}geCts$ *as seen from the example.*

Example 2.8 *Let* $U = V = \{p, o, n, m, l\}$ *with* $U/R = \{\{n\}, \{m, l\}, \{o, p\}\}$, $P = \{l, n\}$, $\tau_R(P) = \{U, \phi, \{n\}, \{l, m\}, \{l, m, n\}\}$ *and* $V/R' = \{\{p\}, \{l, m\}, \{n, o\}\}$, $Q = \{l, n, o\}$, $\sigma_{R'}(Q) = \{V, \phi, \{m, l\}, \{n, o\}, \{o, n, m, l\}\}$. *Then, the identity mappings* $f:(U, \tau_R(P)) \to (V, \sigma_{R'}(Q))$ *and* $g:(V, \sigma_{R'}(Q)) \to (V, \sigma_{R'}(Q))$ *are* $\mathfrak{N}geCts$ *but the composition* $g \circ f$ *is not* $\mathfrak{N}geCts$, *the set* $\{l, n\}$ *is* $\mathfrak{N}c$ *in* V *but* $(g \circ f)^{-1}(\{l, n\}) = \{l, n\}$ *is not* $\mathfrak{N}gec$ *in* U.

Theorem 2.7 *Let* $f:(U, \tau_R(P)) \to (V, \sigma_{R'}(Q))$ *be* $\mathfrak{N}geCts$ *and* $g:(V, \sigma_{R'}(Q)) \to (W, \mu_{R''}(S))$ *be* $\mathfrak{N}Cts$, *then* $g \circ f$ *is* $\mathfrak{N}geCts$.

The map $\mathfrak{N}gMCts$ also satisfy the Theorem 2.7.

Definition 2.2 *A map* $f:(U, \tau_R(P)) \to (V, \sigma_{R'}(Q))$ *is nano generalized* e^* *(resp. nano generalized* $\mathcal{M}^*$*) continuous (briefly,* $\mathfrak{N}ge^*$ *(resp.* $\mathfrak{N}g\mathcal{M}^*$*)* Cts*), if* $\forall$ $\mathfrak{N}ec$ *(resp.* $\mathfrak{N}\mathcal{M}c$*) subset* K *of* V, $f^{-1}(K)$ *is* $\mathfrak{N}gec$ *(resp.* $\mathfrak{N}g\mathcal{M}c$*) subset of* U.

Theorem 2.8 *A map* $f:(U, \tau_R(P)) \to (V, \sigma_{R'}(Q))$ *is* $\mathfrak{N}ge^*Cts$ *(resp.* $\mathfrak{N}g\mathcal{M}^*Cts$*), iff* $f^{-1}(K)$ *is* $\mathfrak{N}geo$ *(resp.* $\mathfrak{N}g\mathcal{M}o$*) in* U $\forall$ $\mathfrak{N}eo$ *set* K *in* V.

Theorem 2.9 *Every* $\mathfrak{N}ge^*Cts$ *(resp.* $\mathfrak{N}g\mathcal{M}^*Cts$*) function is* $\mathfrak{N}geCts$ *(resp.* $\mathfrak{N}g\mathcal{M}Cts$*). But not conversely.*
Proof. Based on every $\mathfrak{N}ec$ set is $\mathfrak{N}gec$.
The other case is also similar

Example 2.9 *Let* $U = \{p, o, n, m, l\}$, $U/R = \{\{p\}, \{m, l\}\{o, n\}\}$, $P = \{l, o, n\}$ & $V = \{p, o, n, m, l\}$, $V/R' = \{\{n\}, \{m, l\}, \{o, p\}\}$, $Q = \{l, n\}$. *Then,* $\tau_R(P) = \{U, \phi, \{m, l\}, \{o, n\}, \{m, l, o, n\}\}$ *and* $\tau_{R'}(Q) = \{V, \phi, \{n\}, \{m, l\}, \{l, m, n\}\}$. *Then the mapping* $f:(U, \tau_R(P)) \to (V, \tau_R'(Q))$ *defined by* $f(l) = l$, $f(m) = m$, $f(n) = n$ *and* $f(o) = o$ *is* $\mathfrak{N}geCts$ *but not* $\mathfrak{N}ge^*Cts$. *The set* $\{m, l, o\}$ *is* $\mathfrak{N}gc$ *in* V *but* $f^{-1}(\{m, l, o\}) = \{m, l, o\}$ *is not* $\mathfrak{N}gec$ *in* U.

