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Weighted Fuzzy Sets

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***Abstract**: The weighted fuzzy subset is to describe the importance of each element in the set through the characteristi3c function. In this paper, we study how an element and its presence in the set, namely, the degree of belongingness plays a role in determining the characteristic level of the set. Presence of an element in the set strengthens the set to a greater extent. Set gets its weightage, because of its elements and its association with the other elements. Association depicts the internal and external factors influencing over an element. In other words, the proposed weighted fuzzy subset has 3-tuple representation such as elements in a set, degree of presence of an elements and degree of impact of the set because of each elements presence.*

***Keywords**: Weighted fuzzy subset, Characteristic level of the set, Strong fuzzy Set, Weak fuzzy set.*

I. INTRODUCTION

In classical set, the characteristic function of a set takes values 0 and 1. The characteristic function in classical set theory describes whether the elements belong to the set or do not belong to the set. Classical set doesn't explain about degree of belongingness or non-belongingness of an element. However, classical structures are developed based on classical set and bi-valued logic which is not able to handle uncertain and partial information. Fuzzy systems have succeeded to provide solutions to many real world problems.

In 1965 Zadeh [7] introduced the concept of fuzzy set theory. A fuzzy set is a generalized set to which objects can belong to a set with various degrees of membership over the interval [0,1]. Zadeh [8] introduced the concept of type-2 fuzzy set and type-n fuzzy set which explains about fuzzy sets and whose membership grades themselves are of type-1 fuzzy sets. Followed by, Atanassov [6] introduced intuitionistic fuzzy sets which explains non-membership and membership of an element in the set. Yager [9] introduced new fuzzy set namely called fuzzy multisets which allow repetition of the membership values. Torra [10] introduced the idea of Hesitancy fuzzy sets which explains about the possible membership values of an each element in the set. T. Pathinathan and E. Mike Dison [3,4] introduced the rotational fuzzy set model which is applied in a real life situation where linguistic predicates may have a relative group of meaning or synonyms. Also rotational fuzzy model describes about two truth functions such as true and false are merged into a single function as a transformation observed from growing in one formal onto the other.

In any real life phenomenon, the degree of belongingness of an element in the set is hard to measure and enumerate. Zadeh's axiomatic theory on fuzzy sets helped us to reduce the complexity. The weightage of an element in a set has greater importance than the belongingness of an element in a set. Conceptually weightage of an element in a set and belongingness of an element in a set differs. Weightage of an element in a set determines the strategy of the set. In other words, presence of a particular element in a set and its impact over its presence strengthen the set characteristics.

Though fuzzy sets give importance to the concept of membership of elements in a particular set, they do not explain the impact of an element in the set. Through this paper, we find weighted fuzzy subset with reference to the membership of the elements in it. We are characterizing a fuzzy subset with the membership of its elements and are studying the weighted set by considering the belongingness of the elements in that set by giving a functional value between the membership of the elements and the existence level of a fuzzy subset. For example, consider earning members in a particular family, suppose maximum number of members in the family earn good amount of money then it shows that family is financially strong. In the same way, maximum number of members in the family do not earn good amount of money then it shows that family is financially weak.

The paper is organized as follows: Section one provides the introduction about various fuzzy sets. Section two presents the preliminaries of different types of fuzzy sets. Section three presents the definition of weighted fuzzy subset and operations of weighted fuzzy subset with numerical examples and then the conclusion.

II. PRELIMINARIES

A. Type-2 fuzzy sets [8]

A type-2 fuzzy set $\underset{\sim}{A}$ is defined by the membership function $\mu_{\underset{\sim}{A}}(a,x)$ is given by, where $a \in X$ and $x \in J_a \subseteq [0,1]$,

$$\underset{\sim}{A} = \left\{ <(a,x), \mu_{\underset{\sim}{A}}(a,x) \backslash > / \forall a \in X, \forall x \in J_a \subseteq [0,1] \right\}$$

in which $0 \le \mu_{\underset{\sim}{A}}(a,x) \le 1$.

Also $\underset{\sim}{A}$ can be expressed as

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$\underset{\sim}{A}=\int_{a\in x}\int_{u\in J_a}\mu_{\underset{\sim}{A}}(a,x)/(a,x), J_a\subseteq[0,1]$, where $\iint$ is denotes the sum of all admissible a and X . For discrete universes of discourse $\int$ is replaced by $\sum$.

