

Improved Decoding of Linear Block Codes by Concatenated Decoders

Moulay Seddiq El Kasmi Alaoui, Issam Abderrahman Joundan, Said Nouh, Abdelaziz Marzak

Abstract: *The use of decoding algorithms allows us to retrieve information after transmitting it over a noisy communication channel. Soft decision decoding is powerful in concatenation schemes that use two or more levels of decoding. In our case, we make a concatenation between the Hartmann & Rudolph (HR) algorithm as symbol-by-symbol decoder and the chase-2 algorithm that is word-to-word decoding algorithm.*

In this paper, we propose to combine two decoding algorithms for constructing a new one with more efficiency and less complexity. This work consists firstly to use the HR with a reduced number of codewords of the dual code then the Chase-2 algorithm which exploits the output of PHR. The simulations results and the comparisons made show that the proposed decoding scheme guarantees very good performance with reduced temporal complexity.

Keywords: *Error correcting codes, Hartmann & Rudolph, Chase-2 algorithm, PHR Chase.*

I. INTRODUCTION

The accelerated use of computers and digital technology in our societies require high quality and reliability in the transmission and storage of data. Controlling information on computer networks, communication systems and storage media is a challenge for researchers and for the designers of modern communication systems; knowing that data is exchanged frequently using communication channels that are not entirely reliable, which can lead to errors. Therefore, the researchers have introduced error correcting codes. These latter add redundant bits in the transferred message to protect the useful data. A variety of error correcting codes are implemented in diverse devices such as Smartphone, compact discs (CDs), digital versatile discs (DVDs), hard disks or packets transferred over Interconnected Network (Internet) or over mobile networks.

Decoding an error correction code is an NP-hard problem [1, 2]. Generally, there are two types of decoders used in

communication systems: hard-decision and soft-decision decoders [3]. Given the difficulty of the problem, several linear code decoding algorithms are used to improve the measured performance as a function of bit error rate (BER). Among these, we find algebraic techniques such as: algorithms developed by solving non linear equations with several variables obtained from the identities of Newton [4]-[6], an algorithm that uses irreducible generator polynomials to decode the quadratic residue (QR) code (47, 24, 11) [7], the Berlekamp-Massey algorithm [8, 9] which is based on the syndrome calculation and the definition of a polynomial error locator. There is also the OSD algorithm of Fossorier et al [10], the algorithm of Chase [11] and the algorithm of Hartmann Rudolf [12]. However, the algebraic methods mentioned above require a large number of calculation operations, in terms of sum and product, in the Galois Field with q element $GF(q)$. This makes their implementation in real-time systems very difficult where algorithms with a speed of correction are required. To address this problem, several researchers have focused their efforts on developing heuristic algorithms to detect and correct transmission errors accurately and within a reasonable time.

Among the methods that exploit non-algebraic techniques we find: methods that exploit permutations [13]-[16], others that use genetic algorithms [17]-[22]. In [23]-[25], the authors proposed the syndrome calculation with Chien search to decode some BCH codes. There are many articles that present deep learning algorithms for error correction [26]-[29]. Several works use hashing techniques to speed up the decoding process and therefore have reduced complexity in execution time [30]-[35]. There are also decoders developed by serial concatenation of two decoders [16, 34, 35, 36].

In this paper, we propose to combine two decoding algorithms for constructing a new one with more efficiency and less complexity to decode linear block codes. The remainder of this paper is structured as follows. In section 2 we present the proposed serial concatenation schema between HR and Chase-2 algorithms. In section 3, we present the experimental results of the proposed decoder and we make comparisons with some competitors. In section 4, we study the temporal complexity of the proposed algorithm. Finally, a conclusion is outlined in section 5.

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II. THE PROPOSED DECODING SCHEME

A. Hartmann & Rudolph decoder

The HR decoder [12] is a symbol by symbol decoder, it is based on a probabilistic study to determine if the bit r_j , of the received sequence r , equals to 0 or 1. To perform this, it exploits all the codewords, of order 2^{n-k} , of the dual code. The use of 2^{n-k} dual codewords makes its temporal complexity very high, of exponential order $O(n^2 2^{n-k})$, and therefore unusable for codes with a reduced code rate. The formula 1 represents the method proposed by HR to decide if the m^{th} bit of the decoded word c' is equal to 1 or 0 from the received sequence r .

$$\begin{cases} c'_m = 0 & \text{if } \sum_{j=1}^{2^{n-k}} \prod_{l=1}^n \left(\frac{1-\phi_l}{1+\phi_l} \right)^{c_{jl}^{\perp} \oplus \delta_{ml}} > 0 \\ c'_m = 1 & \text{otherwise} \end{cases} \quad (1)$$

$$\text{Where } \delta_{ij} = \begin{cases} 1 & \text{if } i = j \\ 0 & \text{otherwise} \end{cases} \quad \text{and } \phi_m = \frac{\Pr(r_m | 1)}{\Pr(r_m | 0)}$$

The bit c_{jl}^{\perp} denotes the l^{th} bit of the j^{th} codeword of the code C^{\perp}

B. Chase-2 algorithm

The Chase-2 [11] decoding algorithm is an efficient soft input hard output decoder. It is based on generation of a binary word h from the real word received r using the formula 2 and creates 2^t test sequences from h by inverting some bits among the t least reliable bits. Each test sequence is then decoded by a hard decision decoder. The codeword selected is the word of smaller metric among the 2^t decoded words.