Remark 2.6 *The composition of two* $\mathfrak{N}ge^*Cts$ *functions need not be* $\mathfrak{N}ge^*Cts$ *as seen from the example.*

Example 2.10 *Let* $U = \{p, o, n, m, l\}$, $U/R = \{\{p\}, \{m, l\}\{o, n\}\}$, $P = \{l, o, n\}$ *and* $V = \{p, o, n, m, l\}$, $V/R' = \{\{n\}, \{m, l\}, \{o, p\}\}$, $Q = \{l, n\}$. *Then,* $\tau_R(P) = \{U, \phi, \{m, l\}, \{o, n\}, \{m, l, o, n\}\}$ & $\tau_{R'}(Q) = \{V, \phi, \{n\}, \{m, l\}, \{m, l, n\}\}$. *Then, the identity mappings* $f:(U, \tau_R(P)) \to (V, \sigma_{R'}(Q))$ & $g:(V, \sigma_{R'}(Q)) \to (V, \sigma_{R'}(Q))$ *are* $\mathfrak{N}ge^*Cts$ *but the composition* $g \circ f$ *is not* $\mathfrak{N}ge^*Cts$, *the set* $\{m, l, o\}$ *is* $\mathfrak{N}gc$ *in* V *but* $(g \circ f)^{-1}(\{m, l, o\}) = \{m, l, o\}$ *is not* $\mathfrak{N}gec$ *in* U.

Theorem 2.10 *Let* $f:(U, \tau_R(P)) \to (V, \sigma_{R'}(Q))$ *be* $\mathfrak{N}ge^*Cts$ *and* $g:(V, \sigma_{R'}(Q)) \to (W, \mu_{R''}(S))$ *be* $\mathfrak{N}eCts$, *then* $g \circ f$ *is* $\mathfrak{N}geCts$.

III NANO GENERALIZED e IRRESOLUTE MAPS

Definition 3.1 *A map* $f:(U, \tau_R(P)) \to (V, \sigma_{R'}(Q))$ *is called nano generalized* e *(resp.* $\mathcal{M}$*) irresolute (briefly,* $\mathfrak{N}geIrr$ *(resp.* $\mathfrak{N}g\mathcal{M}Irr$*)) maps, if* $\forall$ $\mathfrak{N}gec$ *(resp.* $\mathfrak{N}g\mathcal{M}c$*) subset* K *of* V, $f^{-1}(K)$ *is* $\mathfrak{N}gec$ *(resp.* $\mathfrak{N}g\mathcal{M}c$*) subset of* U.

Theorem 3.1 *A map* $f:(U, \tau_R(P)) \to (V, \sigma_{R'}(Q))$ *is [(i)]*

1. $\mathfrak{N}gIrr$, then f is $\mathfrak{N}gCts$.
2. $\mathfrak{N}g\theta Irr$, then f is $\mathfrak{N}g\theta Cts$.
3. $\mathfrak{N}g\delta Irr$, then f is $\mathfrak{N}g\delta Cts$.
4. $\mathfrak{N}g\theta\mathcal{S}Irr$, then f is $\mathfrak{N}g\theta\mathcal{S}Cts$.
5. $\mathfrak{N}g\delta\mathcal{P}Irr$, then f is $\mathfrak{N}g\delta\mathcal{P}Cts$.
6. $\mathfrak{N}g\delta\mathcal{S}Irr$, then f is $\mathfrak{N}g\delta\mathcal{S}Cts$.
7. $\mathfrak{N}geIrr$, then f is $\mathfrak{N}geCts$.
8. $\mathfrak{N}g\mathcal{M}Irr$, then f is $\mathfrak{N}g\mathcal{M}Cts$.