B. Extension Principle [2,8]

Let the fuzzy sets be $P_1,P_2,...,P_r$ respectively in $M_1,M_2,...,M_r$ and $f:M\to N$, $n=f(m_1,m_2,...,m_r)$, then the extension principle makes map to the fuzzy set M to N as follows,

$P=f(Q)$, here Q is the image of P and

$$Q=\{(n,\mu_Q(n))\backslash n=f(m_1,m_2,...,m_r),m_1,m_2,...,m_r\in M\}$$

$$\mu_Q(n)=\begin{cases}\max\limits_{(m_1,m_2,...,m_r)\in f^{-1}(n)}\wedge\left[\mu_{p_1}(m_1),...,\mu_{p_n}(m_n)\right], & f^{-1}(n)\neq 0\\ 0 & ,otherwise\end{cases}$$

III. WEIGHTED FUZZY SUBSET (WFS)

A. Concept of Weighted Fuzzy Subset – An Example

Consider the case: Let X be the set of all 15 cricket players who represent Indian cricket team. Assume that P be the set of 11 players who are selected by combinations and permutations from X . Let $a\in P$ be the set of all playing 11 in the team. Let $\mu_{W(\underset{\sim}{P})}(a)$ be the degree of presence of each player in the playing 11. Then $\eta_{a\in W(\underset{\sim}{P})}$ denotes the degree of impact of the team because of each player's presence. From the above case the set is defined $\{<a,\mu_{W(\underset{\sim}{P})}(a),\eta_{a\in W(\underset{\sim}{P})}>/a\in\underset{\sim}{P}\}$ and $0\leq\mu_{W(\underset{\sim}{P})}(a)\leq 1$ and $0\leq\eta_{a\in W(\underset{\sim}{P})}\leq 1$.

B. Weighted Fuzzy Subset

Let X be a universe of discourse, then a weighted fuzzy subset $W(\underset{\sim}{P})$ in X is defined by

$W(\underset{\sim}{P})=\{<a,\mu_{W(\underset{\sim}{P})}(a),\eta_{a\in W(\underset{\sim}{P})}>/a\in W(\underset{\sim}{P})\}$ and

$\mu_{W(\underset{\sim}{P})}(a):X\to[0,1]$ denotes the degree of presence of an elements

$\eta_{a\in W(\underset{\sim}{P})}:W(\underset{\sim}{A})\to[0,1]$ denotes the degree of impact of the set corresponding to each element presence in the set. Then, a weighted fuzzy subset is defined,

$$W(\underset{\sim}{P})=\begin{cases}1, & if\ \sum\limits_{i=1}^{m}\dfrac{\eta_{a\in W(\underset{\sim}{P})}}{m}\geq\dfrac{1}{2}\\ 0, & if\ \sum\limits_{i=1}^{m}\dfrac{\eta_{a\in W(\underset{\sim}{P})}}{m}<\dfrac{1}{2}\end{cases}$$

Note:

Where m denotes the number of elements in $W(\underset{\sim}{A})$. Here the value 1 represents the weighted fuzzy set is strong when the value is greater than or equal to 0.5 also 0 represents the weighted fuzzy set is week when the value is less than 0.5.

C. Example - Weighted Fuzzy Subset

Let a_1,a_2,a_3,a_4 and a_5 are members in a particular family. We consider earning member in a particular family $\underset{\sim}{P}=\{a_1,a_2,a_3,a_4,a_5\}$. Weighted Fuzzy Subset $\underset{\sim}{P}$ as

$$W(\underset{\sim}{P})=\left\{\begin{matrix}(a_1,0.8,0.7),(a_2,0.6,0.7),(a_3,0.9,0.8),\\(a_4,0.6,0.6),(a_5,0.5,0.6)\end{matrix}\right\}.$$

$$W(\underset{\sim}{P})=\sum_{i=1}^{5}\frac{\eta_{a_i\in W(\underset{\sim}{P})}}{5}=\frac{0.7+0.7+0.8+0.6+0.6}{5}$$

$$=0.68\geq 0.5$$

This shows that the family is financially strong.