From this algorithm, we deduce that the temporal complexity increases exponentially with error correcting capability t of the studied code. The temporal complexity of Chase-2 algorithm is $2^t O(HD)$, where $O(HD)$ is the temporal complexity of the used hard decision decoder.

$$\forall 1 \leq i \leq n, h_i = \begin{cases} 1 & \text{if } r_i > 0 \\ 0 & \text{otherwise} \end{cases} \quad (2)$$

The Chase-2 decoding algorithm works as follows:

```

1  Function Chase(r, t)
   Input
2     ✓ r: Non binary word to decode of length n
   ✓ t: Error correcting capability
   Output
3     ✓ c: Corrected word
   Begin
4     Identify the t least reliable positions
5     Calculate h, the binary version of r
6      $d_{\min} \leftarrow -\text{infinity}$ 
7     For i = 1 to  $2^t$  do
8         Generate  $e_i$ , an error pattern that consider the t
9         identified positions
10         $c_i \leftarrow h + e_i$ 
11         $c_d \leftarrow$  Decode  $c_i$  with a hard decision decoder
12        If  $c_d$  is a code word of C then
13            If  $\text{Ed}(r, c_d) < d_{\min}$  then
14                 $c \leftarrow c_d$ 
15                 $d_{\min} \leftarrow \text{Ed}(r, c_d)$ 
16            EndIf
17        EndIf
18    EndFor
19    End Function

```

Algorithm 1: Chase-2 decoding algorithm function

C. The proposed decoder

Recently, we find several decoding algorithms that are defined by concatenating two serial decoders [16, 33, 35, 36]. In [16], the authors proposed to use the HR algorithm partially with a reduced number of dual code words and correct just symbols whose reliability is below a threshold, then to use a second decoder to complete the decoding of the recovered sequence at the output of the HR decoder. In their work, the authors studied the impact of the number of code word M of the dual code used and the reliability threshold (RT) on error correction performance. Following this study, we deduce that a reduced number of words of the dual code gives very poor results which improve relatively with the increase of the number of words of the dual code; for the reliability threshold, they found that good performance can be achieved with a minimum threshold of 0,35.

By integrating parameters M and RT into formula 1, we propose the following definition of the HR partial decoder function.

```

1  Function PHR(r, RT, M, LCD)
   input :
2     ✓ r : Non binary word to decode of length n
3     ✓ RT : Reliability threshold
4     ✓ M : Number of dual code words to use in
   the decoding with the HR decoder
5     ✓ LCD : List of M dual codewords
   output :
6     ✓ b : Partially decoded word
   Début
7     For i=1 to n do
8         If  $|r[i]| \leq RT$  then
9              $s \leftarrow 0$ 
10            For j=1 to M do
11                 $p \leftarrow 0$ 
12                For k=1 to n do
13                    Compute  $\phi_k$ 
14                    If i=k then
15                         $p \leftarrow p * \left( \frac{1-\phi_k}{1+\phi_k} \right)^{LCD[j][k] \oplus 1}$ 
16                    Else
17                         $p \leftarrow p * \left( \frac{1-\phi_k}{1+\phi_k} \right)^{LCD[j][k] \oplus 0}$ 
18                    End If
19                End For
20                 $s \leftarrow s + p$ 
21            End For
22            If  $s > 0$  then
23                 $b[i] \leftarrow 0$ 
24            Else
25                 $b[i] \leftarrow 1$ 
26            End If
27        End If
28    End For
29    End Function

```

Algorithm 2: HR Partial decoder function

The concatenation scheme proposed in this work is based on the following three steps:

- The first one consists in correcting the low reliability symbols, precisely those whose value is below a reliability threshold: use of the HR decoder in a partial manner.
- The second one is to prepare a real sequence for the Chase-2 decoder. For this, we propose the attribution of the artificial reliabilities to the symbols of the sequence returned by the decoder HR which present an abnormal aspect: use of the formula (3).
- The third step is to definitively decode the sequence prepared in the previous step by the Chase-2 decoder.

The allocation of artificial reliability is carried out according to formula 3:

$$\forall 1 \leq i \leq n, r_{2i} \leftarrow -r_i \times \text{ART if } r_i < 0 \text{ and } b_i = 1 \text{ or } r_i > 0 \text{ and } b_i = 0 \quad (3)$$

Where

- r is the received sequence of length n to be decoded.
- b is the binary version of r returned by PHR function.
- ART is the artificial reliability threshold used to convert b to r_2 .
- r_2 is the non binary sequence of length n resulting from the assignment of reliability thresholds to the different symbols of b .