But not conversely.
Proof. (vii) Let F be $\mathfrak{N}c$ in V, $\Rightarrow$ F is $\mathfrak{N}gec$ in V, since every $\mathfrak{N}cs$ is $\mathfrak{N}gec$. By hypothesis, $f^{-1}(F)$ is $\mathfrak{N}gec$. Therefore, f is $\mathfrak{N}geCts$.
The other cases are similar

Example 3.1 *Let* $U = V = \{p, o, n, m, l\}$, $U/R = \{\{n\}, \{m, l\}, \{o, p\}\}$, $P = \{l, n\}$. *Then* $\tau_R(P) = \{U, \phi, \{n\}, \{m, l\}, \{m, l, n\}\}$ *and* $V/R' = \{\{p\}, \{m, l\}, \{o, n\}\}$, $Q = \{l, o, n\}$. *Then* $\sigma_{R'}(Q) = \{V, \phi, \{m, l\}, \{o, n\}, \{m, l, o, n\}\}$. *Define* $f:(U, \tau_R(P)) \to (V, \sigma_{R'}(Q))$ *as identity mapping. Then [(i)]*

1. f is $\mathfrak{N}gCts$, but f is not $\mathfrak{N}gIrr$, since $f^{-1}(\{l\}) = \{l\}$ which is not $\mathfrak{N}gc$ in U whereas $\{l\}$ is $\mathfrak{N}gc$ in V.
2. f is $\mathfrak{N}g\delta Cts$, but f is not $\mathfrak{N}g\delta Irr$, since $f^{-1}(\{l\}) = \{l\}$ which is not $\mathfrak{N}g\delta c$ in U whereas $\{l\}$ is $\mathfrak{N}g\delta c$ in V.
3. f is $\mathfrak{N}geCts$, but f is not $\mathfrak{N}geIrr$, since $f^{-1}(\{l, n\}) = \{l, n\}$ which is not $\mathfrak{N}gec$ in U whereas $\{l, n\}$ is $\mathfrak{N}gec$ in V.

4. f is $\mathfrak{N}g\mathcal{M}Cts$, but f is not $\mathfrak{N}g\mathcal{M}Irr$, since $f^{-1}(\{l,n\}) = \{l,n\}$ which is not $\mathfrak{N}g\mathcal{M}c$ in U whereas $\{l,n\}$ is $\mathfrak{N}g\mathcal{M}c$ in V.

Example 3.2 *Let* $U = V = \{p,o,n,m,l\}$, $U/R = \{\{p\},\{m,l\},\{o,n\}\}$, $P = \{l,o,n\}$ *Then* $\tau_R(P) = \{U,\phi,\{m,l\},\{o,n\},\{m,l,o,n\}\}$ & $V/R' = \{\{n\},\{m,l\},\{o,p\}\}$, $Q = \{l,n\}$. *Then* $\sigma_{R'}(Q) = \{V,\phi,\{n\},\{m,l\},\{m,l,n\}\}$. *Define* $f:(U,\tau_R(P)) \to (V,\sigma_{R'}(Q))$ *as identity mapping. Then [(i)]*