IV. OPERATIONS ON WEIGHTED FUZZY SUBSETS

Here we verified set operations such as union, intersection and compliment for a weighted fuzzy subset with numerical examples by using Zadeh's [2,8] extension principle.

Let $W(\underset{\sim}{P})$ and $W(\underset{\sim}{Q})$ be two weighted fuzzy subsets. Then

$$W(\underset{\sim}{P})=\{<a,\mu_{W(\underset{\sim}{P})}(a),\eta_{a\in W(\underset{\sim}{P})}>/a\in W(\underset{\sim}{P})\}$$

$$W(\underset{\sim}{Q})=\{<a,\mu_{W(\underset{\sim}{Q})}(a),\eta_{a\in W(\underset{\sim}{Q})}>/a\in W(\underset{\sim}{Q})\}$$

A. Union on Weighted Fuzzy Subsets

Let us take two weighted fuzzy sets $W(\underset{\sim}{P})$ and $W(\underset{\sim}{Q})$, then the union of weighted fuzzy subset as follows

$$W(\underset{\sim}{P}\cup\underset{\sim}{Q})=\left\{\begin{matrix}<a,\left(\mu_{W(\underset{\sim}{P})}\vee\mu_{W(\underset{\sim}{Q})}\right),\\ \sup\limits_{\left(\mu_{W(\underset{\sim}{P})}\vee\mu_{W(\underset{\sim}{Q})}\right)}\left\{\eta_{a\in W(\underset{\sim}{P})}\wedge\eta_{a\in W(\underset{\sim}{Q})}\right\}>\end{matrix}\right\}$$

B. . Example – Union on Weighted Fuzzy Subsets

Let $W(\underset{\sim}{P})=\{(a_1,0.8,1),(a_2,1,0.5),(a_3,0.6,0.4)\}$ and $W(\underset{\sim}{Q})=\{(a_1,0.8,0.7),(a_2,1,0.9),(a_3,0.6,0.3)\}$ are two weighted fuzzy sets, then the union of two fuzzy set is as follows

$$W(\underset{\sim}{P}\cup\underset{\sim}{Q})=\left\{\begin{matrix}<a,\left(\mu_{W(\underset{\sim}{P})}\vee\mu_{W(\underset{\sim}{Q})}\right),\\ \sup\limits_{\left(\mu_{W(\underset{\sim}{P})}\vee\mu_{W(\underset{\sim}{Q})}\right)}\left\{\eta_{a\in W(\underset{\sim}{P})}\wedge\eta_{a\in W(\underset{\sim}{Q})}\right\}>\end{matrix}\right\}$$

We obtain $\left(\mu_{W(\underset{\sim}{P})}\vee\mu_{W(\underset{\sim}{Q})}\right)$ as $0.8,1,0.6$

Let $\left(\mu_{W(\underset{\sim}{P})}\vee\mu_{W(\underset{\sim}{Q})}\right)=0.8$,

$$\mu_{W(\underset{\sim}{P}\cup\underset{\sim}{Q})}(a)=\left\{\begin{matrix}<a,\left(\mu_{W(\underset{\sim}{P})}\vee\mu_{W(\underset{\sim}{Q})}\right),\\ \sup\limits_{\left(\mu_{W(\underset{\sim}{P})}\vee\mu_{W(\underset{\sim}{Q})}\right)}\left\{\eta_{a\in W(\underset{\sim}{P})}\wedge\eta_{a\in W(\underset{\sim}{Q})}\right\}>\end{matrix}\right\}$$

$$\mu_{W(\underset{\sim}{P}\cup\underset{\sim}{Q})}(a_1)=\left\{<a_1,\,0.8,\,\sup_{0.8}[0.7,0.3,0.4]>\right\}$$
$$=(a_1,0.8,0.7)$$

Let $\left(\mu_{W(\underset{\sim}{P})}\vee\mu_{W(\underset{\sim}{Q})}\right)=1$, then

$$\mu_{W(\underset{\sim}{P}\cup\underset{\sim}{Q})}(a_2)=\left\{<a_2,1,\,\sup_{1}[0.9,0.5,0.5,0.3,0.4]>\right\}$$
$$=(a_2,1,0.9)$$