The function BinToReal works as follows:

```

1  Function BinToReal(r, b, ART)
   Input :
2     ✓ r : Non binary received sequence of length n
3     ✓ b : Binary word of length n
4     ✓ ART : Artificial reliability threshold
   Output :
5     ✓ r2 : Non binary word of length n
6  Begin
7     For i=1 to n do
8         If r[i]<0 and b[i]=1 or r[i]>0 and b[i]=0 then
9             r2[i]← -r[i] * ART
10        End If
11    End For
12 End Function

```

Algorithm 3: BinToReal function

Then the proposed algorithm is as follows:

```

Input:
1  ✓ r : Non binary word to decode of length n
2  ✓ RT : Reliability threshold used in the PHR function
3  ✓ ART : Artificial reliability threshold used in the BinToReal function
4  ✓ M : Number of dual code words to use in the decoding with the HR decoder
5  ✓ LCD : List of M dual codewords
6  ✓ t: Error correcting capability
7  Output :
8  ✓ c : Corrected word
9  Begin
10 b ← PHR(r, RT, M, LCD)
11 r2←BinToReal(r, b, ART)
12 c ←Chase(r2, t)
13 End

```

Algorithm 4: The proposed PHR Chase decoding algorithm

III. EXPERIMENT RESULTS AND COMPARISON

In this section, we give the performances of the PHR Chase algorithm for some linear code and a comparison with other decoding algorithms over an AWGN binary channel (Additive White Gaussian Noise) with a BPSK (Binary Phase Shift Keying) modulation is done.

A. What is the good value of the artificial reliability threshold?

In order to apply the PHR Chase algorithm, we propose to study the impact of the choice of the value of the artificial reliability threshold (ART) on the quality of its results. For this, we applied the algorithm for QR(31, 16, 7), BCH(31, 16, 7) and BCH(31, 21, 5) codes. The error correcting performances will be represented in terms of Bit Error Rate (BER) in each Signal to Noise Ratio (SNR= E_b/N_0). Table-I gives the used simulations parameters.

Table-I: Default simulation parameters.

Simulation parameters	value
Channel	AWGN
Modulation	BPSK
Minimum number of residual errors	200
Minimum number of transmitted blocks	1000

In Fig. 1(a), we plot the effect of the variation of ART from -5 to 5 on the quality of the correction results of the QR(31, 16, 7) code. From this Fig. we deduce that the performances of the proposed decoder are very bad with negative values of ART; on the other hand, when they take positive values we notice the great improvement in the results.

Thus, we notice that the best results are those obtained with values of ART between 0 and 1, for that we plot in Fig. 1(b) the performances of the decoder applied to the same code for values of ART between 0 and 1 but with step of 0,25 in this case.

From Fig. 1(b), we can conclude that the best error correction performance is obtained with a value of ART equal to 0,5.

Similarly, in Fig. 2(a) and 2(b), we plot the error correction performance of the proposed decoding scheme applied to BCH(31, 16, 7) code for values of ART between -5 and 5 with step equal to 1 and those between 1 and 4 with step equal to 0,5.

From Fig. 2(a), we confirm the deduced remark by applying the proposed decoder to QR(31, 16, 7) code with negative values of artificial reliability. The best correction results are those obtained with positive values of ART and precisely with those between 1 and 4.

From Fig. 2(b) where we have focused on SNR values between 3 dB and 5 dB, we note that the best Bit Error Rate (BER) are those obtained with an artificial reliability value equal to 2,5.

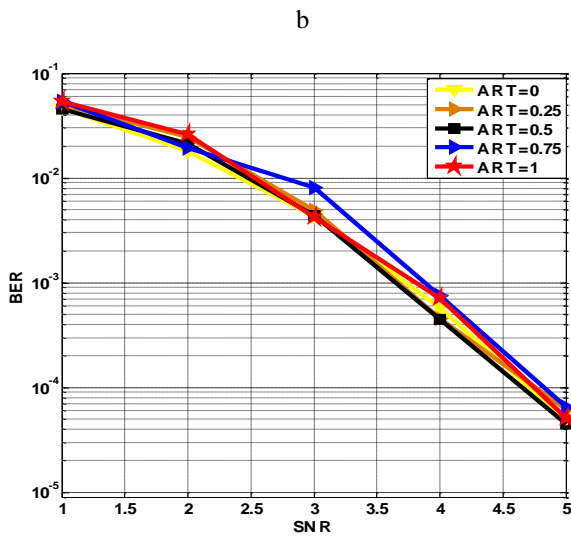
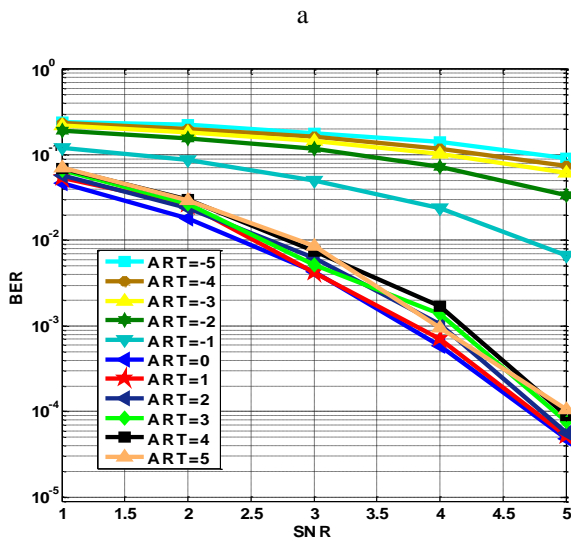


Fig. 1 : Impact of ART on the performances of the PHR Chase decoder applied to QR(31, 16, 7) code for ART between (a) -5 and 5 with step=1; (b) 0 and 1 with step=0.25