1. f is $\mathfrak{N}g\theta Cts$, but f is not $\mathfrak{N}g\theta Irr$, since $f^{-1}(\{l,o\}) = \{l,o\}$ which is not $\mathfrak{N}g\theta c$ in U whereas $\{l,o\}$ is $\mathfrak{N}g\theta c$ in V.
2. f is $\mathfrak{N}g\theta\mathcal{S}Cts$, but f is not $\mathfrak{N}g\theta\mathcal{S}Irr$, since $f^{-1}(\{l,o\}) = \{l,o\}$ which is not $\mathfrak{N}g\theta\mathcal{S}c$ in U whereas $\{l,o\}$ is $\mathfrak{N}g\theta\mathcal{S}c$ in V.
3. f is $\mathfrak{N}g\delta\mathcal{P}Cts$, but f is not $\mathfrak{N}g\delta\mathcal{P}Irr$, since $f^{-1}(\{o,n\}) = \{o,n\}$ which is not $\mathfrak{N}g\delta\mathcal{P}c$ in U whereas $\{o,n\}$ is $\mathfrak{N}g\delta\mathcal{P}c$ in V.
4. f is $\mathfrak{N}g\delta\mathcal{S}Cts$, but f is not $\mathfrak{N}g\delta\mathcal{S}Irr$, since $f^{-1}(\{l,o\}) = \{l,o\}$ which is not $\mathfrak{N}g\delta\mathcal{S}c$ in U whereas $\{l,o\}$ is $\mathfrak{N}g\delta\mathcal{S}c$ in V.

Theorem 3.2 *A map* $f:(U,\tau_R(P)) \to (V,\sigma_{R'}(Q))$ *is* $\mathfrak{N}geIrr$ *(resp.* $\mathfrak{N}g\mathcal{M}Irr$ *) iff for every* $\mathfrak{N}geos$ *(resp.* $\mathfrak{N}g\mathcal{M}os$*)* K *in* V, $f^{-1}(K)$ *is* $\mathfrak{N}geo$ *(resp.* $\mathfrak{N}g\mathcal{M}o$*) in* U.

Proof. Follows from the fact that the complement of $\mathfrak{N}geo$ (resp. $\mathfrak{N}g\mathcal{M}o$) set is $\mathfrak{N}gec$ (resp. $\mathfrak{N}g\mathcal{M}c$) and vice versa.

Theorem 3.3 *If* $f:(U,\tau_R(P)) \to (V,\sigma_{R'}(Q))$ *and* $g:(V,\sigma_{R'}(Q)) \to (W,\mu_{R''}(S))$ *are both* $\mathfrak{N}geIrr$, *then* $g\circ f:(U:\tau_R(P)) \to (W,\mu_{R''}(S))$ *is* $\mathfrak{N}geIrr$.

Proof. Let K be $\mathfrak{N}geo$ in W. Then $g^{-1}(K)$ is $\mathfrak{N}geo$ in V, since g is $\mathfrak{N}geIrr$ and $f^{-1}(g^{-1}(K)) = (g\circ f)^{-1}(K)$ is $\mathfrak{N}geo$ in U, clearly f is $\mathfrak{N}geIrr$. Hence $g\circ f$ is $\mathfrak{N}geIrr$.

The map $\mathfrak{N}g\mathcal{M}Irr$ is also satisfy the Theorem 3.3.

Theorem 3.4 *[(i)]*

1. If $f:(U,\tau_R(P)) \to (V,\sigma_{R'}(Q))$ is $\mathfrak{N}geIrr$ & $g:(V,\sigma_{R'}(Q)) \to (W,\mu_{R''}(S))$ is $\mathfrak{N}geCts$, then $g\circ f:(U,\tau_R(P)) \to (W,\mu_{R''}(S))$ is $\mathfrak{N}geCts$.
2. If $f:(U,\tau_R(P)) \to (V,\sigma_{R'}(Q))$ is $\mathfrak{N}geCts$ & $g:(V,\sigma_{R'}(Q)) \to (W,\mu_{R''}(S))$ is $\mathfrak{N}Cts$, then $g\circ f:(U,\tau_R(P)) \to (W,\mu_{R''}(S))$ is $\mathfrak{N}geCts$.

The map $\mathfrak{N}g\mathcal{M}Irr$ is also satisfy the Theorem 3.4.