Let $\left(\mu_{W(\underset{\sim}{P})}\vee\mu_{W(\underset{\sim}{Q})}\right)=0.6$, then

$$\mu_{W(\underset{\sim}{P}\cup\underset{\sim}{Q})}(a_3)=\left\{<a_3,\,0.6,\,\sup_{0.6}[0.3]>\right\}=(a_3,0.6,0.3)$$
$$W\left(\underset{\sim}{P}\cup\underset{\sim}{Q}\right)=\{(a_1,0.8,0.7),(a_2,1,0.9),(a_3,0.6,0.4)\}.$$

Table- I: Calculation on Union of WFS

$\mu_{W(\underset{\sim}{P})}$	$\mu_{W(\underset{\sim}{Q})}$	$\left(\mu_{W(\underset{\sim}{P})}\vee\mu_{W(\underset{\sim}{Q})}\right)$	$\eta_{a\in W(\underset{\sim}{P})}$	$\eta_{a\in W(\underset{\sim}{Q})}$	$\left(\eta_{a\in W(\underset{\sim}{P})}\wedge\eta_{a\in W(\underset{\sim}{Q})}\right)$
0.8	0.8	0.8	1	0.7	0.7
0.8	1	1	1	0.9	0.9
0.8	0.6	0.8	1	0.3	0.3
1	0.8	1	0.	0.7	0.5
1	1	1	0.5	0.9	0.5
1	0.6	1	0.5	0.3	0.3
0.6	0.8	0.8	0.4	0.7	0.4
0.6	1	1	0.4	0.9	0.4
0.6	0.6	0.6	0.4	0.3	0.3

C. Intersection on Weighted Fuzzy Subsets

Let us take two weighted fuzzy sets $W\left(\underset{\sim}{P}\right)$ and $W\left(\underset{\sim}{Q}\right)$, then the intersection of weighted fuzzy subset as follows

$$W\left(\underset{\sim}{P}\cap\underset{\sim}{Q}\right)=\left\{\begin{array}{l}<a,\left(\mu_{W(\underset{\sim}{P})}\wedge\mu_{W(\underset{\sim}{Q})}\right),\\ \sup\limits_{\left(\mu_{W(\underset{\sim}{P})}\wedge\mu_{W(\underset{\sim}{Q})}\right)}\left\{\eta_{a\in W(\underset{\sim}{P})}\wedge\eta_{a\in W(\underset{\sim}{Q})}\right\}>\end{array}\right\}$$

D. Example – Intersection on Weighted Fuzzy Subsets

Let $W\left(\underset{\sim}{P}\right)=\{(a_1,0.8,1),(a_2,1,0.5),(a_3,0.6,0.4)\}$ and $W\left(\underset{\sim}{Q}\right)=\{(a_1,0.8,0.7),(a_2,1,0.9),(a_3,0.6,0.3)\}$ are two weighted fuzzy sets, then the intersection of two weighted fuzzy subset as follows

We obtain $\left(\mu_{W(\underset{\sim}{A})}\wedge\mu_{W(\underset{\sim}{B})}\right)$ as $0.8,1,0.6$.

Let us take $\left(\mu_{W(\underset{\sim}{A})}\wedge\mu_{W(\underset{\sim}{B})}\right)=0.8$, then

$$\mu_{W(\underset{\sim}{P}\cap\underset{\sim}{Q})}(a)=\left\{\begin{array}{l}<a,\left(\mu_{W(\underset{\sim}{P})}\wedge\mu_{W(\underset{\sim}{Q})}\right),\\ \sup\limits_{\left(\mu_{W(\underset{\sim}{P})}\wedge\mu_{W(\underset{\sim}{Q})}\right)}\left\{\eta_{a\in W(\underset{\sim}{P})}\wedge\eta_{a\in W(\underset{\sim}{Q})}\right\}>\end{array}\right\}$$

$$\mu_{W(\underset{\sim}{P}\cap\underset{\sim}{Q})}(a_1)=\left\{<a_1,0.8,\sup_{0.8}[0.7,0.9,0.5]>\right\}$$
$$=(a_1,0.8,0.9)$$

Let us take $\left(\mu_{W(\underset{\sim}{A})}\wedge\mu_{W(\underset{\sim}{B})}\right)=1$, then

$$\mu_{W(\underset{\sim}{P}\cap\underset{\sim}{Q})}(a_2)=\left\{<a_2,1,\sup_{1}[0.5]>\right\}$$
$$=(a_2,1,0.5)$$