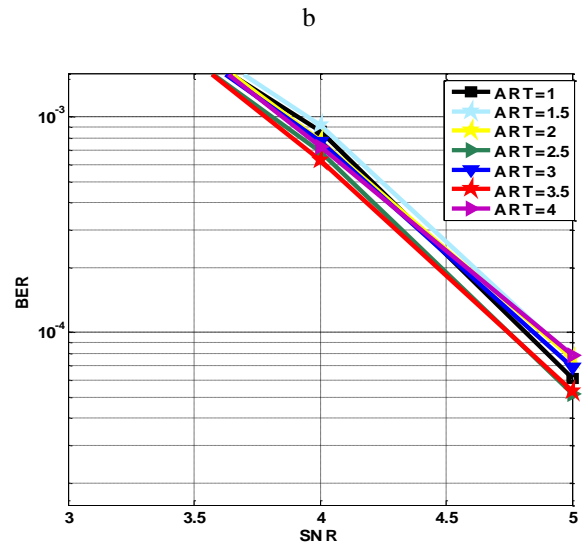
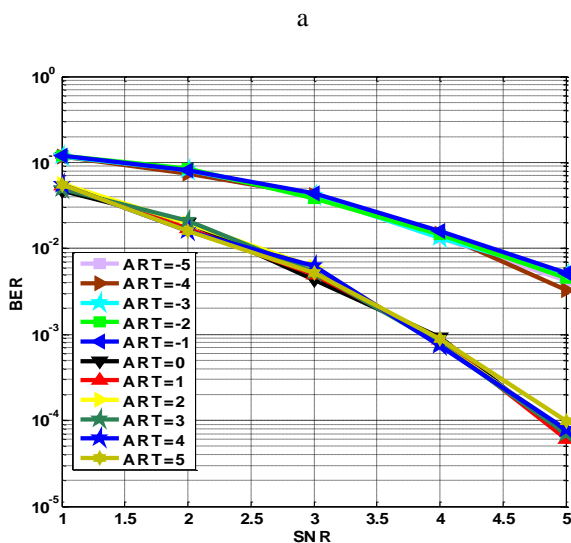


Fig. 2: Impact of ART on the performances of the PHR Chase decoder applied to BCH(31, 16, 7) code for ART between (a) -5 and 5 with step=1; (b) 1 and 4 with step=0.5

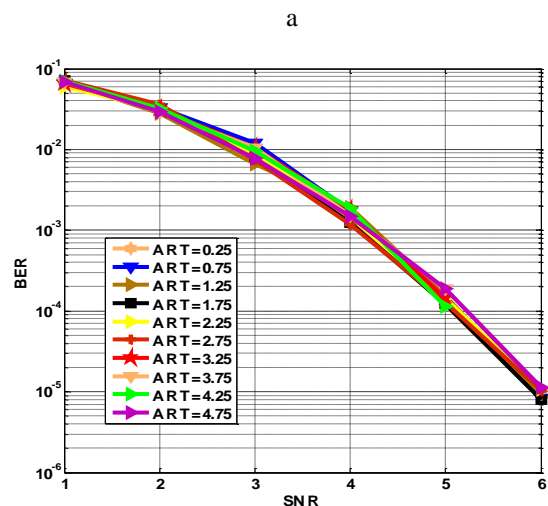


Fig. 3 : Impact of ART on the performances of the PHR Chase decoder applied to BCH(31, 21, 5) code for ART between 0.25 et 4.75 with step = 0.5 for SNR values (a) between 1 dB and 6 dB ; (b) between 3.5 dB and 6 dB.

In Fig. 3(a) and 3(b), we plot the impact of parameter ART on the performances of the PHR Chase decoder applied to the BCH(31, 21, 5) code for ART values between 0,25 and 4,75 with step equal to 0,5. From Fig. 3(b), we notice that the best performances are obtained with ART value equal to 1,75.

From Fig. 1, 2 and 3, we deduce that the best error correction performances are those obtained with positive values of artificial reliability between 0,5 and 2,5.

B. Simulation results and comparison with other decoders

In this section, we present the simulation results of PHR Chase algorithm applied to several linear codes and we compare its performances with some competitor decoders.

Fig. 4(a) and 4(b) respectively represent the PHR Chase performances for some BCH codes of lengths 31 and 63. These Fig. show that the coding gain guaranteed by PHR Chase is approximately 3,7 dB for BCH(31, 21, 5), 3,9 dB for BCH(31, 16, 7), 4,1 dB for BCH(63, 51, 5), 4,6 dB for BCH(63, 45, 7) and 4,9 dB for BCH(63, 39, 9). We also note that from SNR = 2, BCH(31, 16, 7) code gives a coding gain of about 0,2 dB comparing to BCH(31, 21, 5). As well as the BCH(63, 39, 9) guarantees performances which respectively exceed those of BCH(63, 45, 7) by 0,3 dB and those of BCH(63, 51, 5) by 0.8 dB for BER=10⁻⁵.