Theorem 3.5 *If* $f:(U,\tau_R(P)) \to (V,\sigma_{R'}(Q))$ *is* $\mathfrak{N}geIrr$ & $g:(V,\sigma_{R'}(Q)) \to (W,\mu_{R''}(S))$ *is* $\mathfrak{N}eCts$, *then* $g\circ f:(U,\tau_R(P)) \to (W,\mu_{R''}(S))$ *is* $\mathfrak{N}geCts$.

Proof. Let K be $\mathfrak{N}o$ in W. Then $g^{-1}(K)$ is $\mathfrak{N}eo$ in V, clearly g is $\mathfrak{N}eCts$. Thus $g^{-1}(K)$ is $\mathfrak{N}geo$ in V, clearly every $\mathfrak{N}eo$ set is $\mathfrak{N}geo$ and $f^{-1}(g^{-1}(K)) = (g\circ f)^{-1}(K)$ is $\mathfrak{N}geo$ in U, since f is $\mathfrak{N}geIrr$. Hence $g\circ f$ is $\mathfrak{N}geCts$.

The map $\mathfrak{N}g\mathcal{M}Irr$ is also satisfy the Theorem 3.5.

Theorem 3.6 *If* $f:(U,\tau_R(P)) \to (V,\sigma_{R'}(Q))$ *is* $\mathfrak{N}geIrr$ & $g:(V,\sigma_{R'}(Q)) \to (W,\mu_{R''}(S))$ *is* $\mathfrak{N}gCts$, *then* $g\circ f:(U,\tau_R(P)) \to (W,\mu_{R''}(S))$ *is* $\mathfrak{N}geCts$.

Proof. Proof is similar from the Theorem 3.5.

Proposition 3.1 *Let* $f:(U,\tau_R(P)) \to (V,\sigma_{R'}(Q))$ *be a function. If* f *is [(i)]*

1. $\mathfrak{N}geIrr$, then it is $\mathfrak{N}ge^*Cts$.
2. $\mathfrak{N}ge^*Cts$, then it is $\mathfrak{N}geCts$.

Proof. The proof is immediate.

Remark 3.1 *Every* $\mathfrak{N}eIrr$ *function is* $\mathfrak{N}ge^*Cts$. *But not conversely.*

Example 3.3 *In Example 3.1,* f *is* $\mathfrak{N}ge^*Cts$, *but* f *is not* $\mathfrak{N}ge^*Irr$, *since* $f^{-1}(\{l,n\}) = \{l,n\}$ *which is not* $\mathfrak{N}gec$ *in* U *whereas* $\{l,n\}$ *is* $\mathfrak{N}gec$ *in* V.

Theorem 3.7 *Let* $f:(U,\tau_R(P)) \to (V,\sigma_{R'}(Q))$ & $g:(V,\sigma_{R'}(Q)) \to (W,\mu_{R''}(S))$ *be any two maps. Then if* f *is* $\mathfrak{N}ge^*Cts$ & g *is* $\mathfrak{N}eIrr$, *then* $g\circ f:(U,\tau_R(P)) \to (W,\mu_{R''}(S))$ *is* $\mathfrak{N}ge^*Cts$.

Theorem 3.8 *Let* $f:(U,\tau_R(P)) \to (V,\sigma_{R'}(Q))$ & $g:(V,\sigma_{R'}(Q)) \to (W,\mu_{R''}(S))$ *be any two functions. [(i)]*

1. If f is $\mathfrak{N}geIrr$ & g is $\mathfrak{N}geCts$, then $g\circ f$ is $\mathfrak{N}geCts$.
2. If f is $\mathfrak{N}geIrr$ & g is $\mathfrak{N}ge^*Cts$, then $g\circ f$ is $\mathfrak{N}ge^*Cts$.

CONCLUSION

In this paper, $\mathfrak{N}geCts$, $\mathfrak{N}g\mathcal{M}Cts$ and their respective irresoluteness are studied and many interesting examples in various forms are discussed.

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