Let us take $\left(\mu_{W(\underset{\sim}{A})}\wedge\mu_{W(\underset{\sim}{B})}\right)=0.6$, then

$$\mu_{W(\underset{\sim}{P}\cap\underset{\sim}{Q})}(a_3)=\left\{<a_3,0.6,\sup_{0.6}[0.3,0.3,0.4,0.4,0.3]>\right\}$$
$$=(a_3,0.6,0.4)$$
$$W\left(\underset{\sim}{P}\cap\underset{\sim}{Q}\right)=\{(a_1,0.8,0.9),(a_2,1,0.5),(a_3,0.6,0.4)\}$$

Table- II: Calculation on Intersection of WFS

$\mu_{W(\underset{\sim}{P})}$	$\mu_{W(\underset{\sim}{Q})}$	$\left(\mu_{W(\underset{\sim}{A})}\wedge\mu_{W(\underset{\sim}{B})}\right)$	$\eta_{a\in W(\underset{\sim}{P})}$	$\eta_{a\in W(\underset{\sim}{Q})}$	$\left(\eta_{a\in W(\underset{\sim}{P})}\wedge\eta_{a\in W(\underset{\sim}{Q})}\right)$
0.8	0.8	0.8	1	0.7	0.7
0.8	1	0.8	1	0.9	0.9
0.8	0.6	0.6	1	0.3	0.3
1	0.8	0.8	0.5	0.7	0.5
1	1	1	0.5	0.9	0.5
1	0.6	0.6	0.5	0.3	0.3

0.6	0.8	0.6	0.4	0.7	0.4
0.6	1	0.6	0.4	0.9	0.4
0.6	0.6	0.6	0.4	0.3	0.3

E. Compliment on Weighted Fuzzy Subsets

Let $W(\underset{\sim}{P})$ be a weighted fuzzy set, then its complement of weighted fuzzy set is defined as follows

$$W(\underset{\sim}{P}^{C}) = \left\{ < a, 1-\mu_{W(\underset{\sim}{P})}(a), 1-\eta_{a\in W(\underset{\sim}{P})}(\underset{\sim}{P}) > / a \in W(\underset{\sim}{P}) \right\}$$

where,

$1-\mu_{W(\underset{\sim}{P})} : X \to [0,1]$ denotes the non-membership degree,

$1-\eta_{a\in W(\underset{\sim}{P})} : W(\underset{\sim}{P}) \to [0,1]$ denotes the non - membership degree of the set corresponding to each element in the set.

F. Example - Compliment on Weighted Fuzzy Subsets

Let $W(\underset{\sim}{P}) = \left\{ \begin{matrix} (a_1, 0.8, 0.7), (a_2, 0.6, 0.7), (a_3, 0.9, 0.8), \\ (a_4, 0.6, 0.6), (a_5, 0.5, 0.6) \end{matrix} \right\}$

is a weighted fuzzy set then the compliment of a set defined as

$$W(\underset{\sim}{P}^{C}) = \left\{ \begin{matrix} (a_1, 0.2, 0.3), (a_2, 0.4, 0.3), (a_3, 0.1, 0.2), \\ (a_4, 0.4, 0.4), (a_5, 0.5, 0.4) \end{matrix} \right\}.$$

V. INTERPRETATION OF THE RESULT

The concept of weighted fuzzy sets has been extended from Zadeh's fuzzy set. In general, existing fuzzy sets accompany the degree of membership and non-membership of elements. The proposed weighted fuzzy sets discuss degree of sets. In other words, the degree of impact of the set corresponding to each element presence in the set. The advantage of weighted fuzzy sets is to identify whether the existing fuzzy sets is strong or weak fuzzy sets. The proposed concept also verified with union, intersection and compliment operations by Zadeh's extension principle.

VI. CONCLUSION

In this article, the weighted fuzzy subset has been introduced and verified with union, intersection and compliment operations with numerical examples. Study on the weighted fuzzy subset will explain more about its characteristics relating to weather the given fuzzy subset is strong or weak whereas the existing fuzzy subset explains only the membership or non-membership of an elements.

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