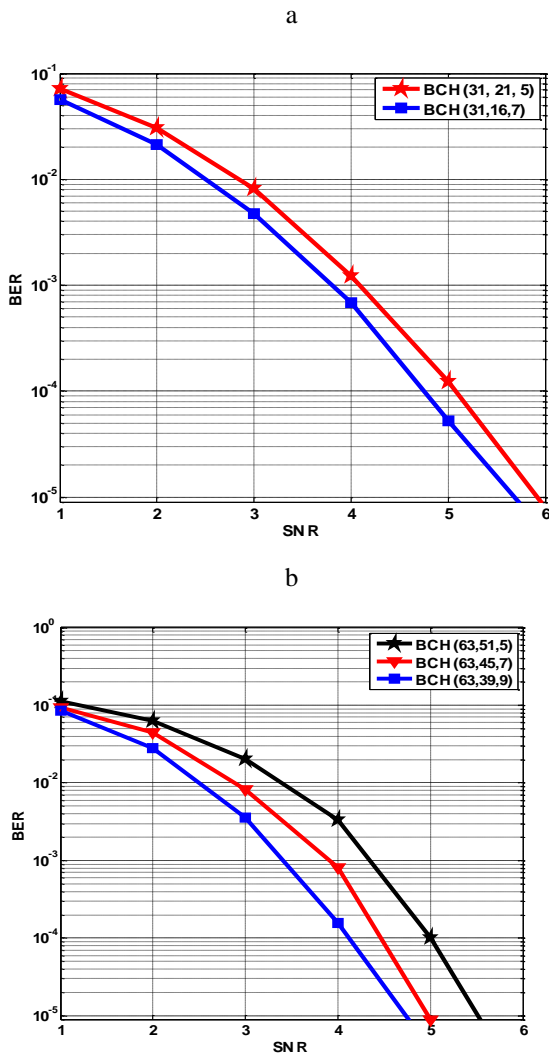


Fig. 4 : PHR Chase performances for some BCH codes of lengths (a) 31; (b) 63.

In Fig. 5, we present the performances of our PHR Chase decoder for some quadratic residue (QR) codes of lengths up to 71 and we deduce that the coding gain is about 3,8 dB for QR(23, 12, 7), 4 dB for QR(31, 16, 7), 4,8 dB for QR(47, 24, 11) and 5.1 dB for QR(71, 36, 11). Thus, we deduce that the correction performances improve with the length of the code, for example the QR(71, 36, 11) code guarantees a coding gain of 1,3 dB comparing to QR(23, 12, 7) code.

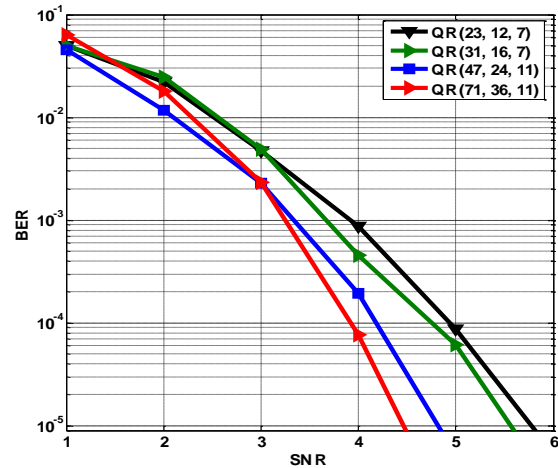


Fig. 5: PHR Chase performances for some QR codes of lengths up to 71.

In order to show the efficiency of our proposed concatenation scheme, we plot the results of performance comparisons guaranteed by PHR Chase with those of several other competitors. In Fig. 6(a), we plot the error correction performances of PHR Chase, SD1 [23], PHR-ARDecGA [35], Chase-HSDec [33], PHR-SPDA [35] and ocGAD [37] for the BCH(63, 51, 5) code. From this Fig. we deduce that from SNR equal to 4 dB, the correction performances of PHR Chase exceed those of all other competitors studied for this code. Also, a coding gain of 0,5 dB is guaranteed by PHR Chase compared to the first successor who is ocGAD.

The performance comparison results of the PHR Chase decoder with those of cGAD, PHR-HSDec, PHR-SPDA, PHR-BM and SDHT for BCH(63, 45, 7) code have been plotted in Fig. 6(b); this Fig. shows that the correction performances provided by PHR Chase are the same as those of PHR-HSDec and they exceed those guaranteed by the other decoders. For example for a BER=10⁻⁵, PHR Chase ensures a coding gain of about 1,1 dB compared to SDHT and PHR-SPDA decoders and for a BER=10⁻⁴, PHR Chase guarantees a coding gain of approximately 1.6 dB compared to cGAD decoder.

In Fig. 7(a) and 7(b), we represent respectively a comparison of the performances of the PHR Chase, PHR-SPDA and Chase-HSDec decoders for the BCH(63, 39, 9) code and another for PHR Chase, Chase HSDec, HR and SDHT for the QR(31, 16, 7) code.

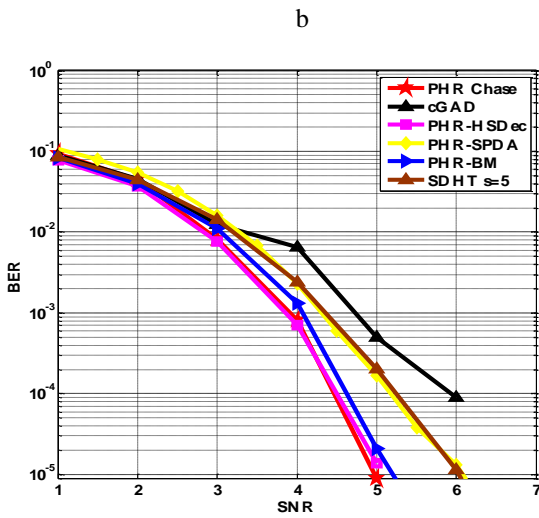
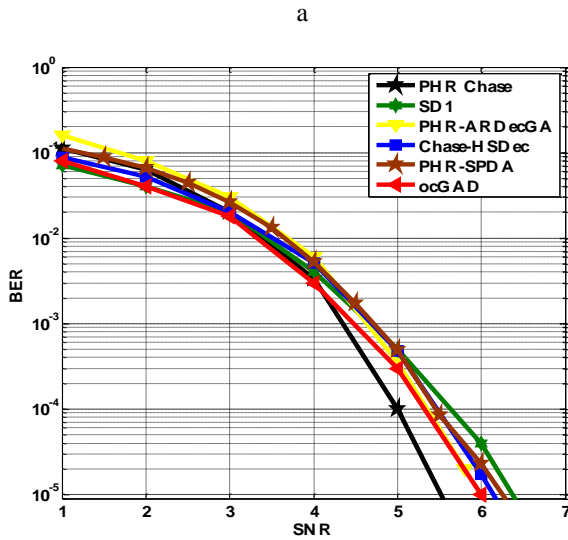


Fig. 6 : Performances comparison of PHR Chase with (a) SD1, PHR-ARDecGA, Chase-HSDec, PHR-SPDA and ocGAD decoders for BCH(63, 51, 5) code; (b) cGAD, PHR-HSDec, PHR-SPDA, PHR-BM, SDHT $s = 5$ decoders for BCH(63, 45, 7) code

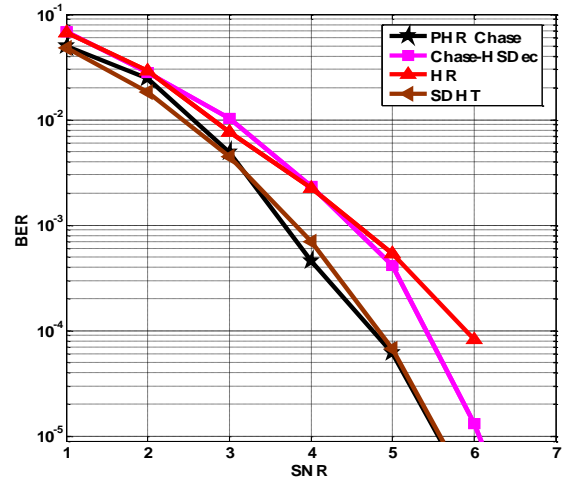
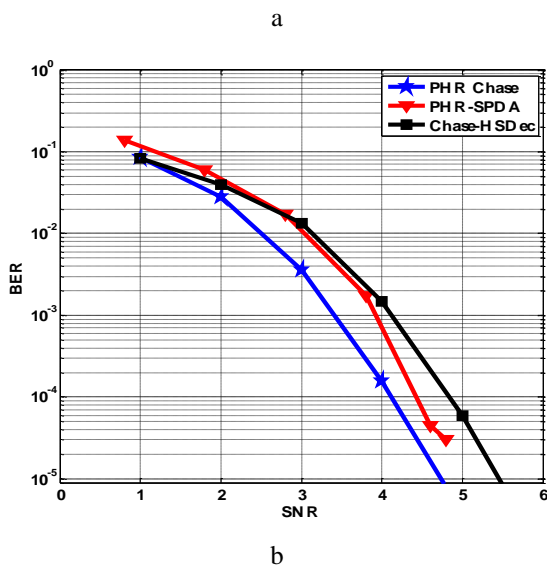


Fig. 7 : Performances comparison of the PHR Chase with (a) PHR-SPDA and Chase-HSDec decoders for BCH(63, 39, 9) code; (b) Chase-HSDec, HR and SDHT decoders for QR(31, 16, 7) code.

In Fig. 8, we plot the comparison results of the error correction performance of the PHR Chase, PHR-SPDA, cGAD, and Shakeel [38] decoders for the quadratic residue code of length 71. From this Fig., we deduce that the PHR Chase and PHR-SPDA decoders guarantee the same performances for the studied code and their performances far exceed that of other competitors. For example, the coding gain guaranteed by PHR Chase and PHR-SPDA is about 1,6 dB for BER=10⁻⁴ compared to competitors.

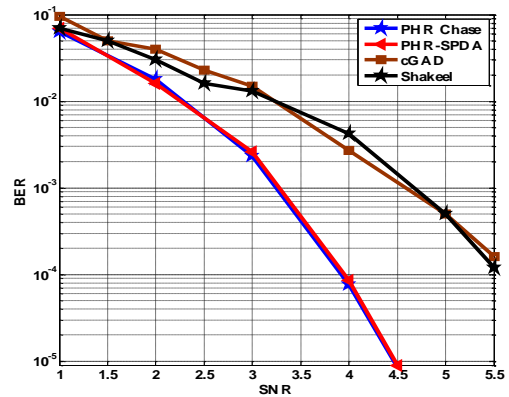


Fig. 8 : Performances comparison of the PHR Chase with PHR-SPDA, cGAD and Shakeel decoders for QR(71, 36, 11) code.

In order to show the importance of the proposed concatenation, we have studied the performances of the HR without Chase, those of Chase without HR and those of the proposed concatenation for BCH(31, 21, 5), BCH(31, 16, 7), BCH(63, 51, 5) and QR(31, 16, 7) codes. In Fig. 9(a), 9(b), 10(a) and 10(b), we compare the decoding qualities of HR, Chase-2 and PHR Chase decoders respectively for BCH(31, 21, 5), BCH(31, 16, 7), BCH(63, 51, 5) and QR(31, 16, 7) codes.

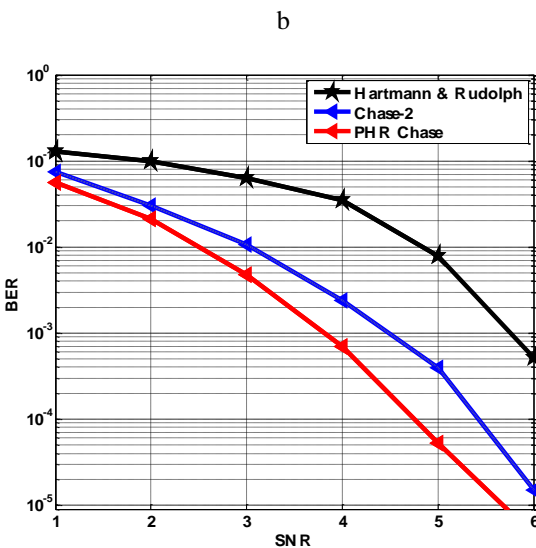
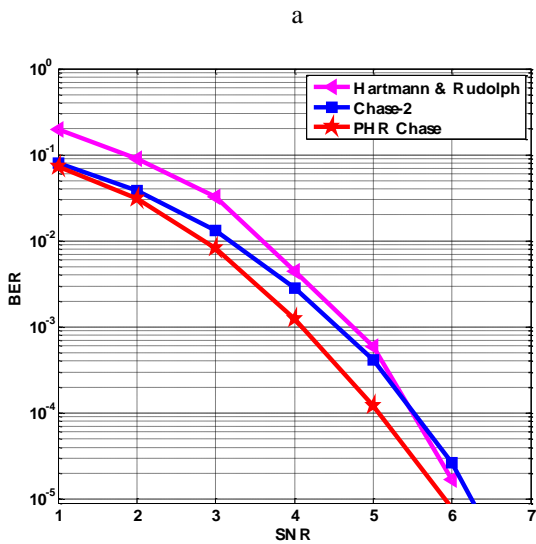


Fig. 9 : Performances comparison of HR, Chase-2 and PHR Chase decoders for (a) BCH(31, 21, 5) code; (b) BCH(31, 16, 7) code

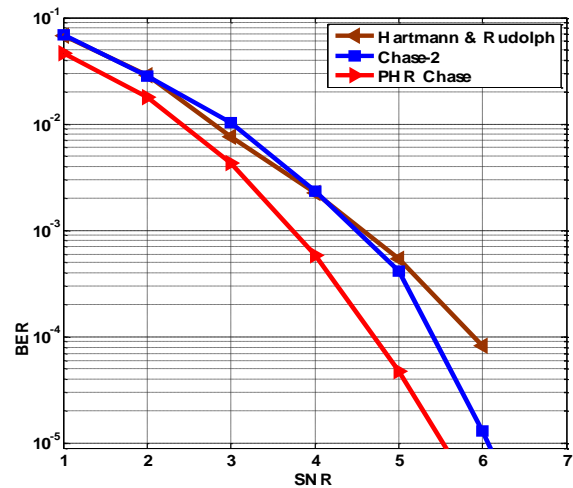
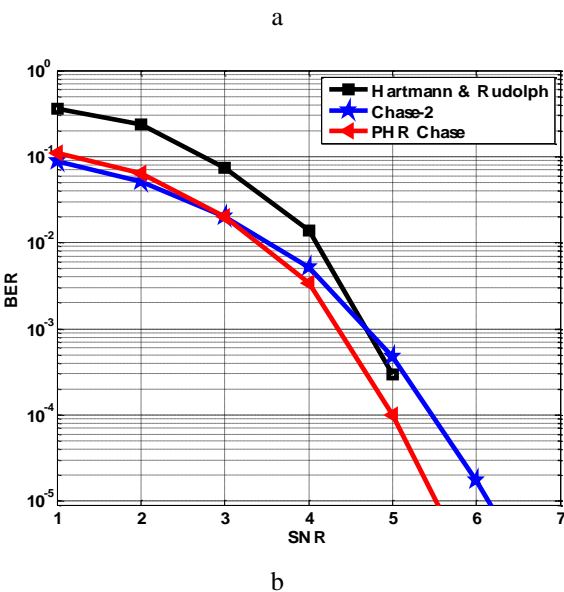


Fig. 10 : Performances comparison of HR, Chase-2 and PHR Chase decoders for (a) BCH(63, 51, 5) code; (b) QR(31, 16, 7) code

From Fig. 9 and 10, we deduce that with the serial use of the two decoders we can amply improve their error correction performances compared to the case where each decoder is used individually.

The best performance guaranteed by the concatenation scheme and also the added value of using two serial decoders clearly show the success of this idea.

IV. TEMPORAL COMPLEXITY STUDY

Fig. 9 and 10 show that we have been able to guarantee much better performances than in the case of using Chase-2 or HR algorithms individually. The temporal complexity of the PHR Chase decoder equals the sum of those of the decoders that compose it. The complexity of PHR Chase equals to the sum of the complexities of HR partially exploited and that of Chase-2.

Let M be the number of dual code word used in the HR decoding, t the code correction capability and $C(HD)$ is the complexity of the HIHO decoder used in the Chase-2 algorithm, hence the temporal complexity of the proposed decoding scheme is:

$$C(\text{PHR Chase}) = C(\text{PHR}) + C(\text{Chase-2}) = O(Mn^2 + 2^t C(HD)) \quad (4)$$

From the formula 4, we deduce that the complexity depends on the three parameters M , t and $C(HD)$, for this reason we propose to study practically the required execution time (E.T.) to execute the decoder resulting from the concatenation and that of HR decoder executed individually i.e. with all the code words of the dual code.

The results of this study are shown in Fig. 11 and 12 where we have respectively plotted the evolution of the execution time ratio (ETR)

$$\text{ETR} = \frac{\text{E.T. of PHR Chase algorithm}}{\text{E.T. of HR algorithm}}, \text{ and the rate of}$$

reduction of the execution time (RRET) guaranteed by the proposed decoding scheme for the BCH(31, 21, 5), BCH(31, 16, 7), BCH(63, 51, 5) and QR(31, 16, 7) codes with SNR values vary between 1 dB and 5 dB.

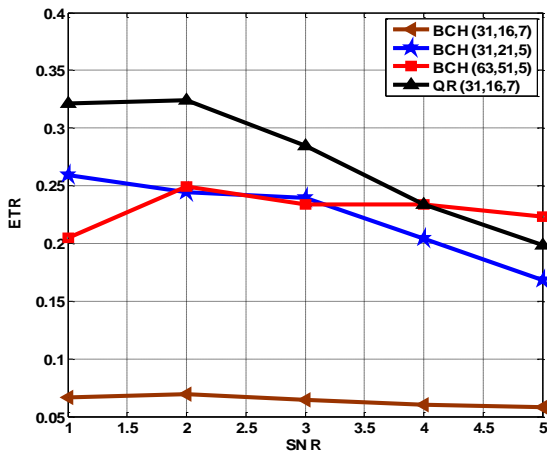


Fig. 11 : ETR evolution for BCH(31, 21, 5), BCH(31, 16, 7), BCH(63, 51, 5) and QR(31, 16, 7) codes.

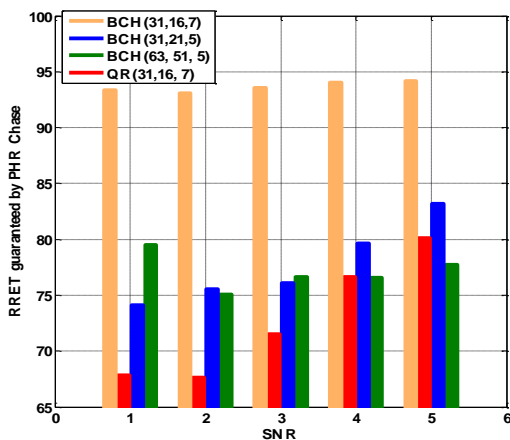


Fig. 12 : RRET guaranteed by PHR Chase for BCH(31, 16, 7), BCH(31, 21, 5), BCH(63, 51, 5) and QR(31, 16, 7) codes

From Fig. 12, we can notice that with the proposed concatenation scheme, the rate of reduction of the execution time (RRET) is:

- Reduced by about 94% for BCH(31, 16, 7) code, and it has a stable appearance with SNR variation.
- Between 74% and 83% for BCH(31, 21, 5) code, and it is growing in parallel with the values of SNR.
- Between 68% and 81% for QR(31, 16, 7) code, and it is growing in parallel with the values of SNR.
- Between 75% and 79% for BCH(63, 51, 5) code.

The RRET comparison results clearly show the effect of the proposed concatenation on the execution time of the HR algorithm for all codes studied.

From the analysis of error correction performance, comparisons with competitors and the study of the temporal complexity of the proposed decoding scheme we can confirm the great success of the concatenation idea.

V. CONCLUSION

In this paper, we have presented a fast and efficient decoder developed from a serial concatenation between the Hartmann and Rudolph algorithm and Chase-2 algorithm; we have applied it successfully to decode several linear codes. The simulation and comparison results show that the proposed

PHR Chase guarantees very good performances compared to some competitors. The number of used codewords in the decoding process is very small, which has allowed us to alleviate in a very powerful way the temporal complexity; for example, by applying our decoding scheme to the BCH(31, 16, 7) code, we were able to reduce the execution time of the HR algorithm by 94%. The best results of PHR Chase will open new way for the artificial intelligence algorithms in the coding theory field.

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